STRUCTURAL OPTIMIZATION:
FROM CONTINUUM AND GROUND STRUCTURES TO ADDITIVE MANUFACTURING

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LCCV – UFAL
2014

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UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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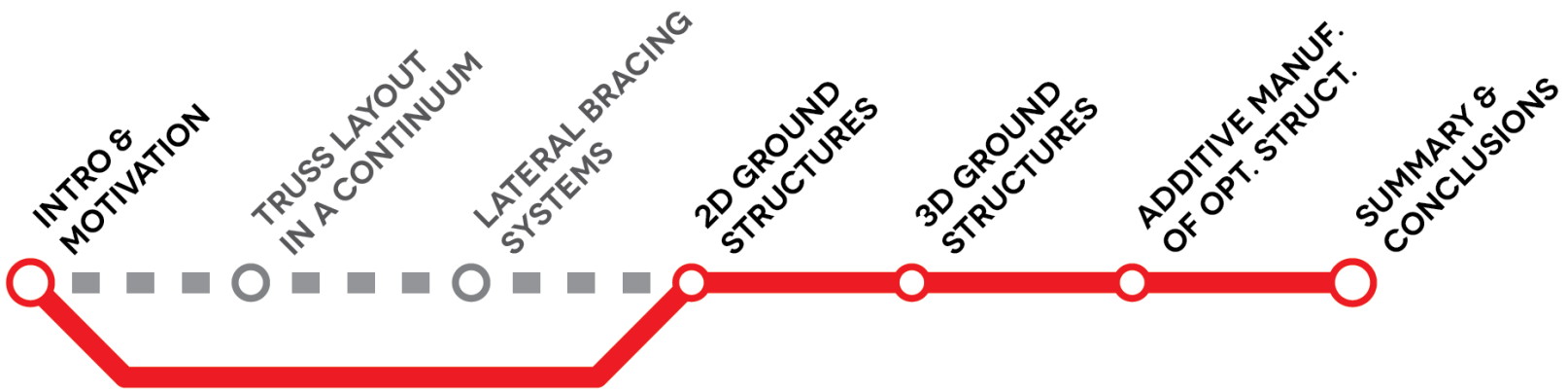
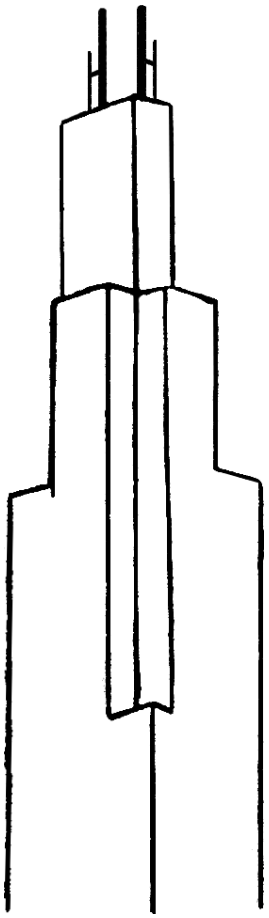

INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES
ADDITIVE MANUF. OF OPT. STRUCT.
SUMMARY & CONCLUSIONS

ROADMAP

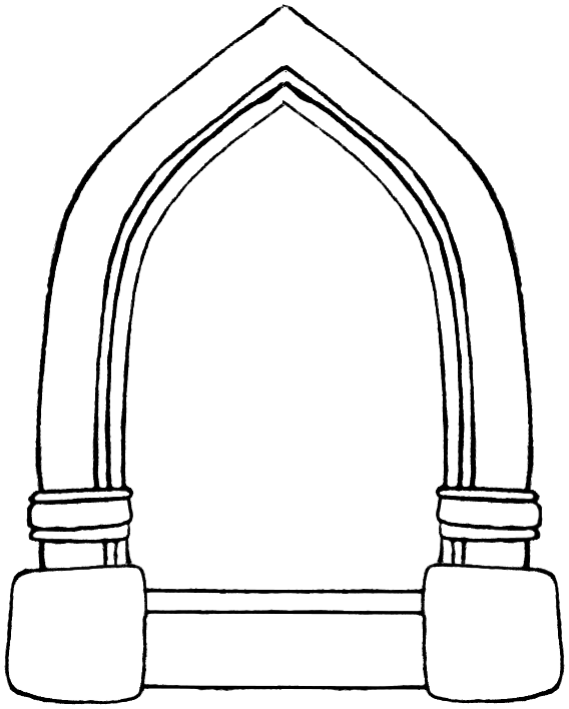
1) INTRO & MOTIVATION

- WHY USE STRUCTURAL OPTIMIZATION?

LIMITED RESOURCES



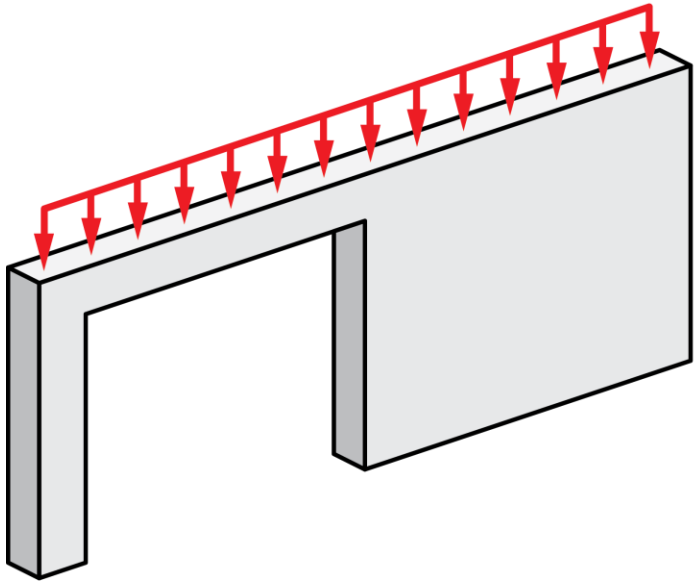
EXTREME STRUCTURES



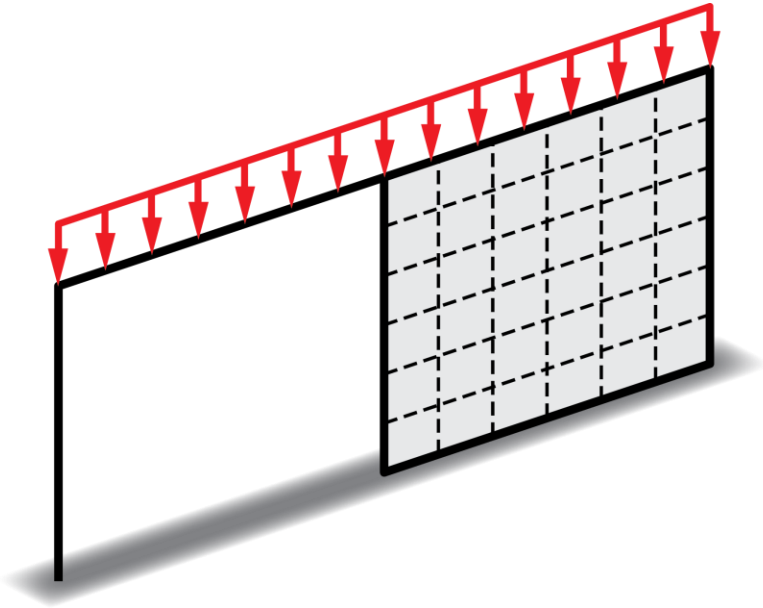
FUNCTIONAL

1) INTRO & MOTIVATION

- WHY DISCRETE—CONTINUUM?
  - LIMITED MODELING CAPABILITY
  - REASONABLE SIMPLIFICATIONS OF REALITY



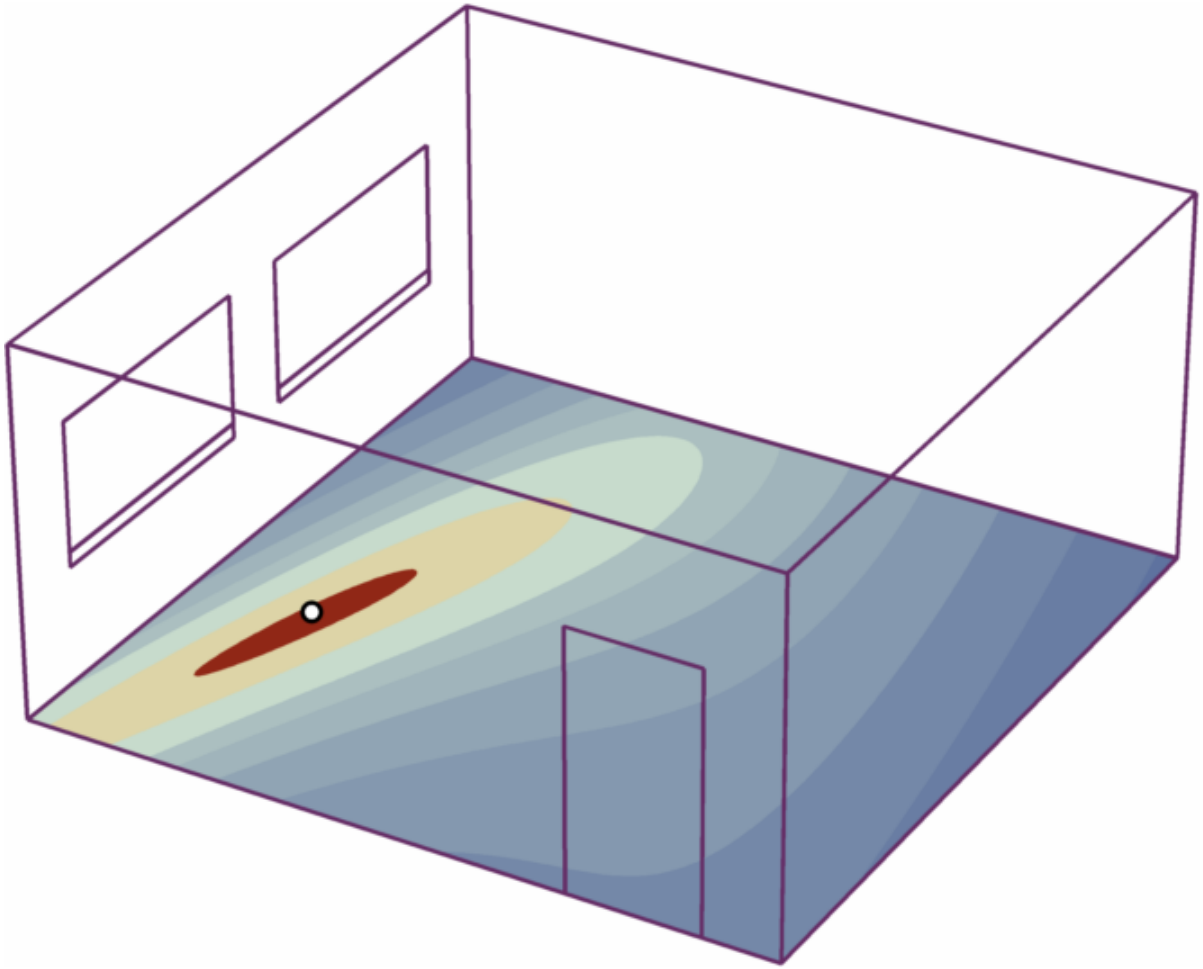
REAL FRAME



SIMPLIFIED FRAME MODEL

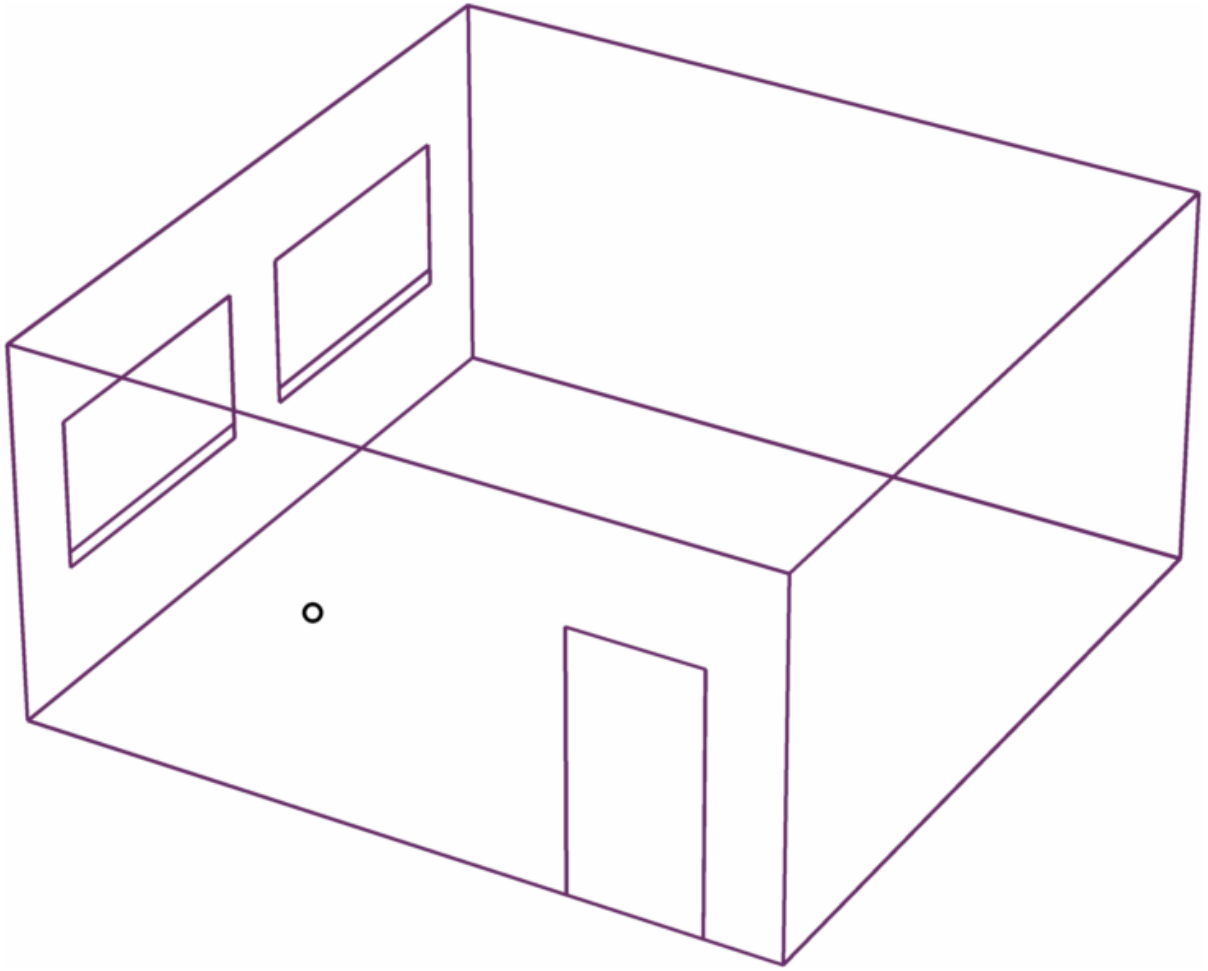
1) INTRO & MOTIVATION

- GRADIENT VS. NON-GRADIENT



1) INTRO & MOTIVATION

- GRADIENT VS. NON-GRADIENT



1) INTRO & MOTIVATION

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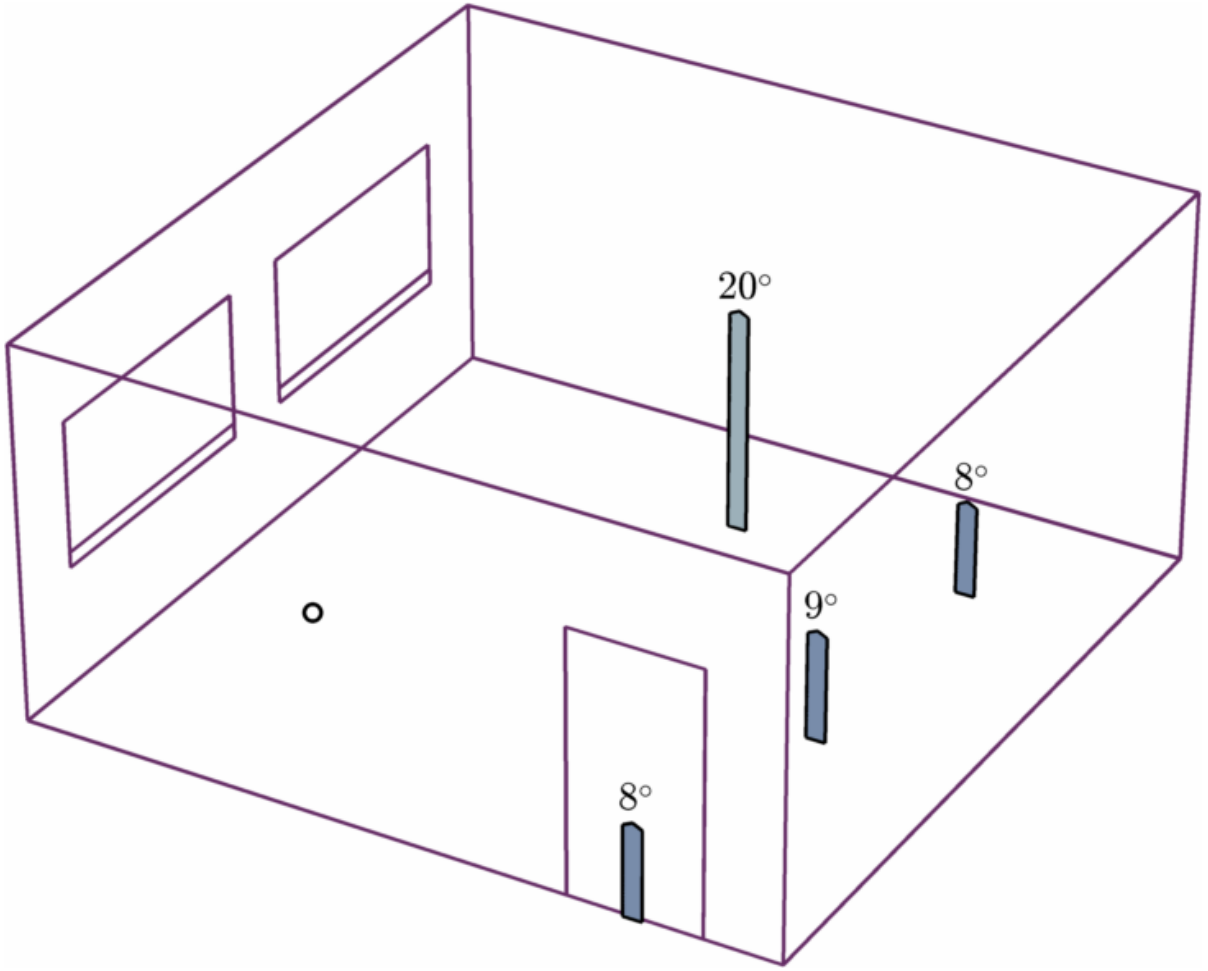
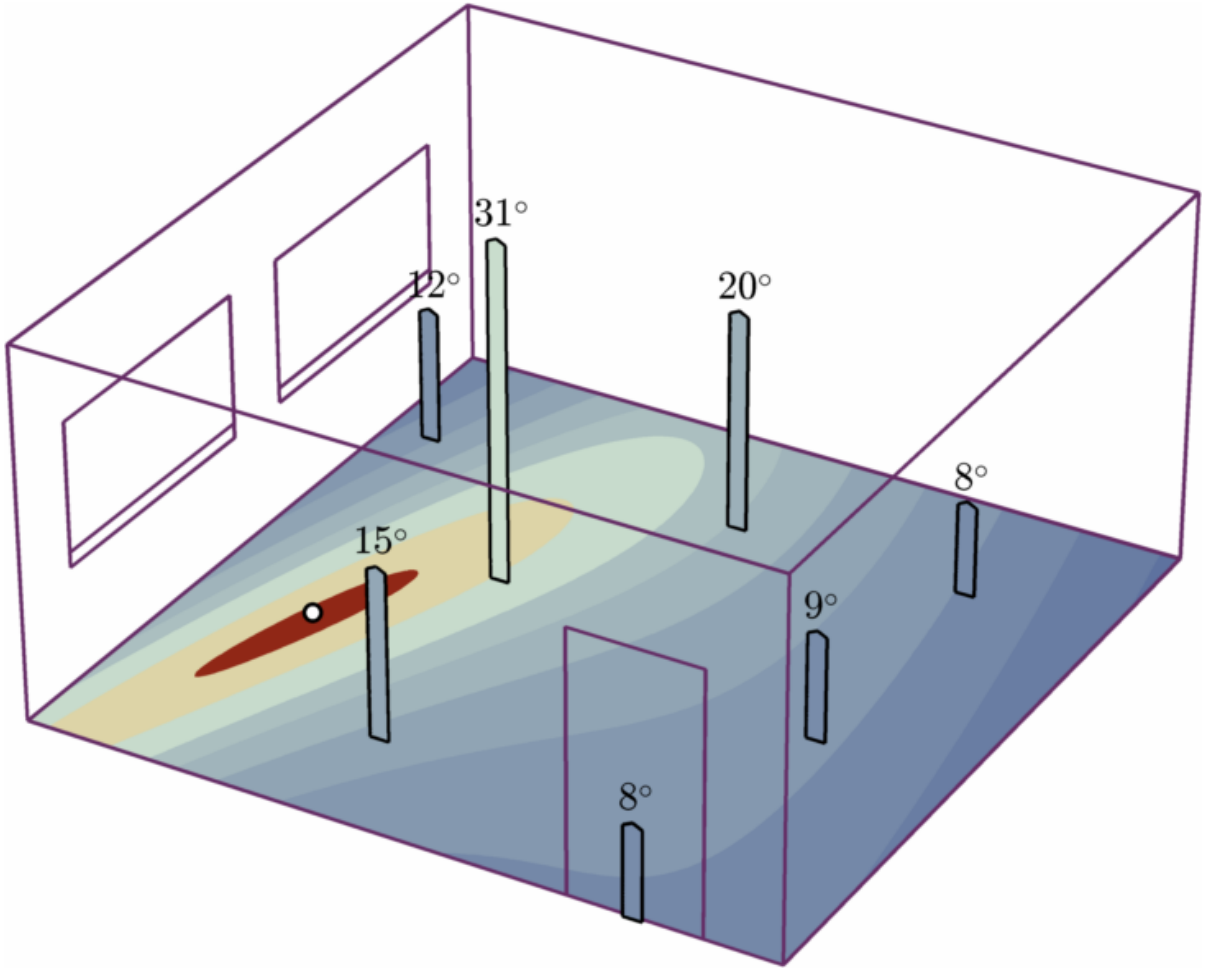


1) INTRO & MOTIVATION

- GRADIENT VS. NON-GRADIENT



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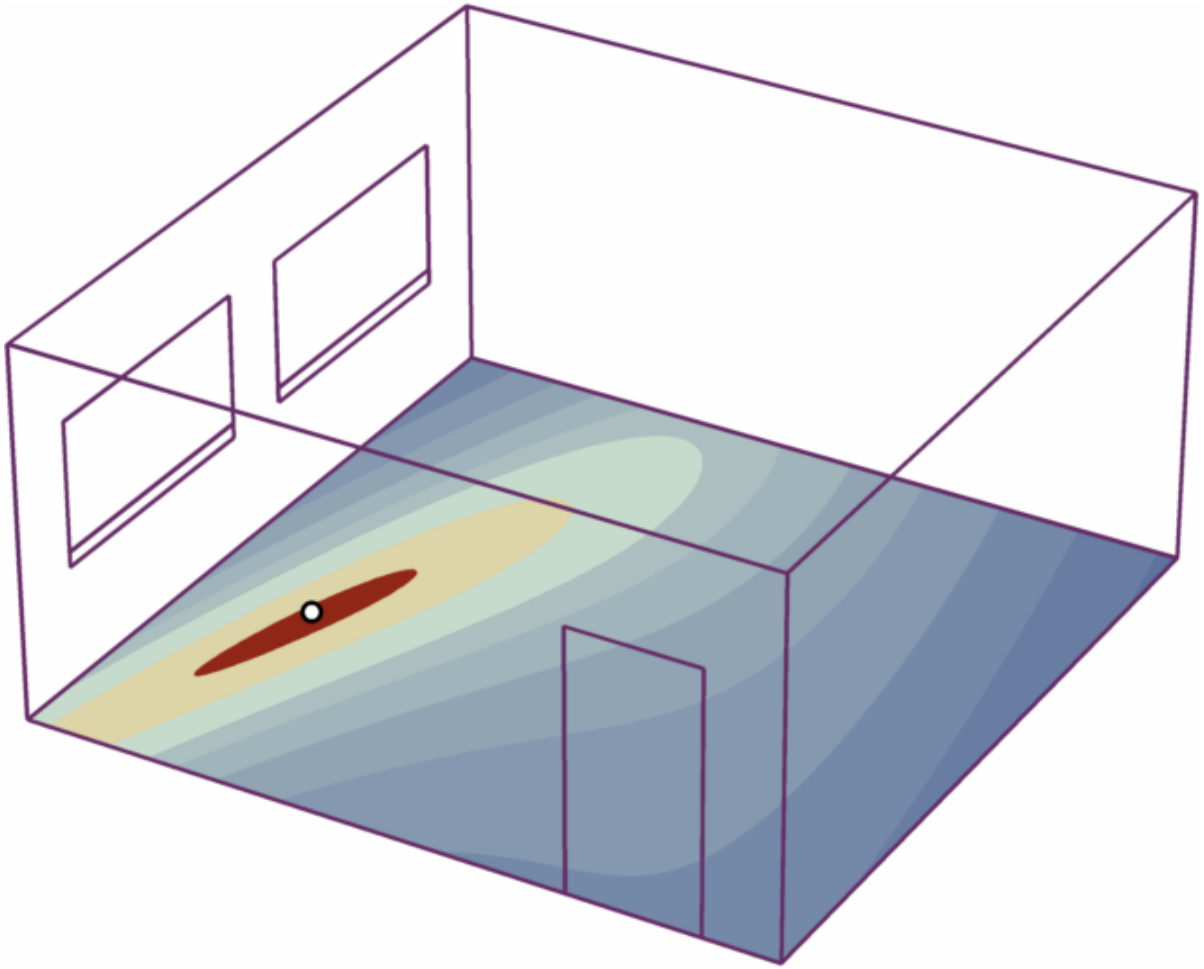
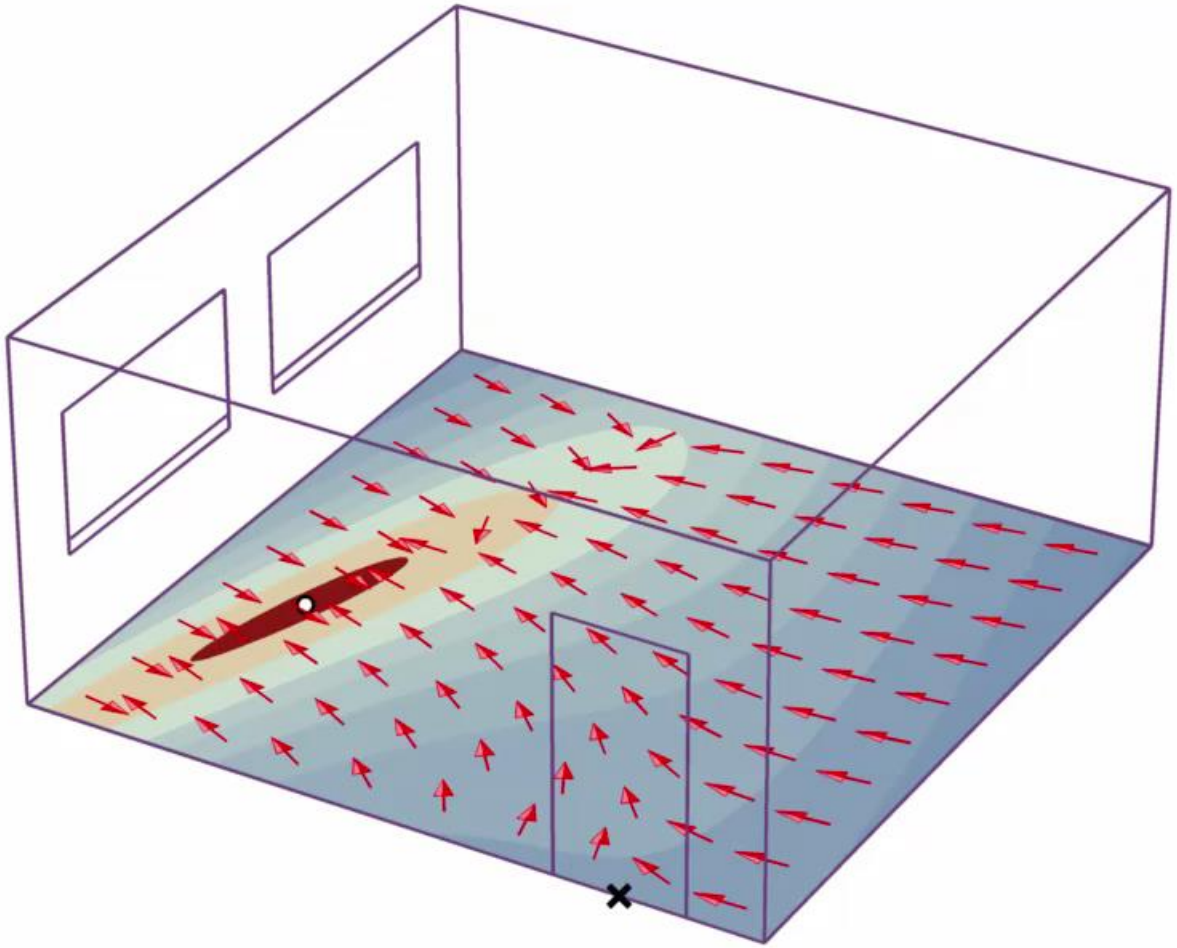
- GRADIENT VS. NON-GRADIENT



1) INTRO & MOTIVATION

- GRADIENT VS. NON-GRADIENT



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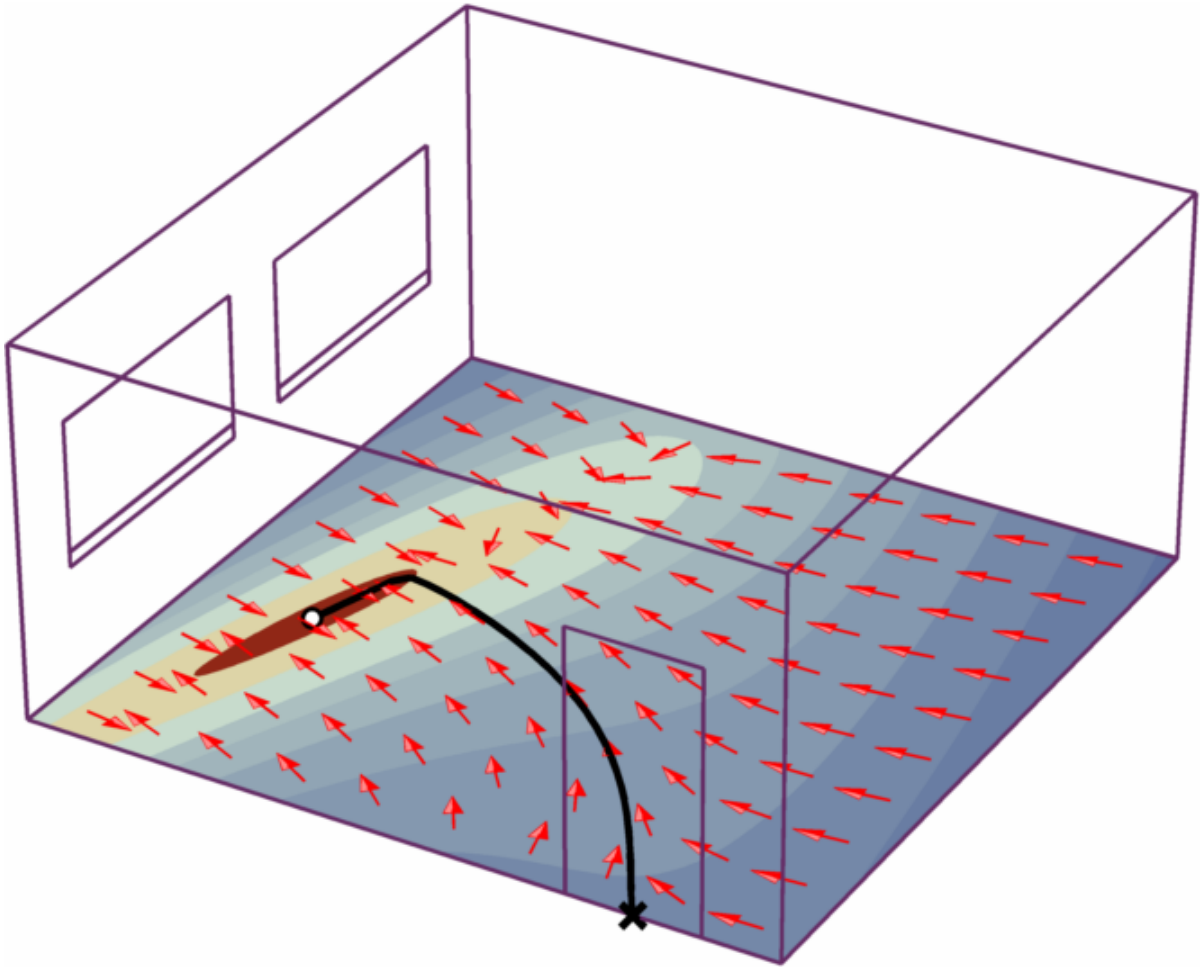
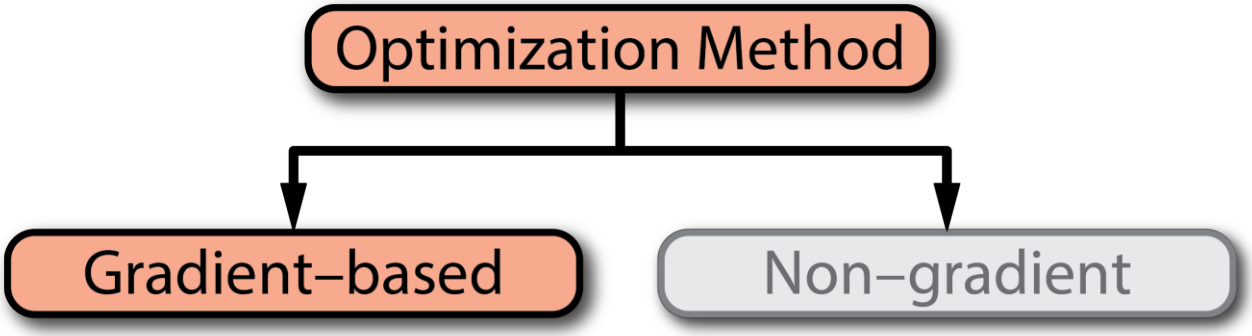
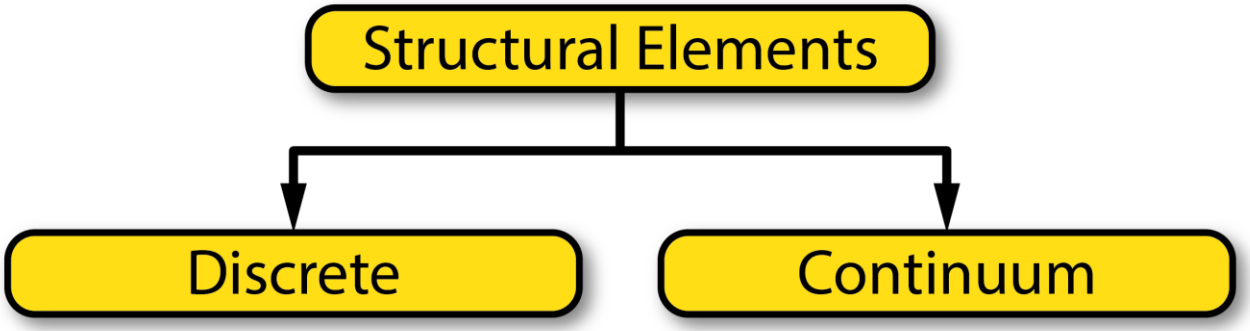
- GRADIENT VS. NON-GRADIENT



1) INTRO & MOTIVATION

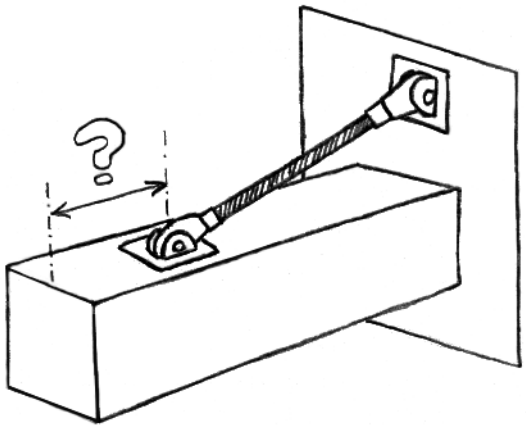
- STRUCTURAL OPTIMIZATION METHODS



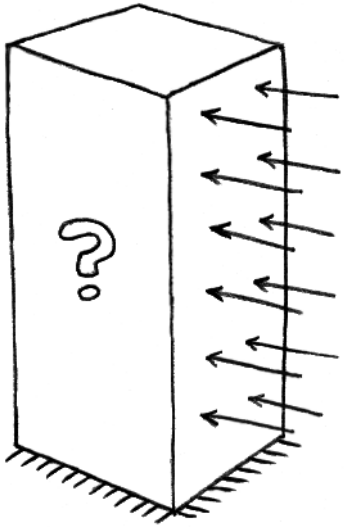


1) INTRO & MOTIVATION

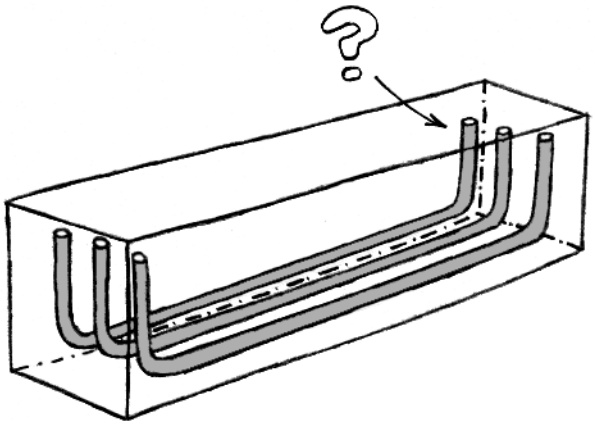
- POTENTIAL APPLICATIONS



ANCHOR POINT LOCATION

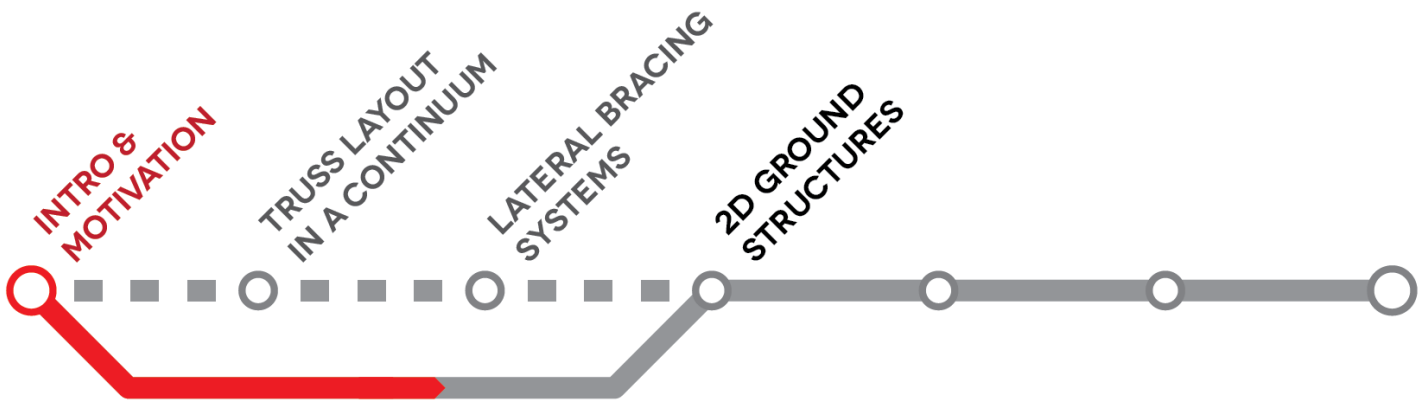


LATERAL BRACING SYSTEM



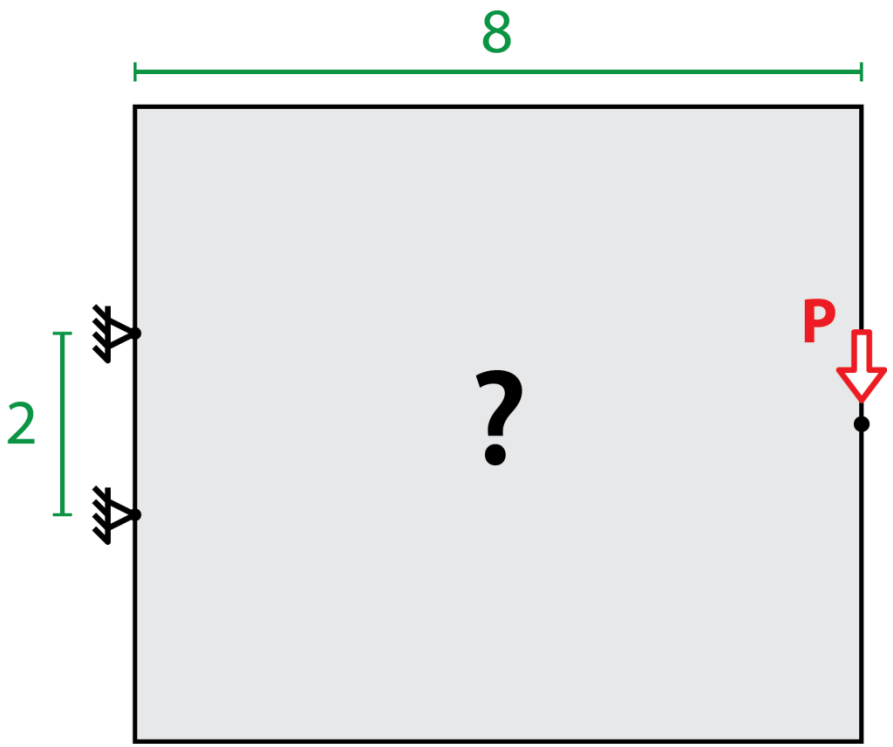
REINFORCEMENT LAYOUT

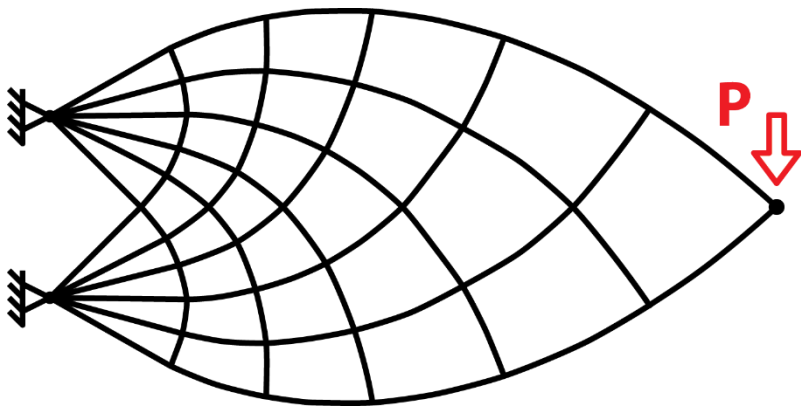
ROADMAP


INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES

4) GROUND STRUCTURES IN 2D

- TRUSS LAYOUT OPTIMIZATION IS HIGHLY NONLINEAR: **GEOMETRY** AND **SIZING**





MAZUREK A (2012) – "GEOMETRICAL ASPECTS OF OPTIMUM TRUSS LIKE STRUCTURES FOR THREE–FORCE PROBLEM", STRUCT AND MULTIDISCIPLINARY OPT 45(1), PP 21–32

4) GROUND STRUCTURES IN 2D

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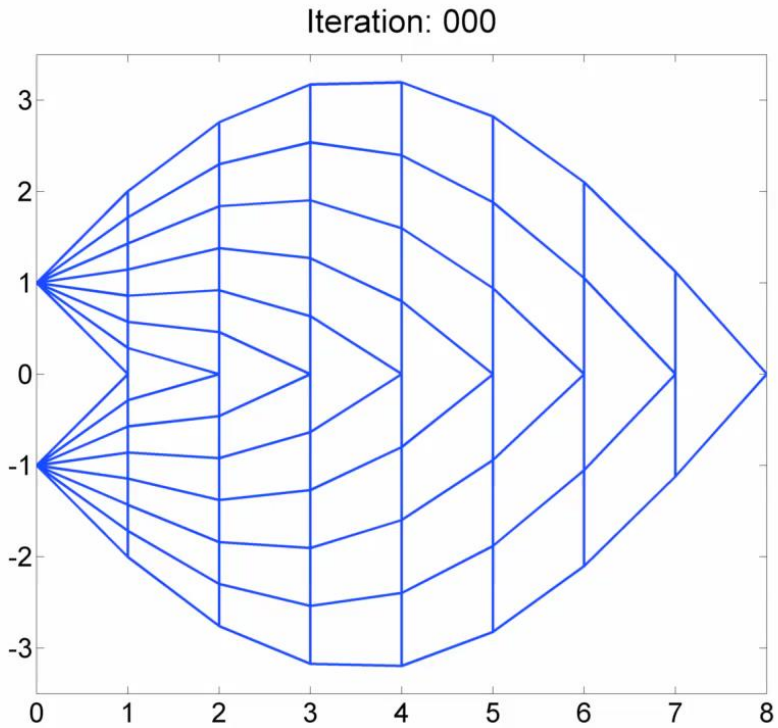

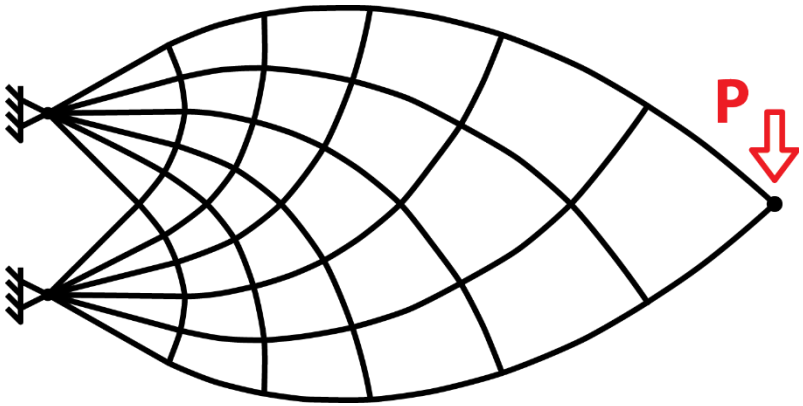

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4) GROUND STRUCTURES IN 2D

- MAIN IDEA:
  CONVERT A GEOMETRY AND SIZE OPTIMIZATION TO A SIZING-ONLY PROBLEM

- PLASTIC FORMULATION:

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \\
& a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{aligned}
$$

DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES", JOURNAL DE MECANIQUE 3(1), PP 25–52

4) GROUND STRUCTURES IN 2D

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \\
& a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{aligned}
$$

VANISHING CONSTRAINT

- MULTIPLYING THE INEQUALITY BY CROSS-SECTIONAL AREA

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C a_i \leq n_i \leq \sigma_T a_i
\end{aligned}
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4) GROUND STRUCTURES IN 2D

$$\begin{aligned} \min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\ \text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\ & -\sigma_C a_i \leq n_i \leq \sigma_T a_i \end{aligned}$$

- INTRODUCING SLACK VARIABLES

$$\left.\begin{aligned} n_i + 2\frac{\sigma_0}{\sigma_C} s_i^- &= \sigma_T a_i \\ -n_i + 2\frac{\sigma_0}{\sigma_T} s_i^+ &= \sigma_C a_i \\ \sigma_0 &= (\sigma_T + \sigma_C)/2 \end{aligned}\right\} \quad \begin{aligned} a_i &= \frac{s_i^+}{\sigma_T} + \frac{s_i^-}{\sigma_C} \\ n_i &= s_i^+ - s_i^- \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{s}^+, \mathbf{s}^-} \quad & V = \mathbf{l}^T \left(\frac{\mathbf{s}^+}{\sigma_T} + \frac{\mathbf{s}^-}{\sigma_C} \right) \\ \text{s.t.} \quad & \mathbf{B}^{\mathbf{T}} \left(\mathbf{s}^+ - \mathbf{s}^- \right) = \mathbf{f} \\ & s_i^+, s_i^- \geq 0 \end{aligned}$$

4) GROUND STRUCTURES IN 2D

- REMARKS
  - DESIGN VARIABLES DOUBLED: S^+ AND S^-
  - NO MORE VANISHING CONSTRAINT
  - DIFFERENT LIMITS IN TENSION AND COMPRESSION
  - LINEAR PROGRAM

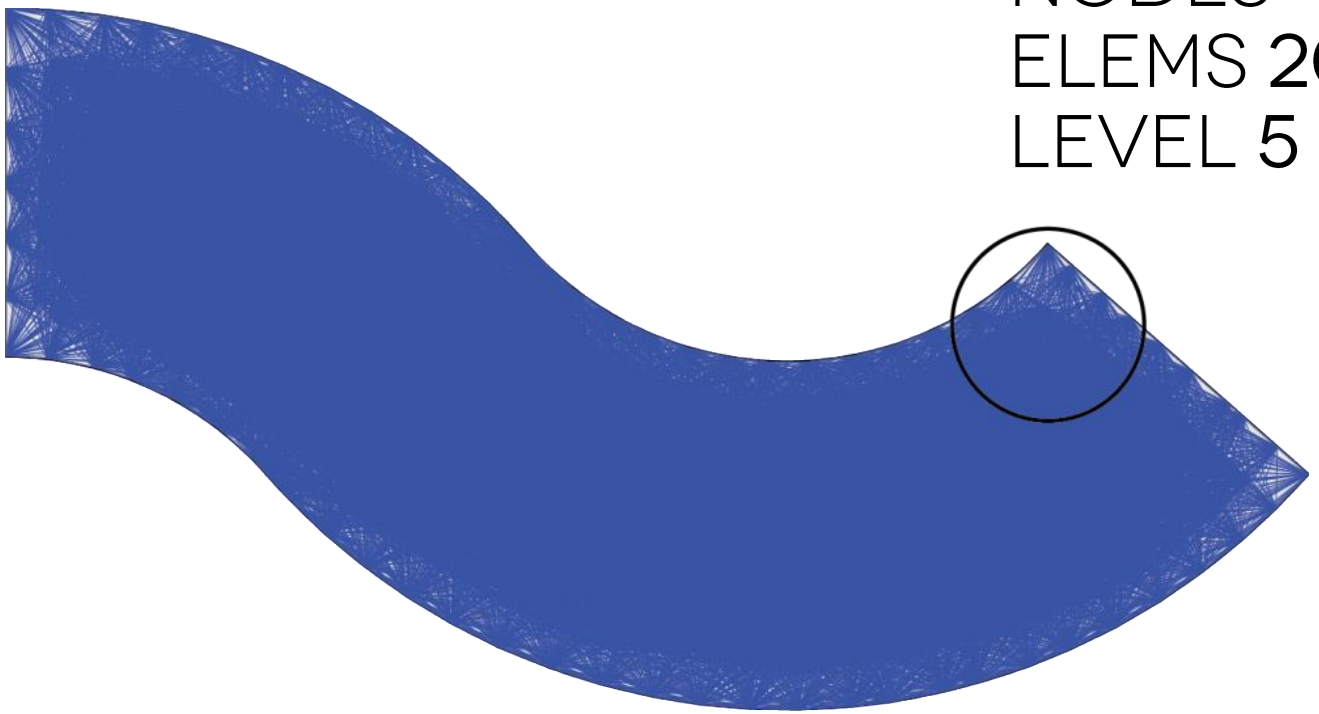
KARMARKAR N (1984) "A NEW POLYNOMIAL–TIME ALGORITHM FOR LINEAR PROGRAMMING." COMBINATORICA, 4(4):373–395.

WRIGHT MH (2004) "THE INTERIOR–POINT REVOLUTION IN OPTIMIZATION: HISTORY, RECENT DEVELOPMENTS, AND LASTING CONSEQUENCES." BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, 42(1):39–56.

$$\begin{aligned} \min_{\mathbf{s}^+,\mathbf{s}^-} \quad & V = \mathbf{l}^T \left(\frac{\mathbf{s}^+}{\sigma_T} + \frac{\mathbf{s}^-}{\sigma_C} \right) \\ \text{s.t.} \quad & \mathbf{B^T} \left(\mathbf{s}^+ - \mathbf{s}^- \right) = \mathbf{f} \\ & s_i^+, s_i^- \geq 0 \end{aligned}$$

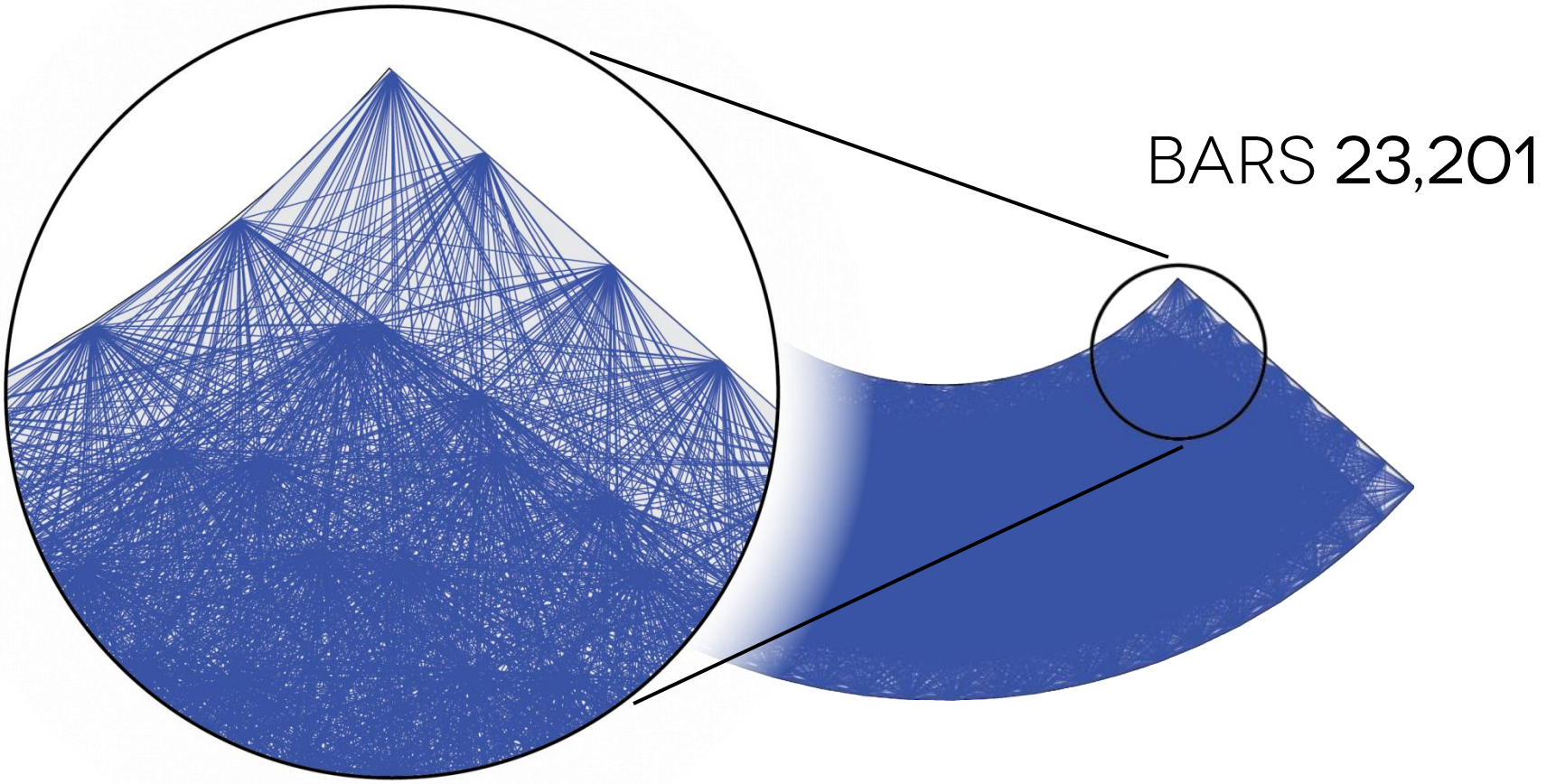
4) GROUND STRUCTURES IN 2D

- SIZING OF A HIGHLY **INTERCONNECTED** AND **REDUNDANT** TRUSS



4) GROUND STRUCTURES IN 2D

- SIZING OF A HIGHLY **INTERCONNECTED** AND **REDUNDANT** TRUSS



4) GROUND STRUCTURES IN 2D

- UNIQUE SOLUTION — NO COLLINEAR BARS

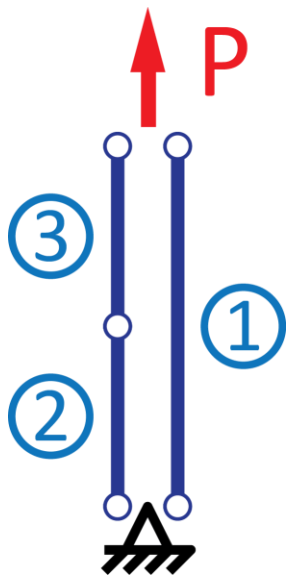
GIVEN $\sigma_T = 1$ AND $P = 1$

$a_1 = 1.0$

$a_1 = 1.0 \quad a_2 = a_3 = 0.0$

$a_1 = 0.0 \quad a_2 = a_3 = 1.0$

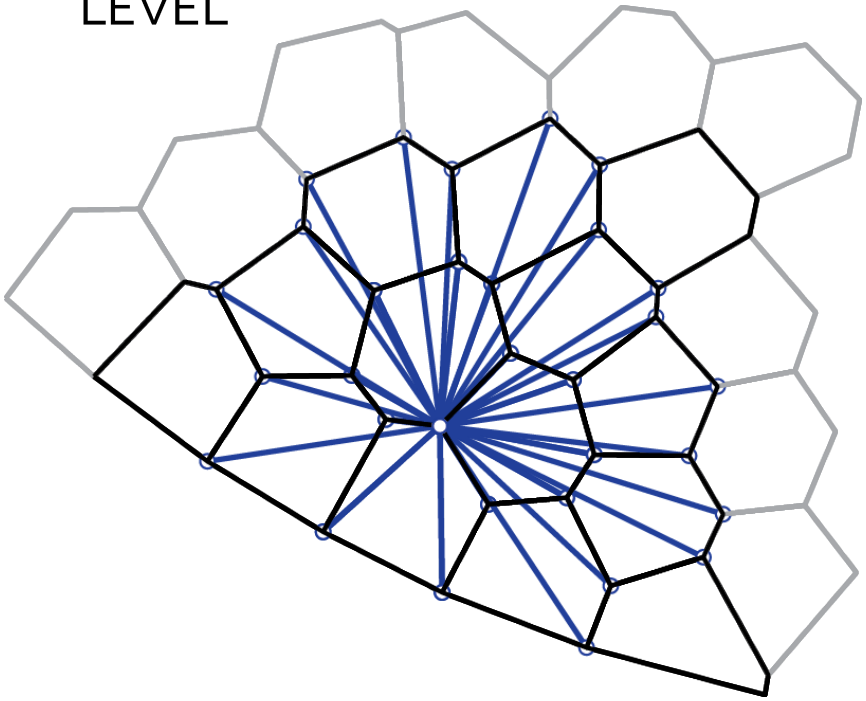
$a_1 = 0.5 \quad a_2 = a_3 = 0.5$

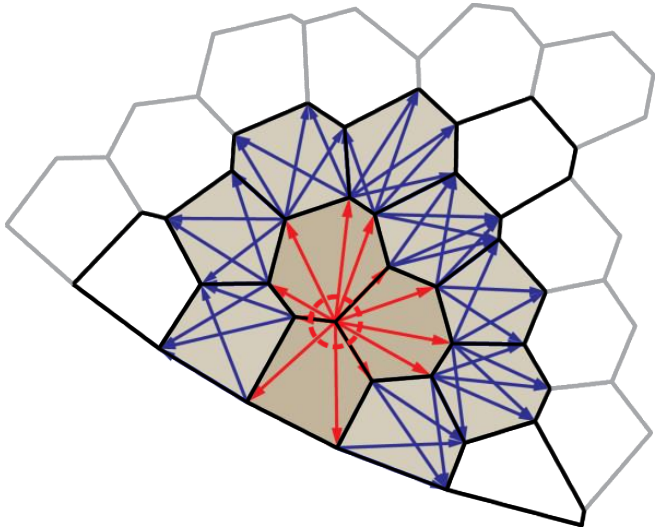


4) GROUND STRUCTURES IN 2D

- HIGHLY INTERCONNECTED TRUSS
  - CONNECTIVITY GENERATION

TRUSS MEMBERS
AT THIS CONNECTION
LEVEL

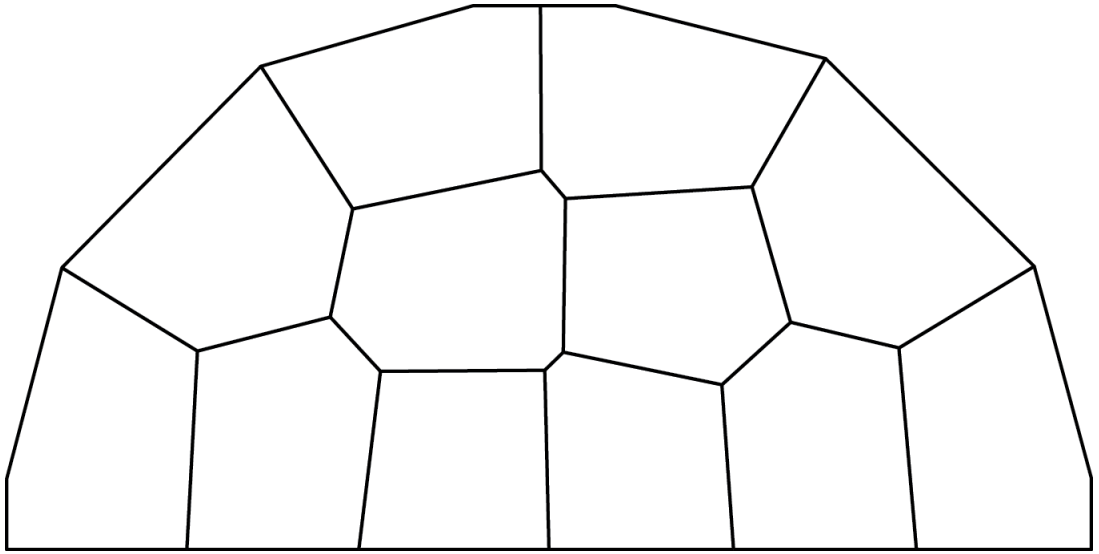




CONNECTION LEVEL: ZERO

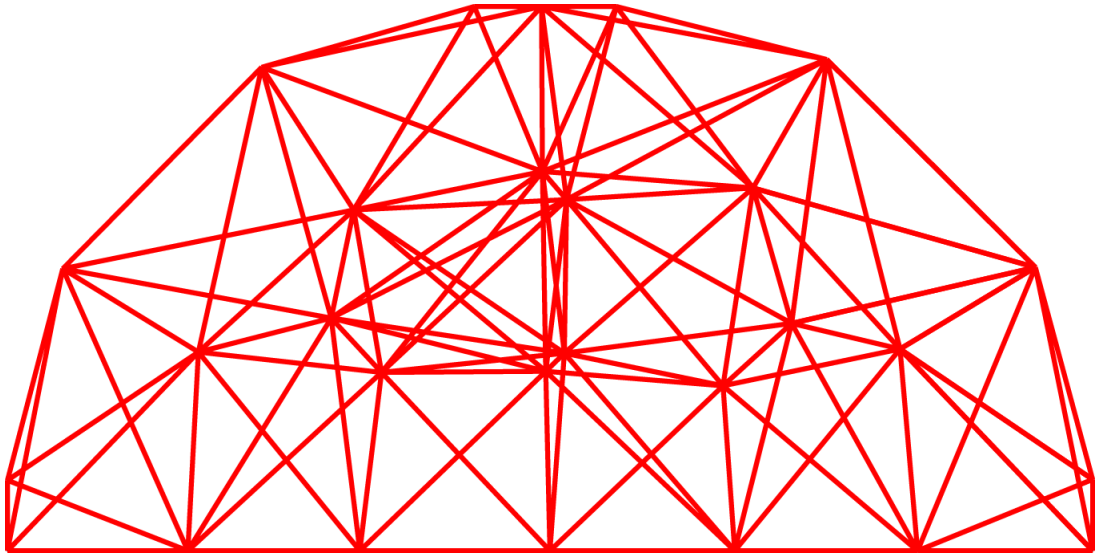
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - BASE MESH



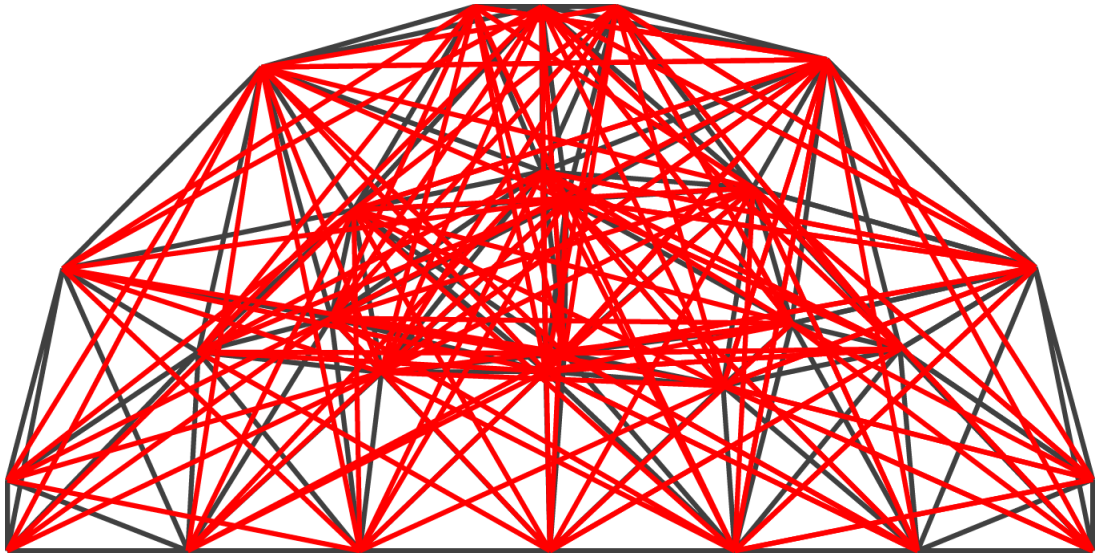
GROUND STRUCTURE METHOD

- EXAMPLE
  - CONNECTIVITY: LEVEL 1



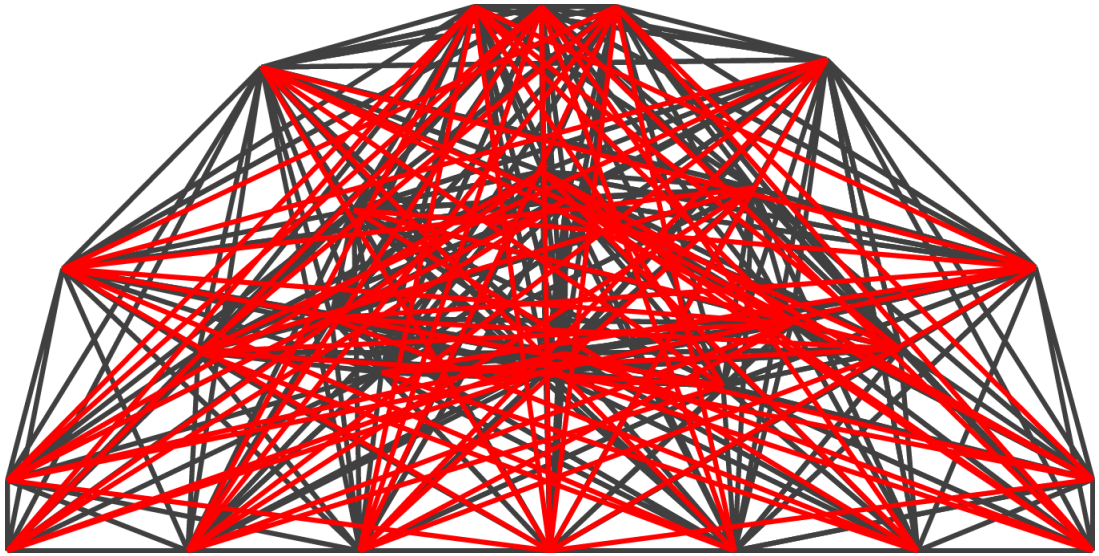
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 2



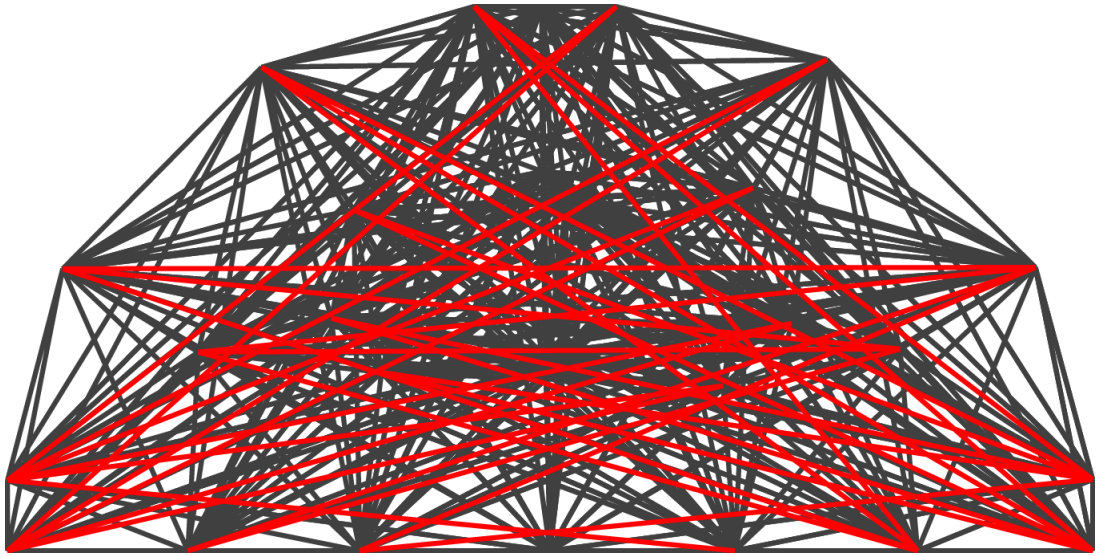
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 3



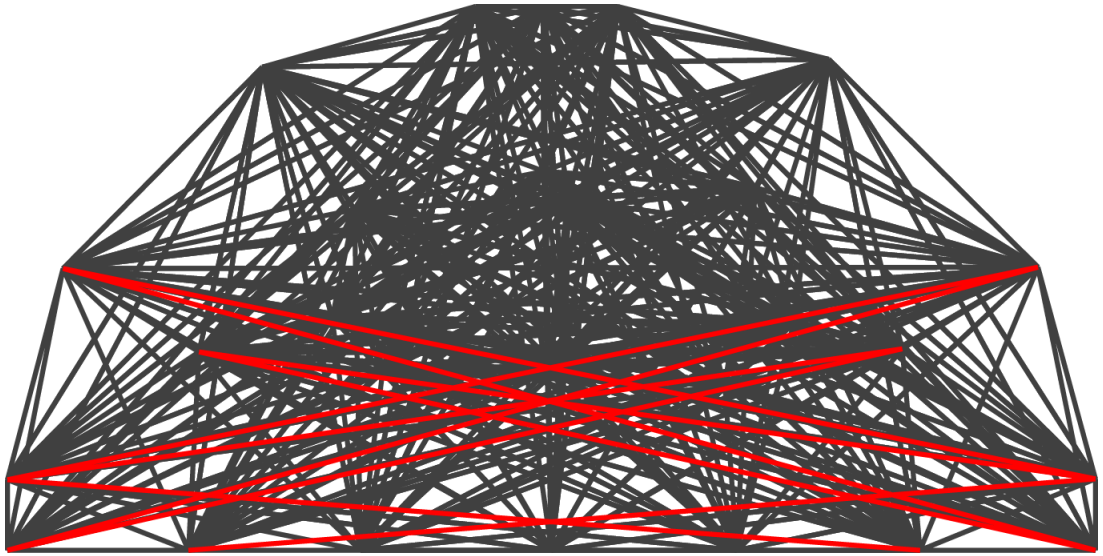
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 4



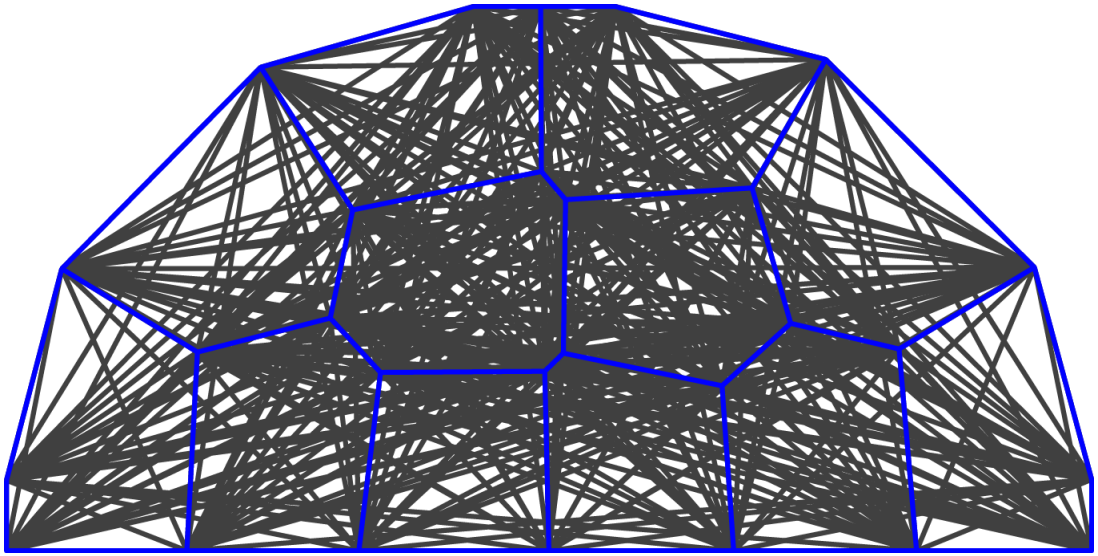
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 5



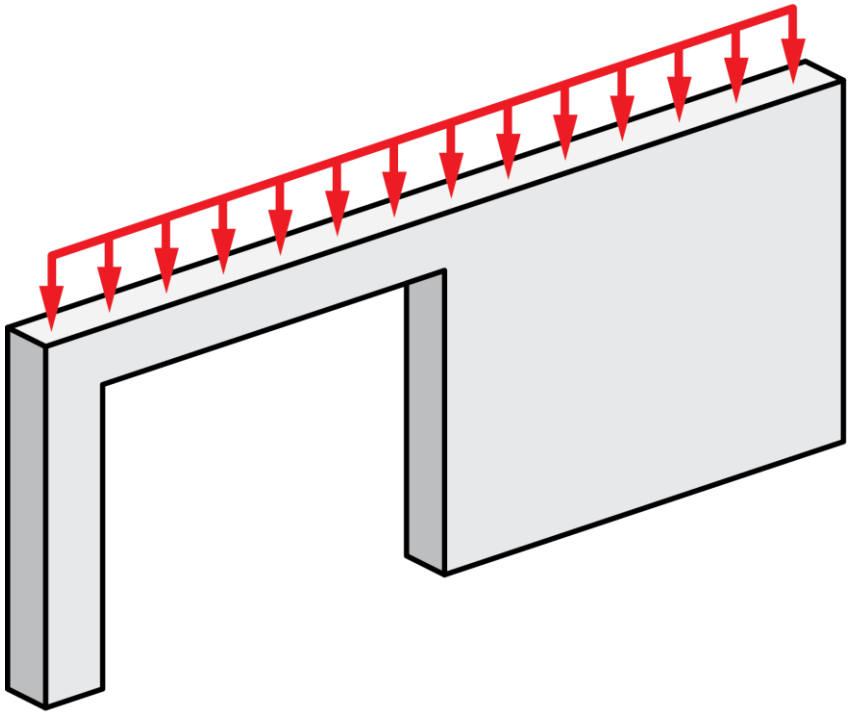
4) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 5



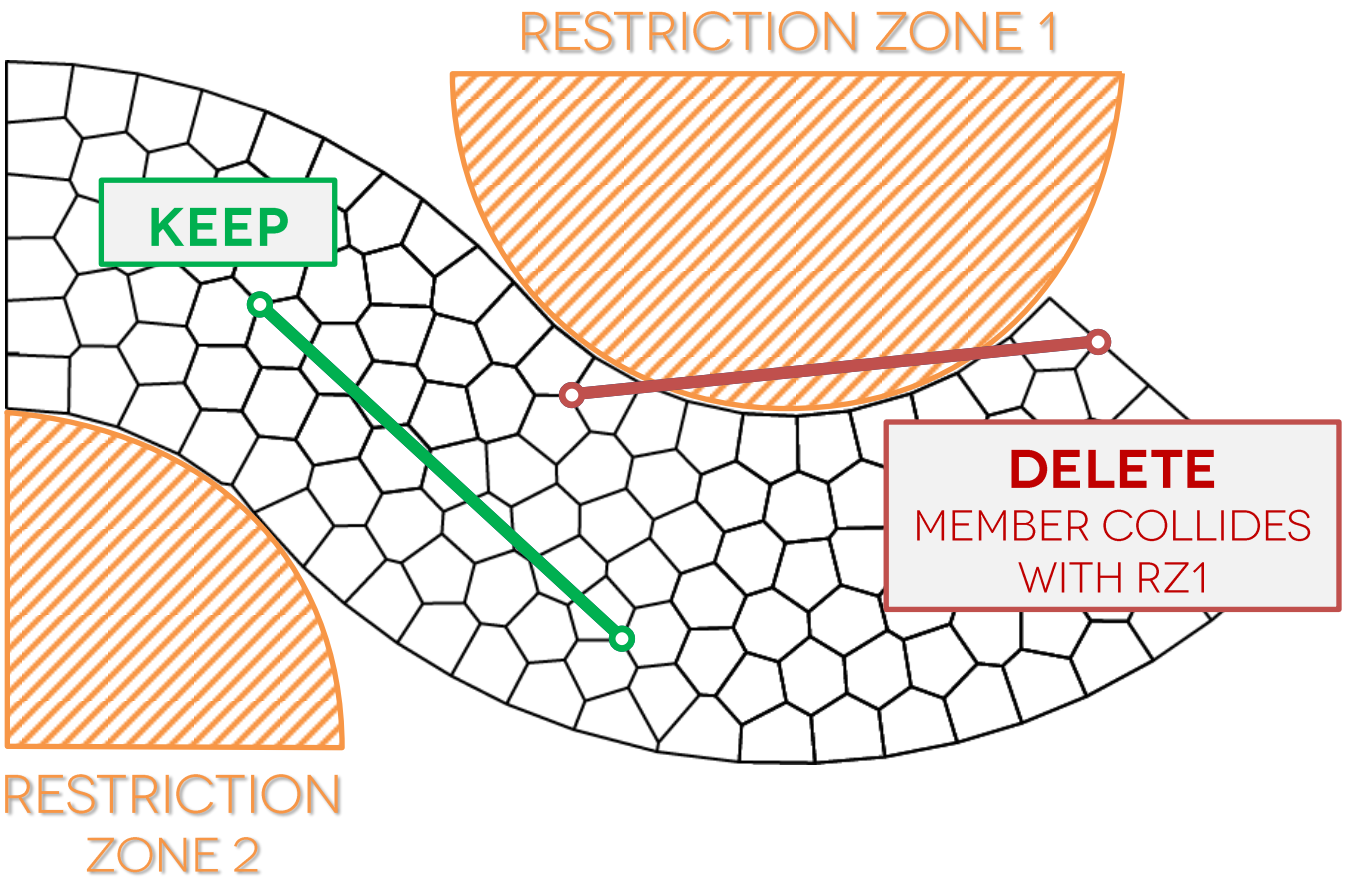
4) GROUND STRUCTURES IN 2D

- THERE CANNOT BE BARS **EVERYWHERE**
  - DEFINE ZONES WHERE NO BARS CAN BE



4) GROUND STRUCTURES IN 2D

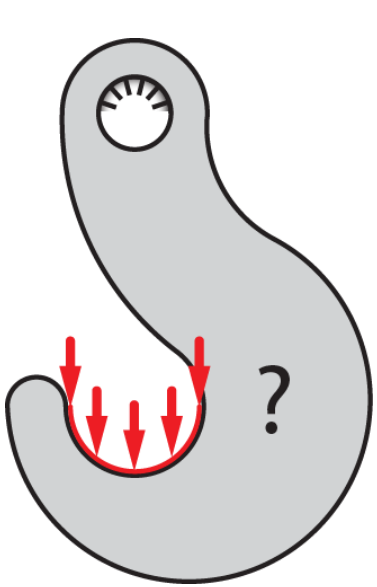
- INTERSECTION TESTS FROM VIDEO-GAME AND COMPUTER GRAPHICS INDUSTRY



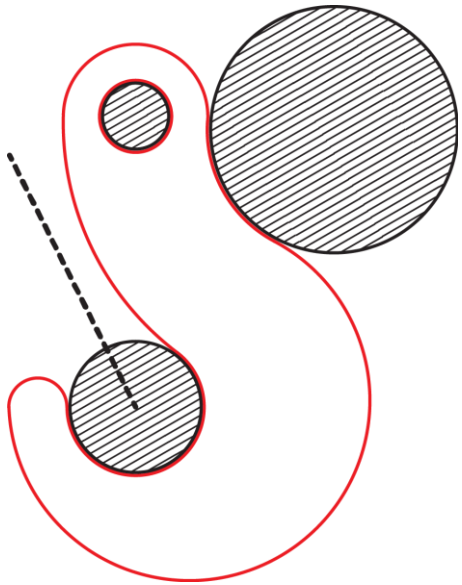
4) GROUND STRUCTURES IN 2D

- RESTRICTION ZONE PRIMITIVES
  - CIRCLE
  - SEGMENT (LINE)
  - RECTANGLE
  - POLYGON

- CAN BE COMBINED…

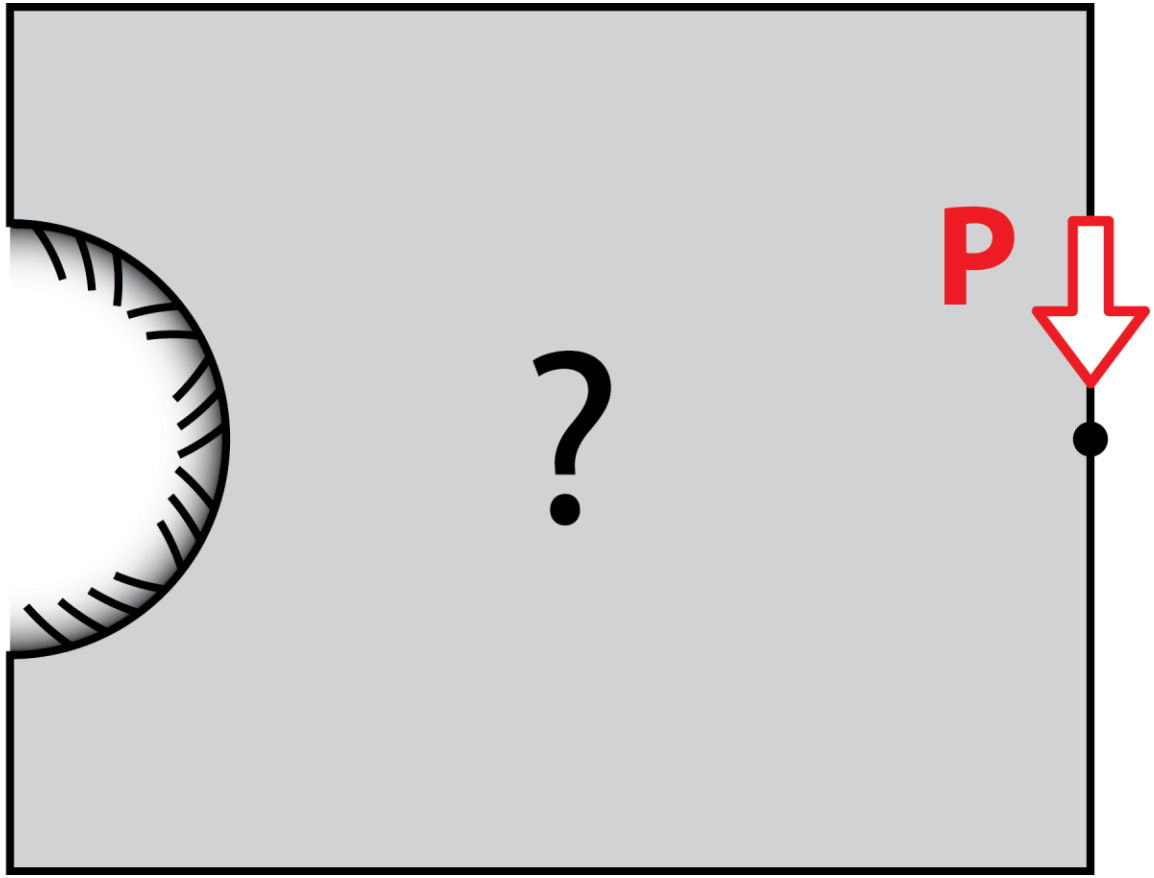


3 CIRCLES
+
1 SEGMENT



4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER



MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

4) GROUND STRUCTURES IN 2D

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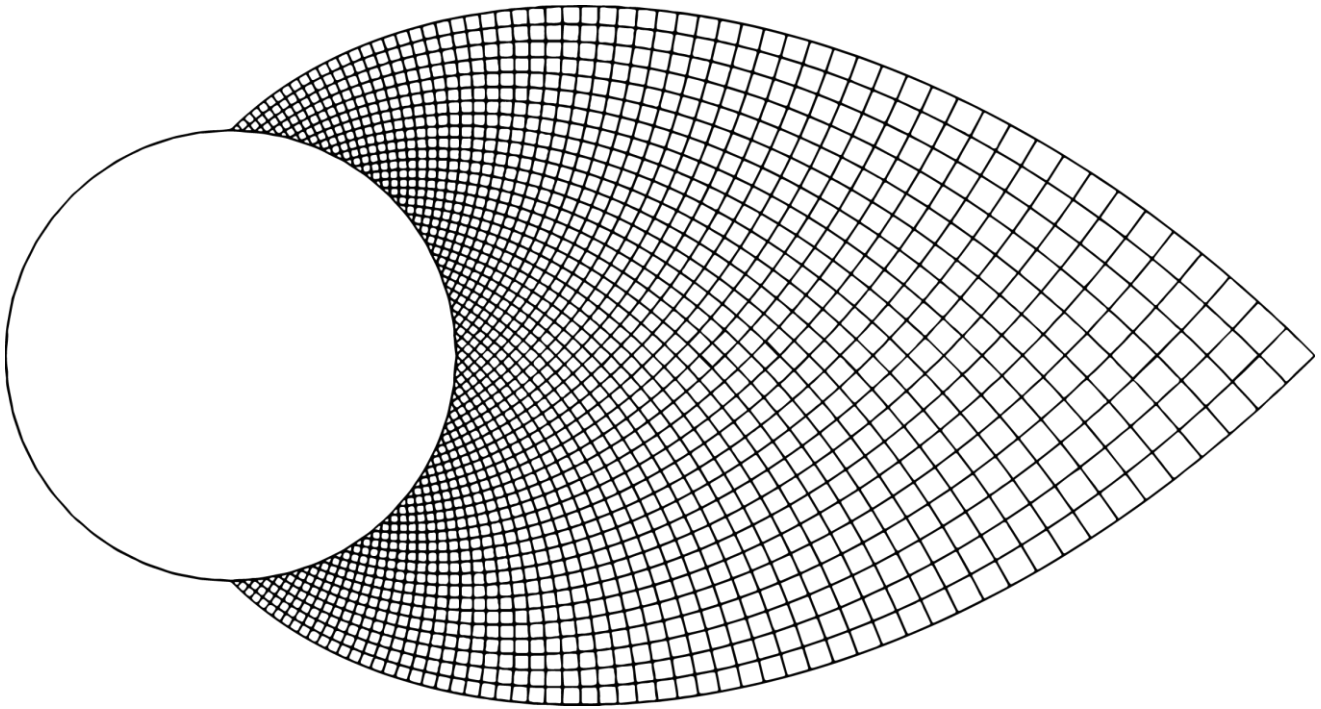

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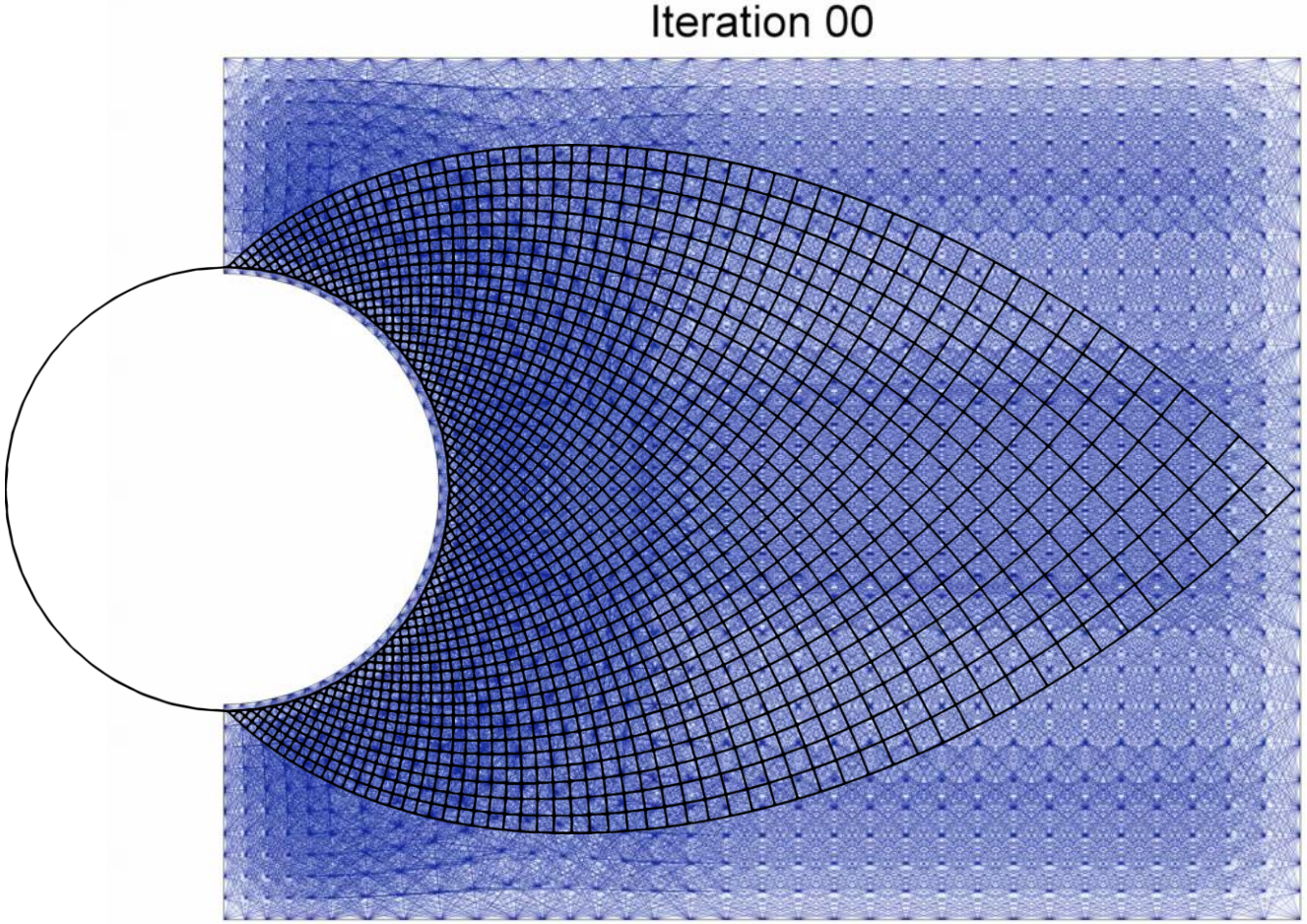
4) GROUND STRUCTURES IN 2D

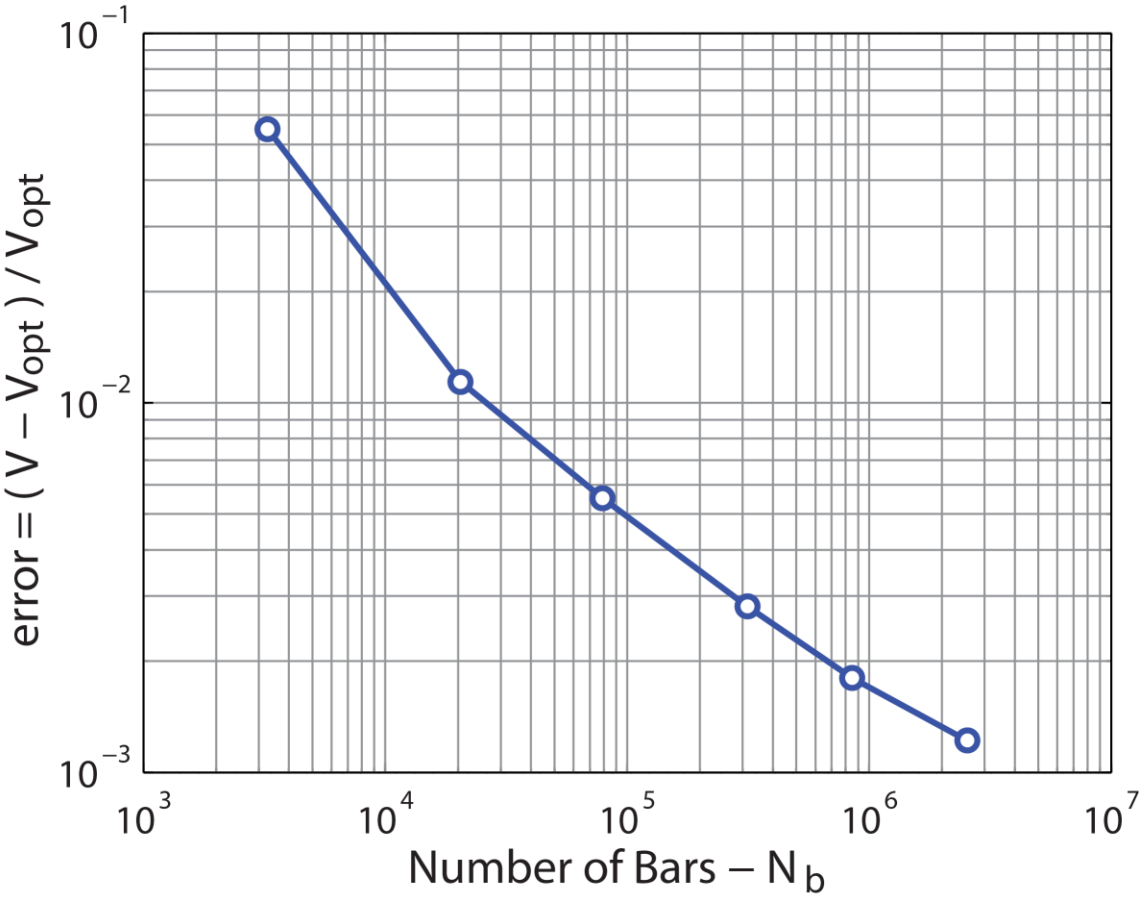
- MICHELL CANTILEVER 28,256 BARS



MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

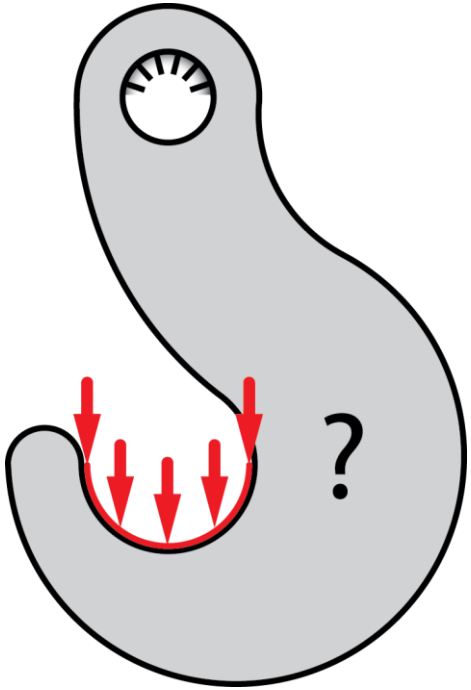
4) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER

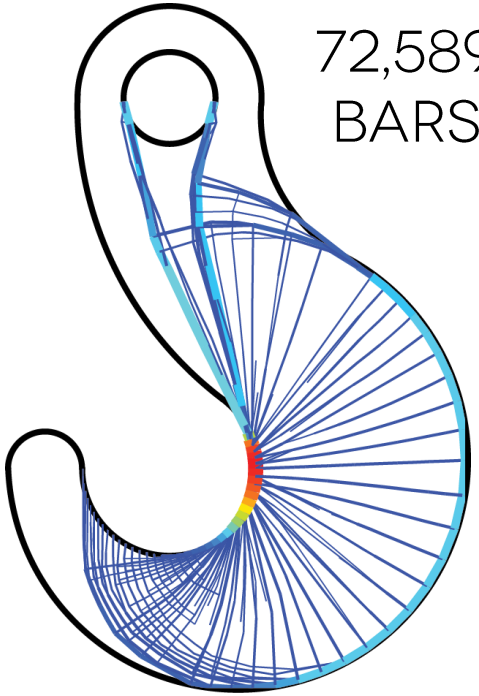


4) GROUND STRUCTURES IN 2D

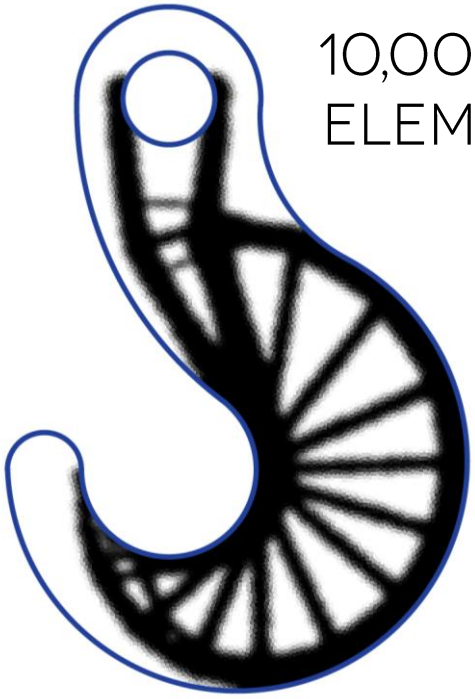
- HOOK PROBLEM



DOMAIN & BCs



GROUND STRUCTURES

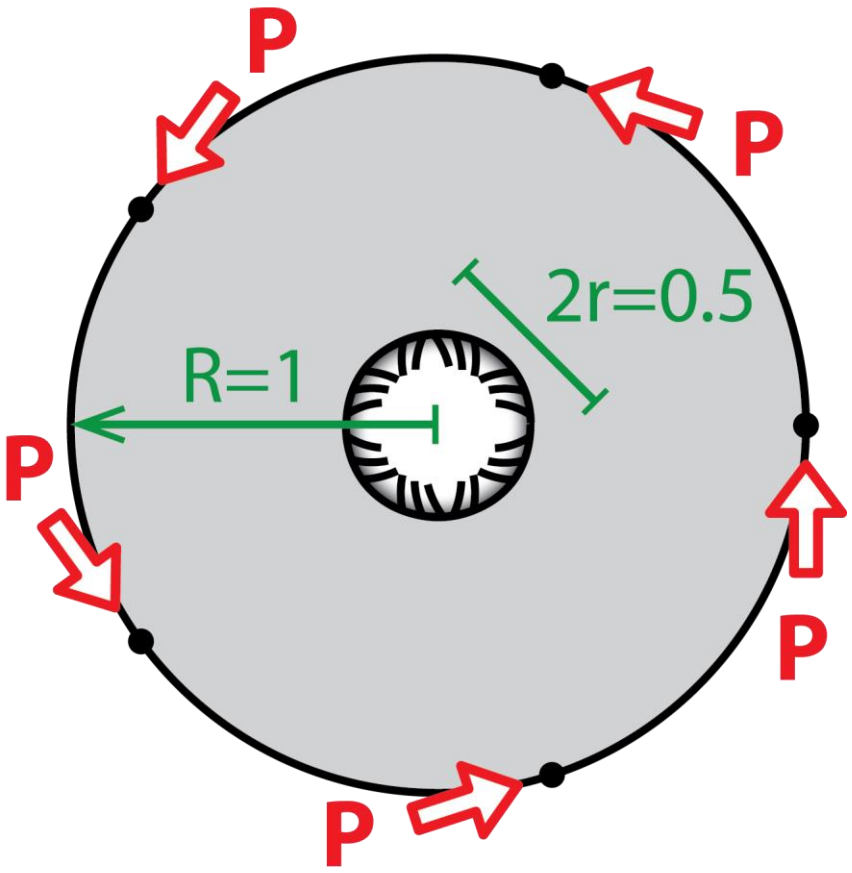


DENSITY–BASED METHOD ◇

◇ TALISCHI C, PAULINO GH, PEREIRA A, MENEZES IFM (2012), "POLYTOP: A MATLAB IMPLEMENTATION OF A GENERAL TOPOLOGY OPTIMIZATION FRAMEWORK USING UNSTRUCTURED POLYGONAL FINITE ELEMENT MESHES", STRUCT. MULTIDISC. OPTIM. 45(3), 329–357

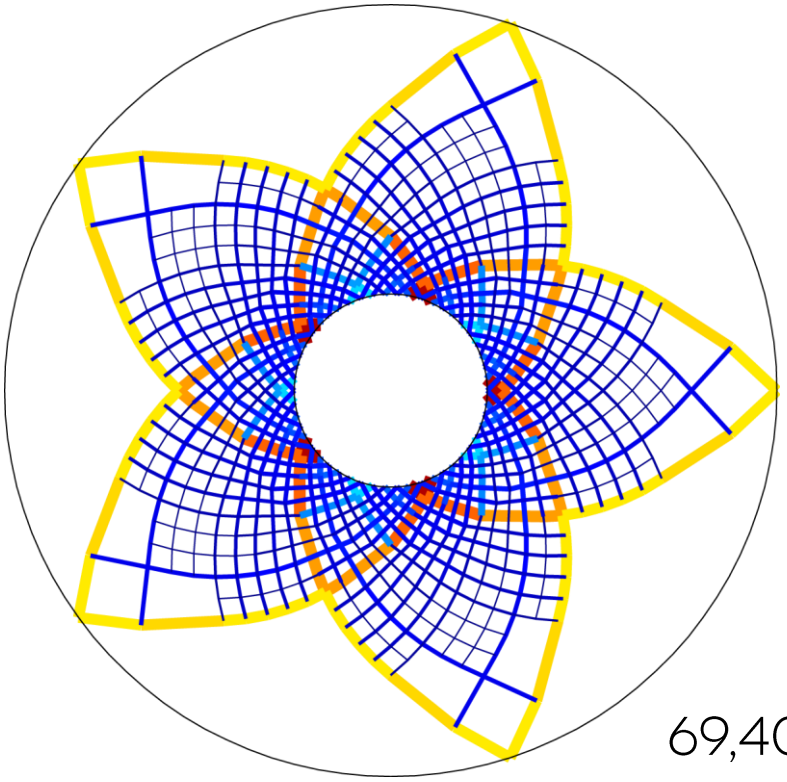
4) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM



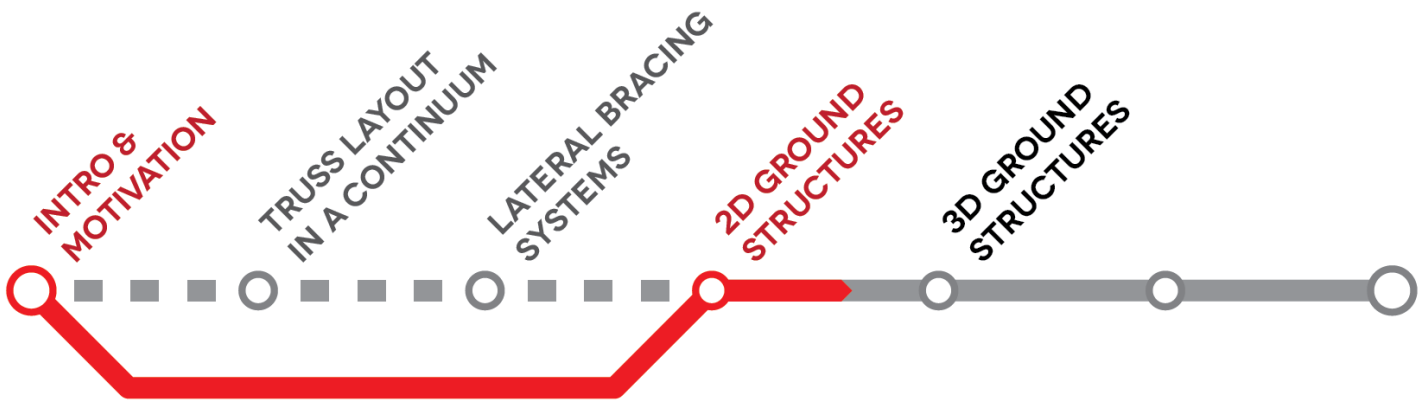
4) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM



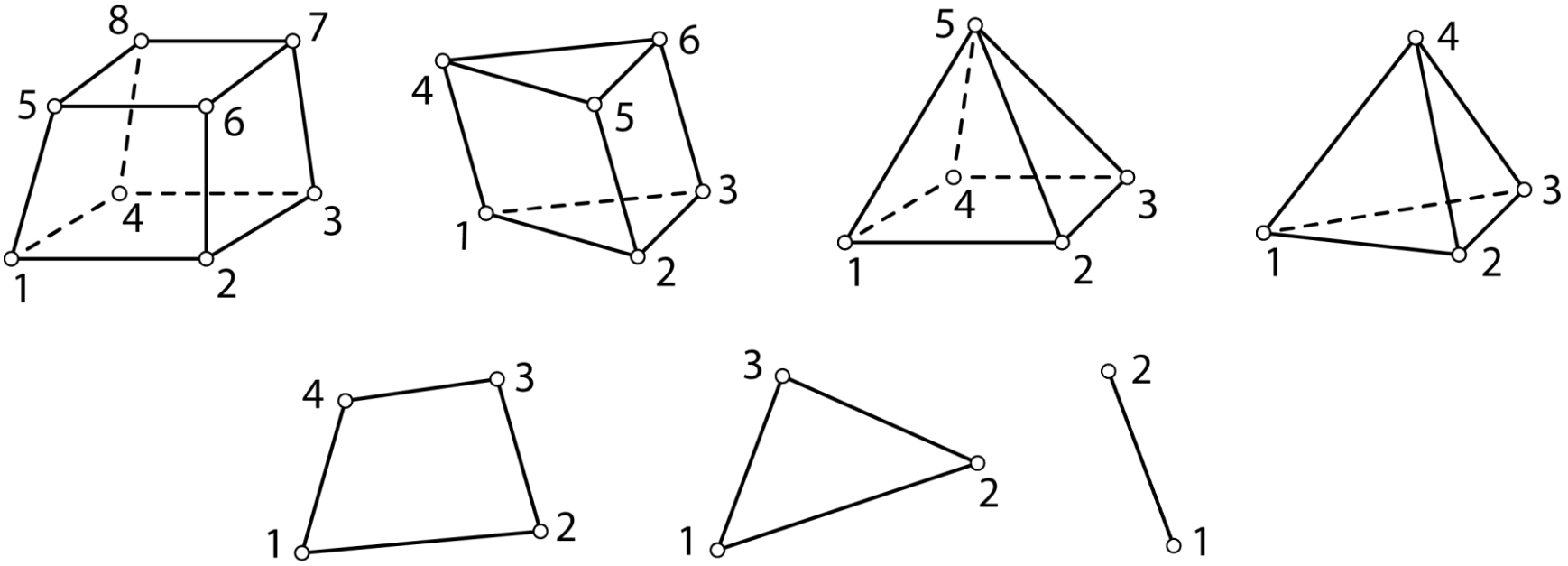
69,400 BARS

ROADMAP


INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES

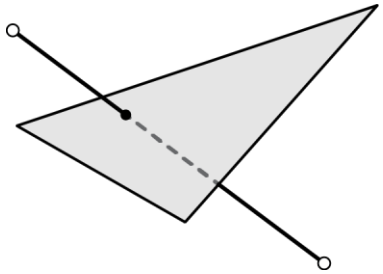
5) GROUND STRUCTURES IN 3D

- BASE-MESH DEFINITION
  - GROUND STRUCTURE ALGORITHM SUPPORTS ANY CONVEX POLYTOPE
  - IMPLEMENTATION IS RESTRICTED TO 7 ELEMENTS: MESH GENERATION AND PLOTTING PURPOSES

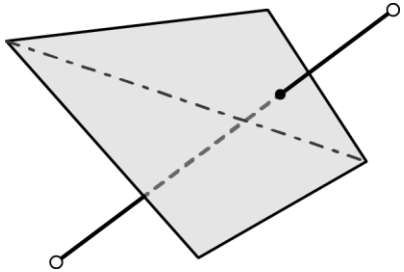


5) GROUND STRUCTURES IN 3D

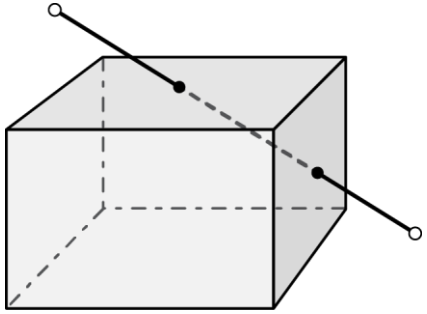
- RESTRICTION PRIMITIVES:



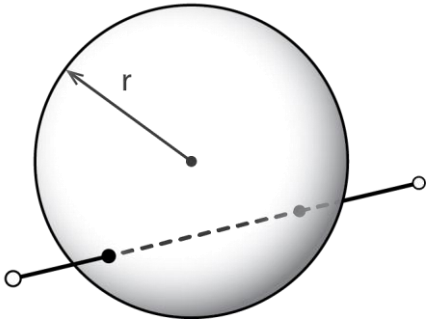
TRIANGLE



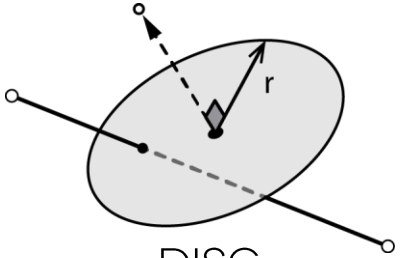
QUAD



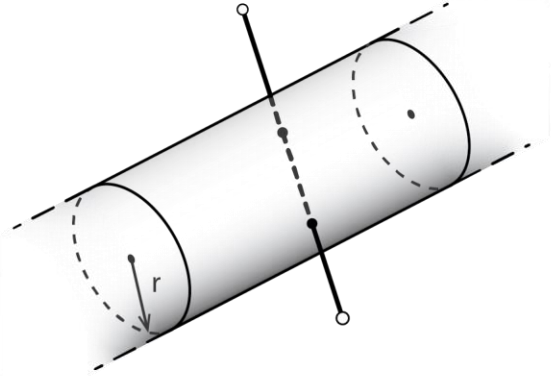
BOX



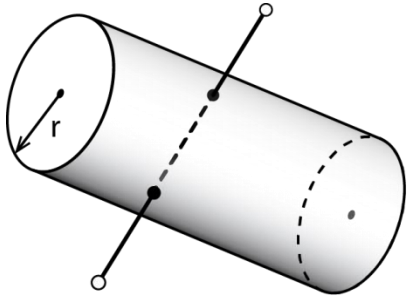
SPHERE



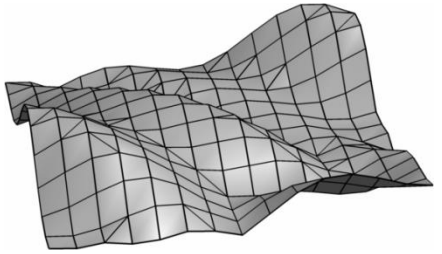
DISC



CYLINDER



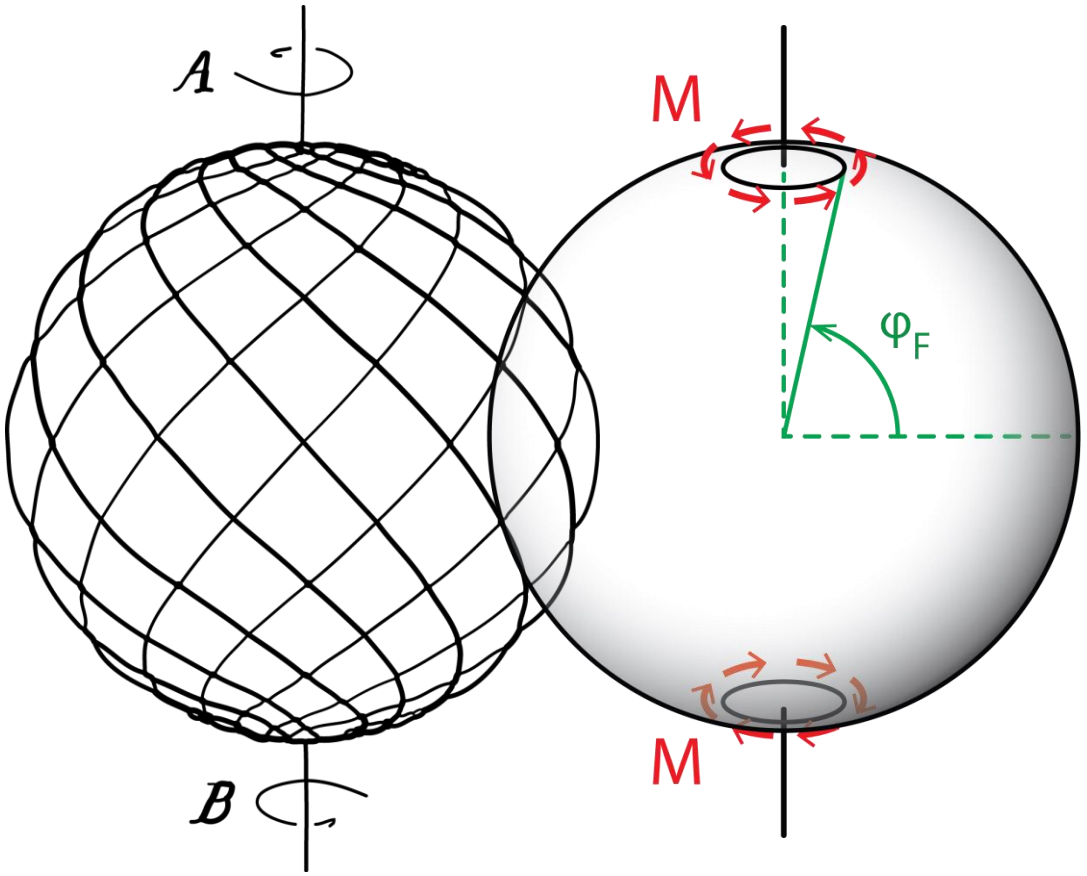
ROD



SURFACE

5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

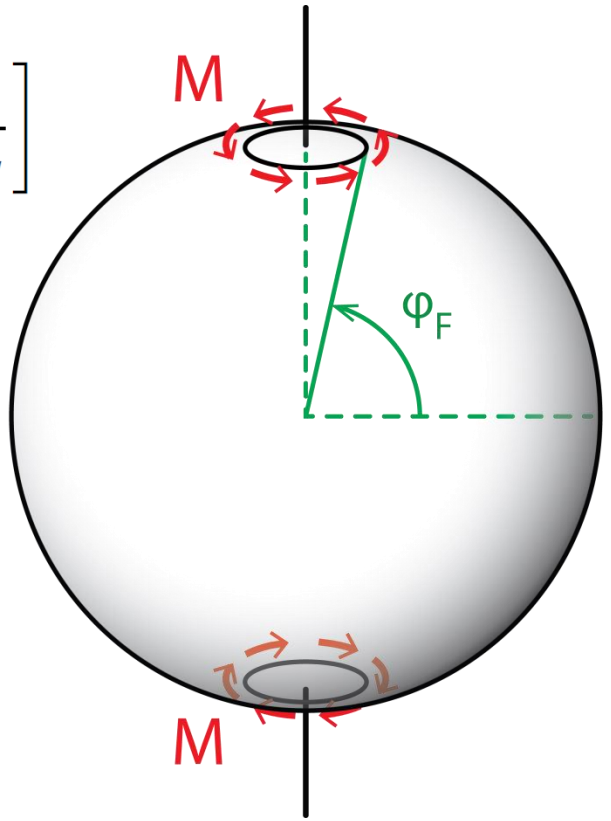


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

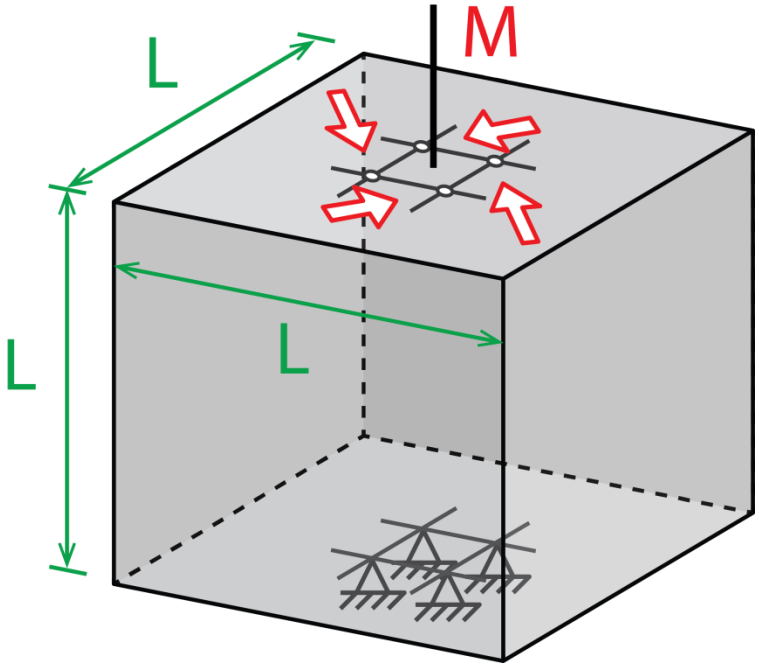
$$V_{opt} = 2M \log\left(\tan\left\{\frac{\pi}{4} + \frac{\phi_F}{2}\right\}\right)\left[\frac{1}{\sigma_T} + \frac{1}{\sigma_C}\right]$$

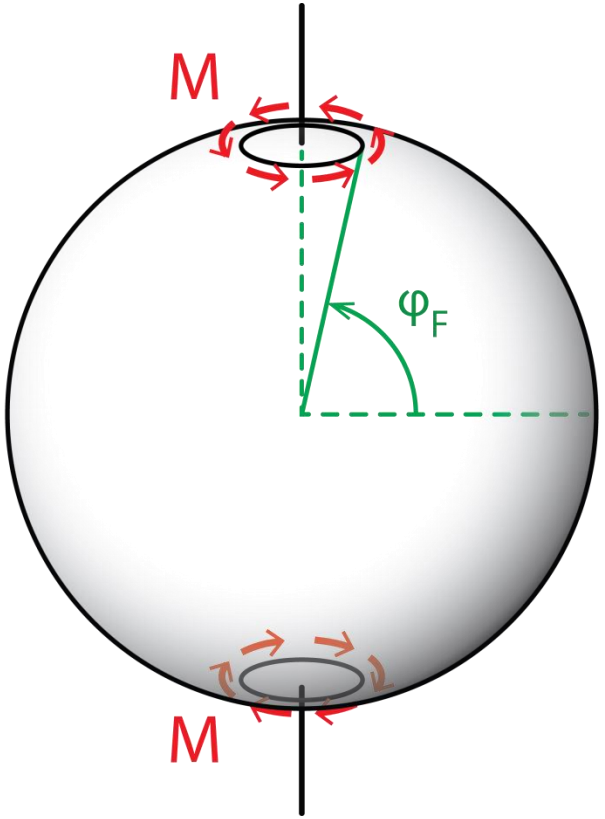


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

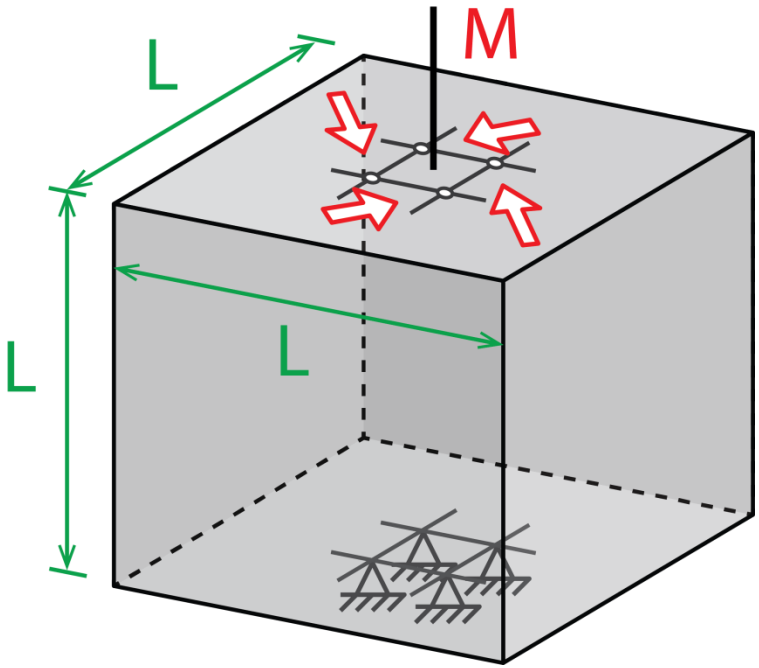


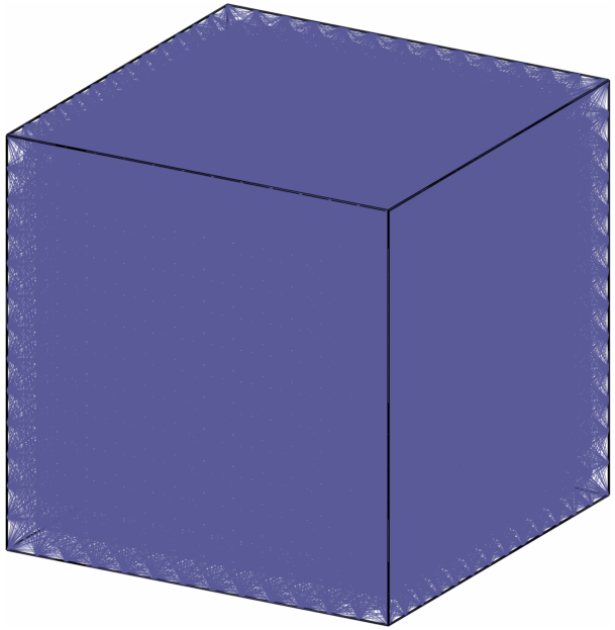


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES", PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

5) GROUND STRUCTURES IN 3D

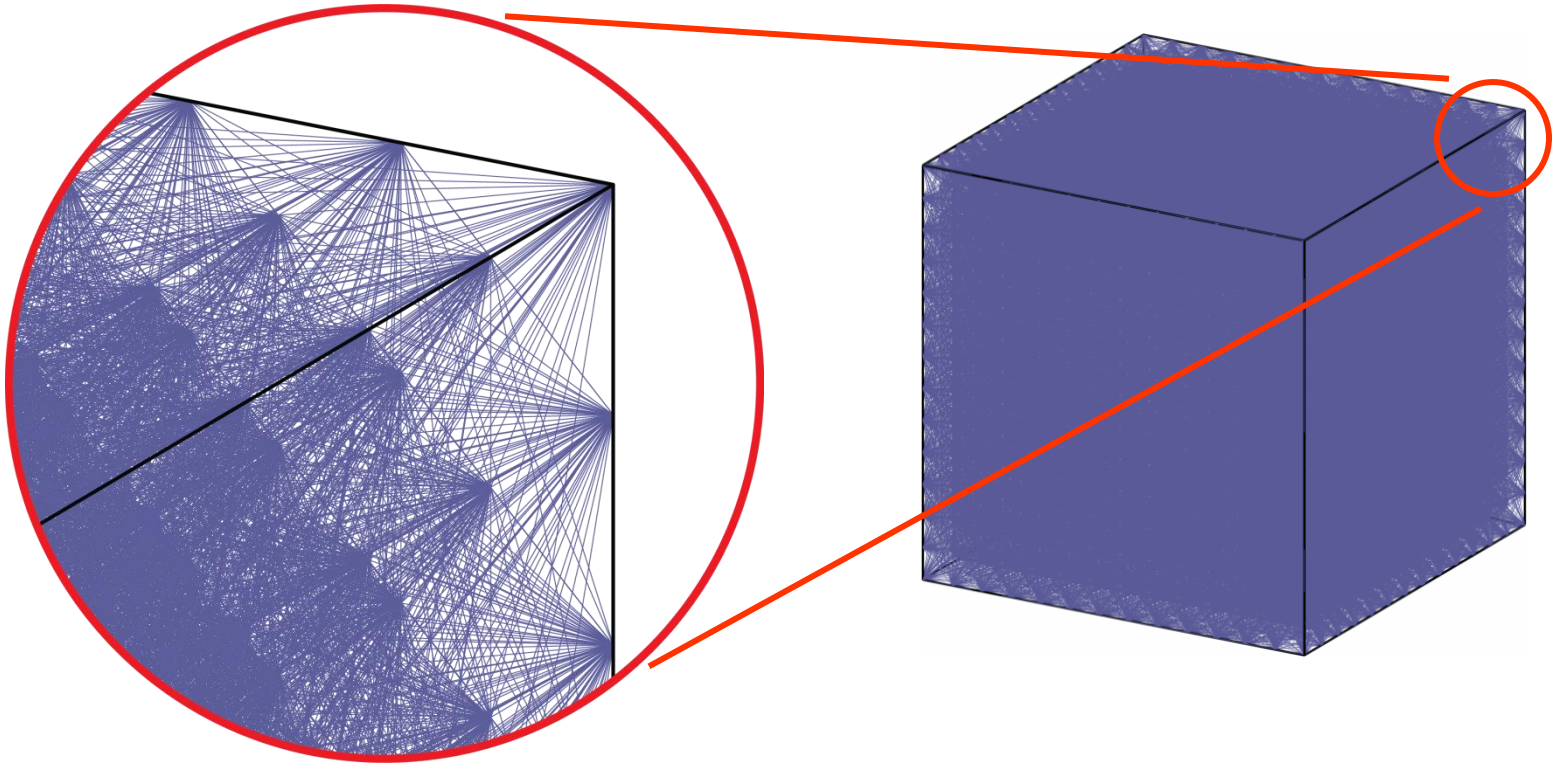
- TORSION BALL PROBLEM





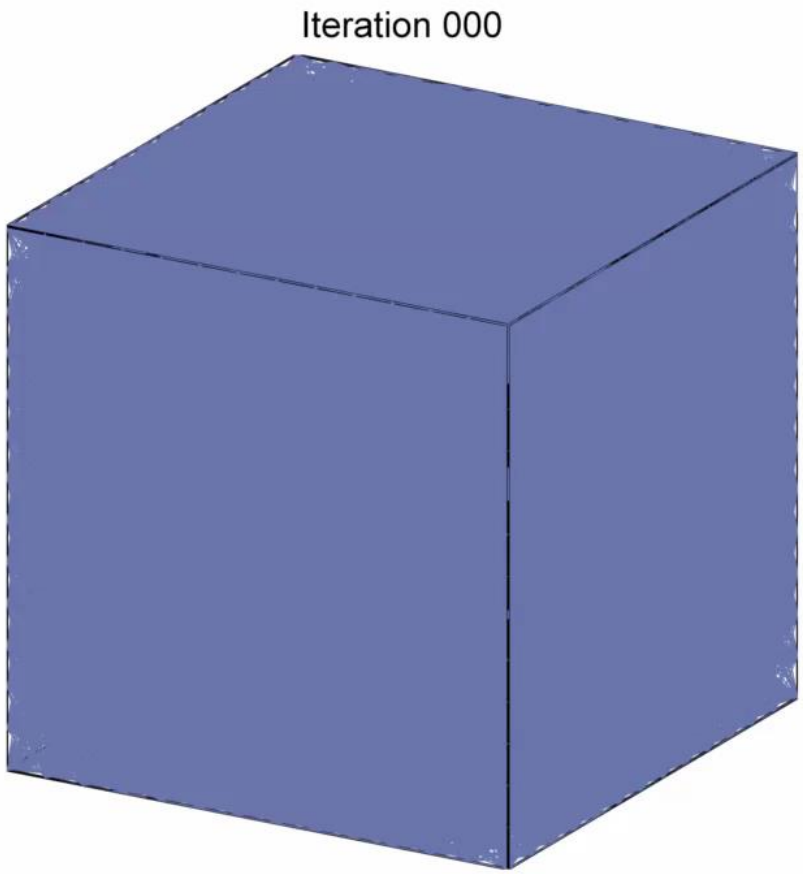
5) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM



5) GROUND STRUCTURES IN 3D

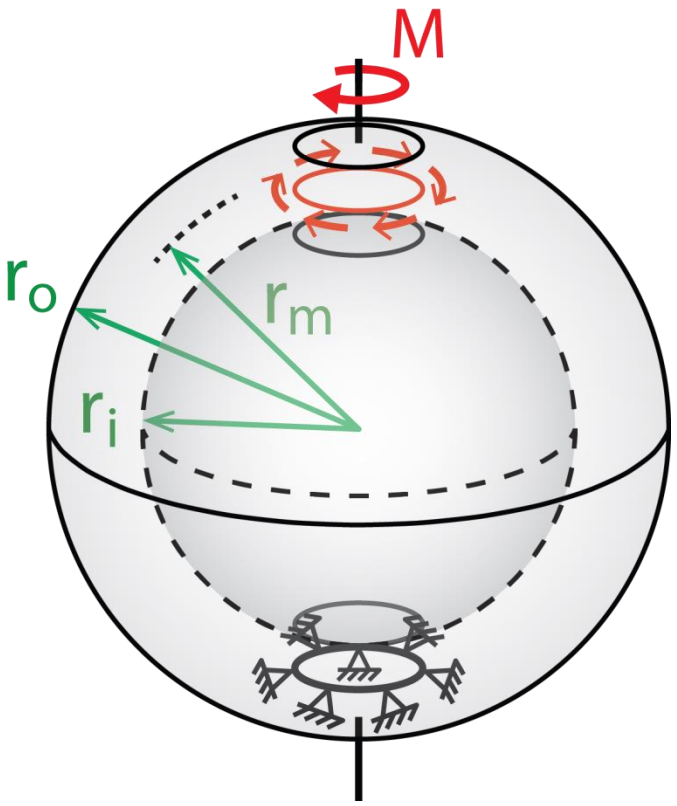
- TORSION BALL PROBLEM

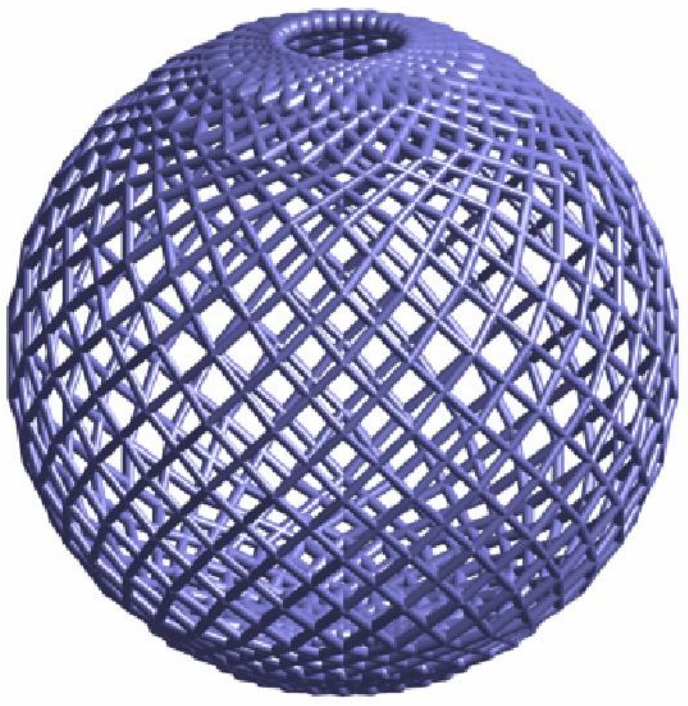


268,636 BARS

5) GROUND STRUCTURES IN 3D

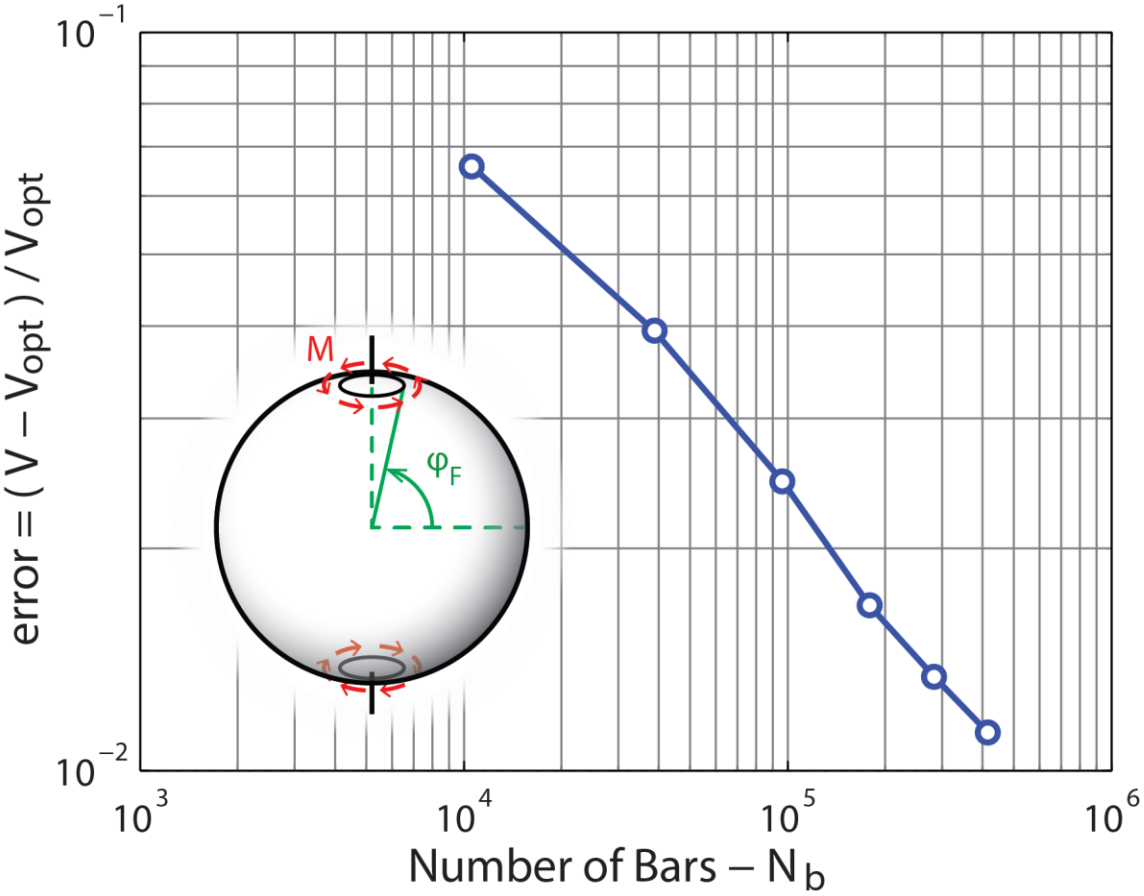
- TORSION BALL
  IMPROVING THE BASE MESH: SPHERICAL COORDINATES





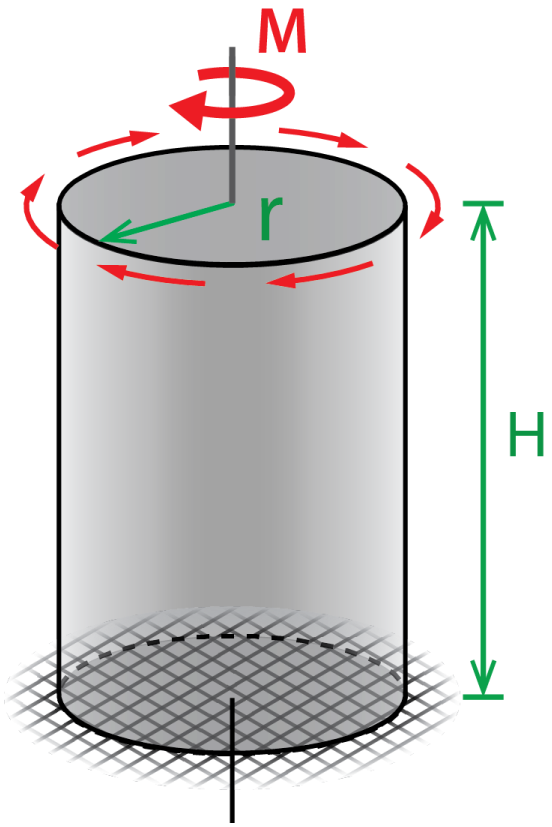
5) GROUND STRUCTURES IN 3D

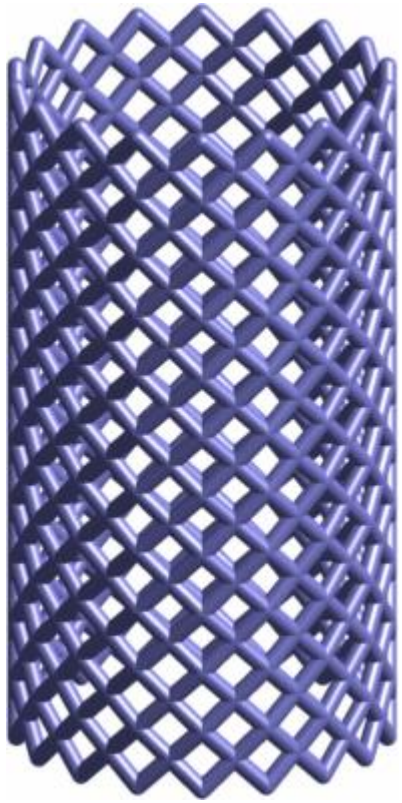
- TORSION BALL
  IMPROVING THE BASE MESH: SPHERICAL COORDINATES



5) GROUND STRUCTURES IN 3D

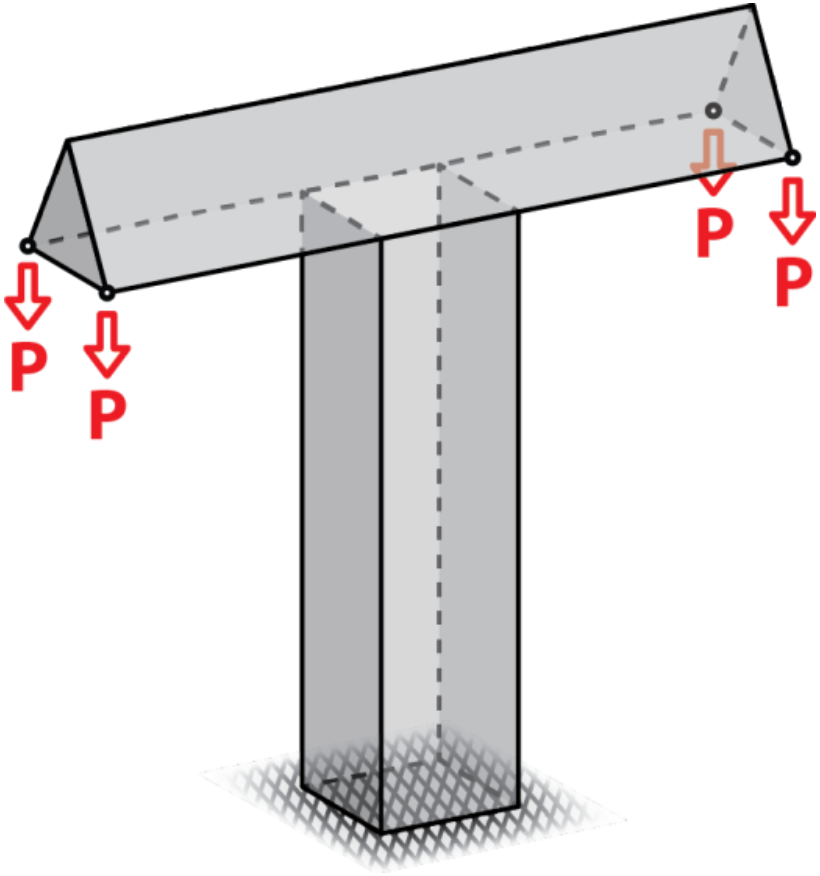
- OTHER KNOWN SOLUTIONS?

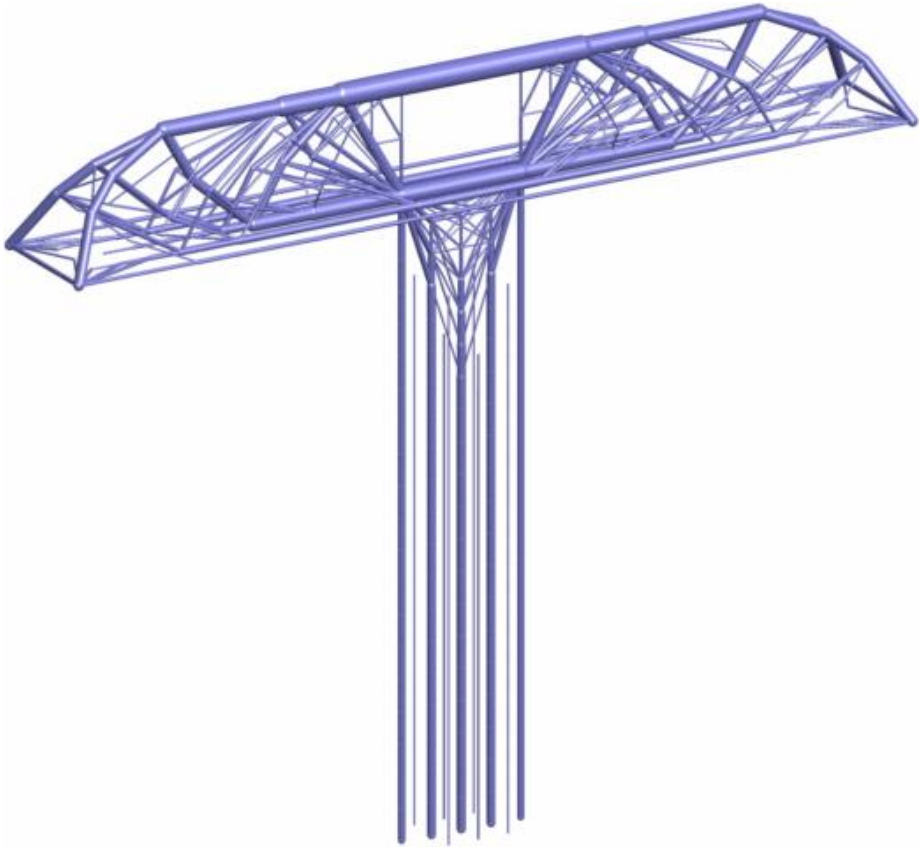




5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

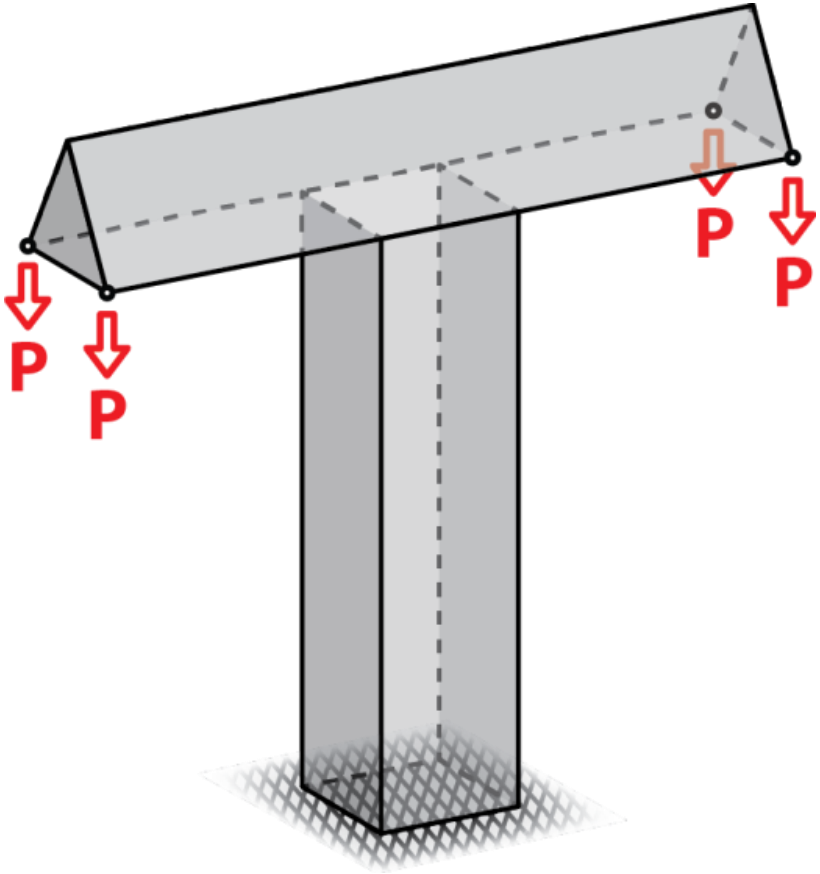


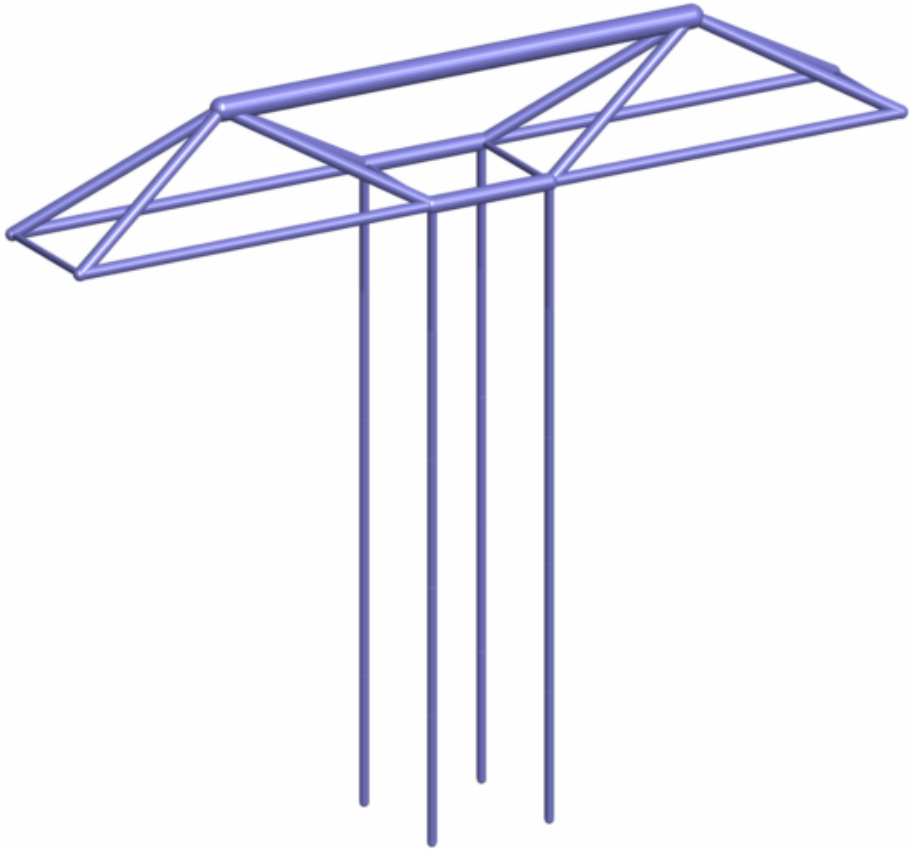


SMITH ODS (1998). "GENERATION OF GROUND STRUCTURES FOR 2D AND 3D DESIGN DOMAINS." ENGINEERING COMPUTATIONS, 15(4):462–500.

5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

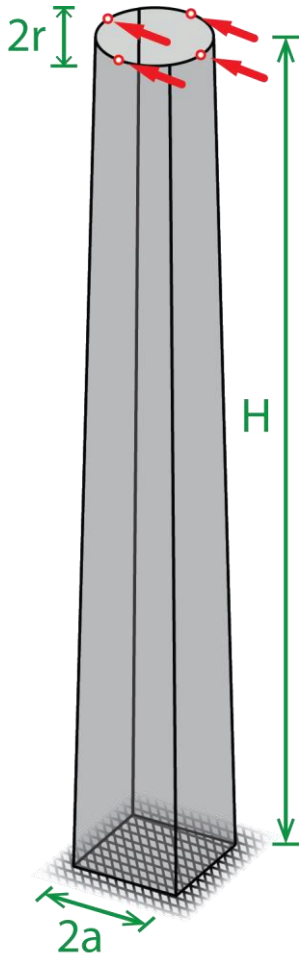


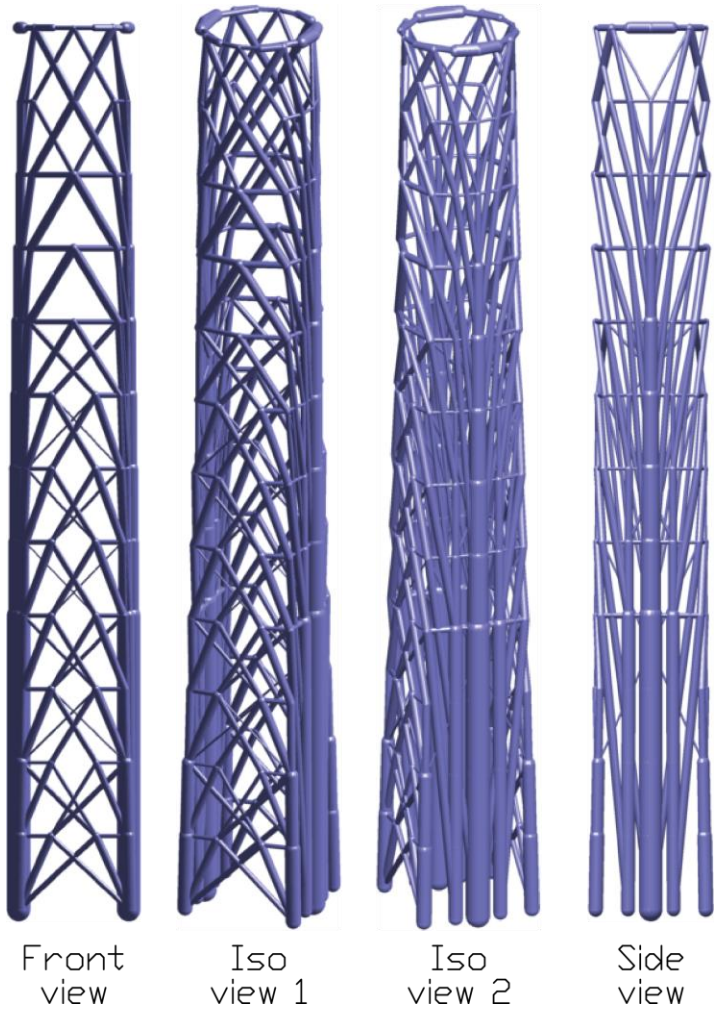


SMITH ODS (1998). "GENERATION OF GROUND STRUCTURES FOR 2D AND 3D DESIGN DOMAINS." ENGINEERING COMPUTATIONS, 15(4):462–500.

5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?

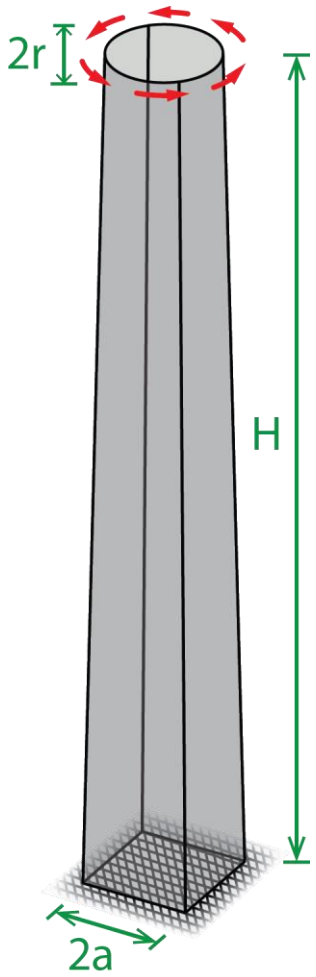




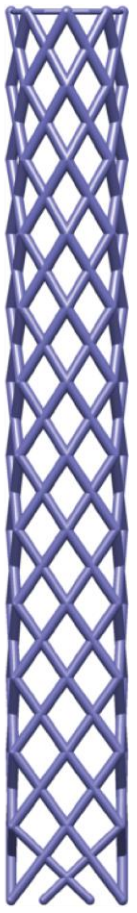
4,100 BARS

5) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?



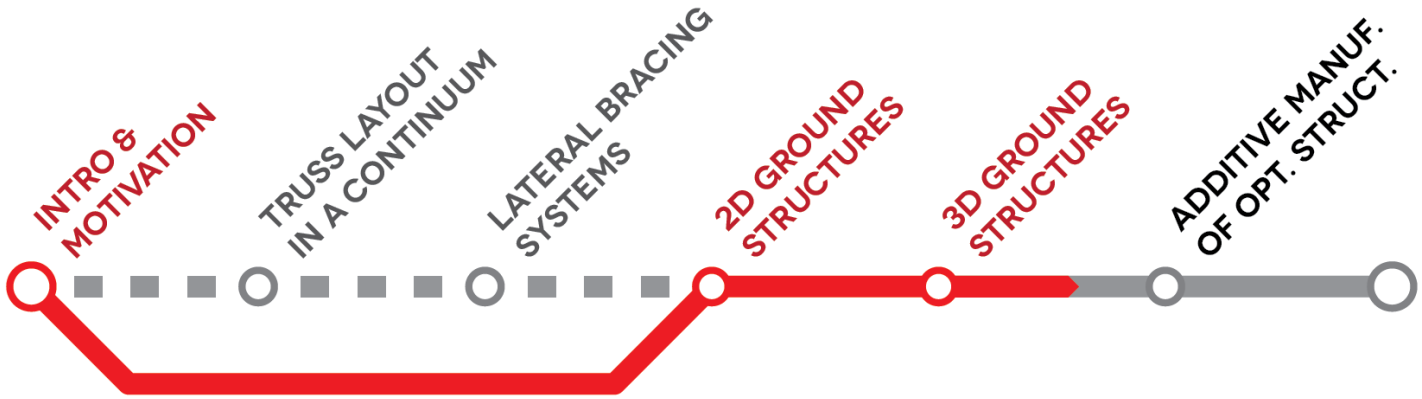
Iso view



Front view

4,100 BARS

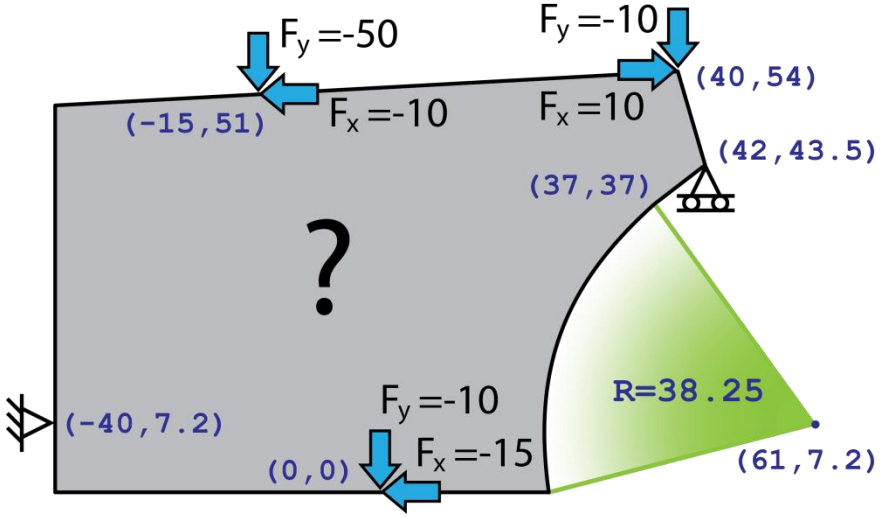
ROADMAP


INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES
ADDITIVE MANUF. OF OPT. STRUCT.

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- INTRODUCTION TO DENSITY–BASED TOPOLOGY OPTIMIZATION

CANNONDALE CAPO
(URBAN COMMUTER BIKE)

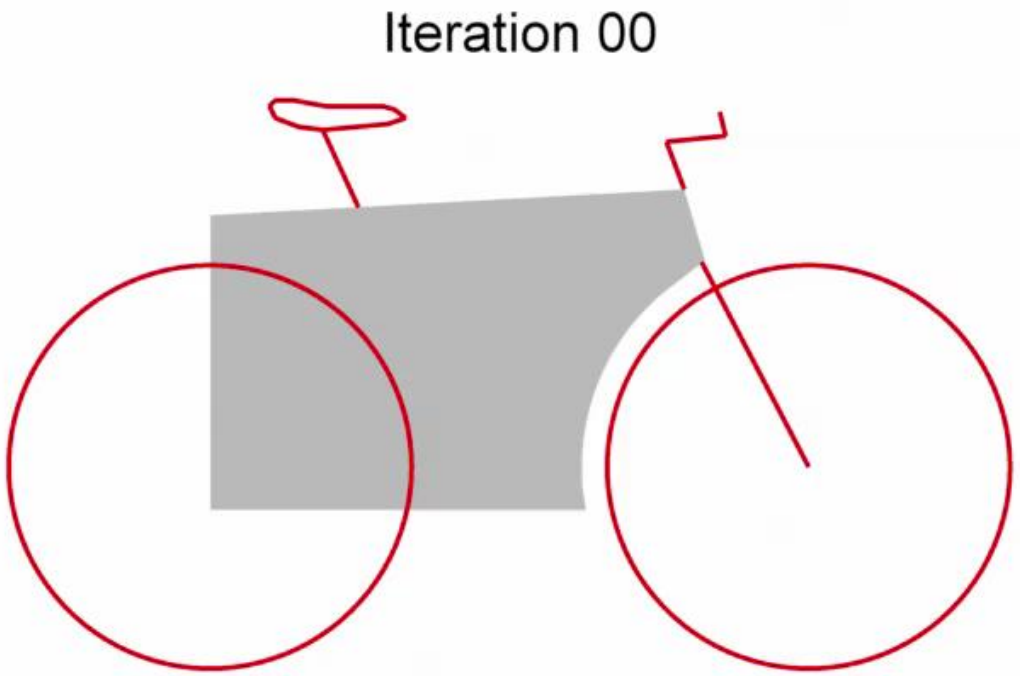


BIKE DOMAIN AND LOADS

ZEGARD T, PAULINO GH (2013) "TOWARD GPU ACCELERATED TOPOLOGY OPTIMIZATION ON UNSTRUCTURED MESHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 48(3):473–485

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- INTRODUCTION TO DENSITY-BASED TOPOLOGY OPTIMIZATION



ZEGARD T, PAULINO GH (2013) "TOWARD GPU ACCELERATED TOPOLOGY OPTIMIZATION ON UNSTRUCTURED MESHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 48(3):473–485

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- DENSITY-BASED (NESTED) FORMULATION:
  - USING A DENSITY FILTER[1]
  - MODIFIED SIMP[2][3][4]

$$
\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & J\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \\
\text{s.t.} \quad & \overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho} \qquad \text{FILTERING} \\
& \textstyle\sum_i^{N_e} \overline{\rho}_i v_i - \left(f\right)\left(V_0\right) \leq 0 \qquad \text{VOLUME CONSTRAINT} \\
& g_i\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \leq 0 \qquad i = 1 \ldots N_c \\
& 0 \leq \rho_j \leq 1 \qquad j = 1 \ldots N_e \qquad \text{1 = SOLID, 0 = VOID} \\
& E_k\left(\overline{\rho}_k\right) = E_{min} + \overline{\rho}_k^p \left(E_0 - E_{min}\right) \qquad k = 1 \ldots N_e \qquad \text{MOD-SIMP} \\
\text{with} \quad & \mathbf{K}\left(\overline{\rho}\right)\mathbf{u} = \mathbf{f}
\end{aligned}
$$

1) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143–2158

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193–202

3) ZHOU M, ROZVANY G (1991) "THE COC ALGORITHM, PART II: TOPOLOGICAL, GEOMETRICAL AND GENERALIZED SHAPE OPTIMIZATION. "COMP METH APPL MECH ENGRG 89:309–336

4) SIGMUND O (2007) "MORPHOLOGY-BASED BLACK AND WHITE FILTERS FOR TOPOLOGY OPTIMIZATION." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 33(4–5):401–424.

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- DENSITY-BASED (NESTED) FORMULATION:
  - USING A DENSITY FILTER[1]
  - MODIFIED SIMP[2][3][4]

$$
\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & J\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \\
\text{s.t.} \quad & \overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho} \\
& \textstyle\sum_i^{N_e} \overline{\rho}_i v_i - \left(f\right)\left(V_0\right) \leq 0 \\
& g_i\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \leq 0 \qquad i = 1 \ldots N_c \\
& 0 \leq \rho_j \leq 1 \qquad j = 1 \ldots N_e \\
& E_k\left(\overline{\rho}_k\right) = E_{min} + \overline{\rho}_k^p \left(E_0 - E_{min}\right) \qquad k = 1 \ldots N_e \\
\text{with} \quad & \mathbf{K}\left(\overline{\rho}\right)\mathbf{u} = \mathbf{f}
\end{aligned}
$$

P=1
VARIABLE THICKNESS SHEET PROBLEM (CONVEX)

1) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143–2158

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193–202

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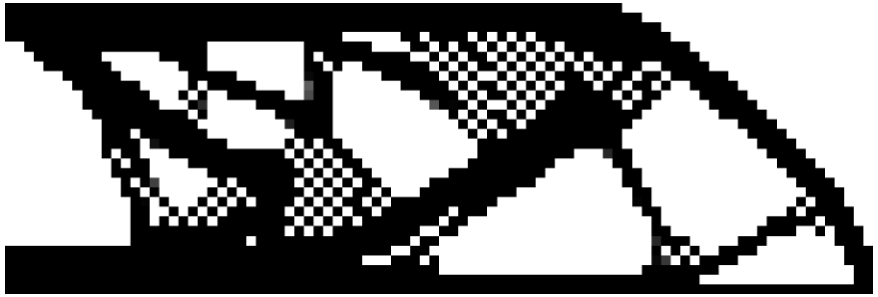
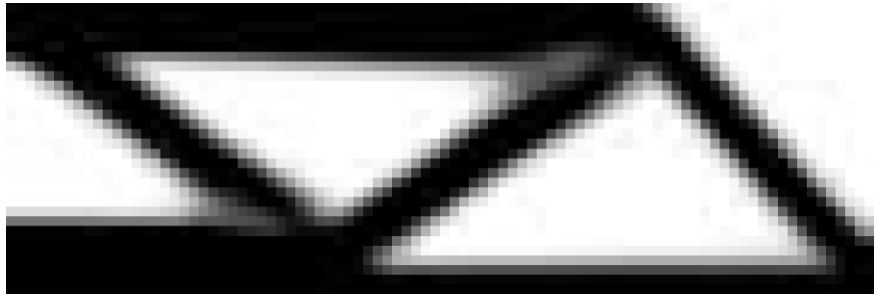
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- FILTERS IN DENSITY-BASED FORMULATION:
  - SENSITIVITY FILTER (1-FIELD)
  - DENSITY FILTER (2-FIELDS)
  - PROJECTION FILTER (3-FIELDS)

USED IN THIS WORK



UNFILTERED
(CHECKERBOARD)



FILTERED

REVIEW ON FILTERING:
SIGMUND O, MAUTE K (2013) "TOPOLOGY OPTIMIZATION APPROACHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 48(6):1031-1055

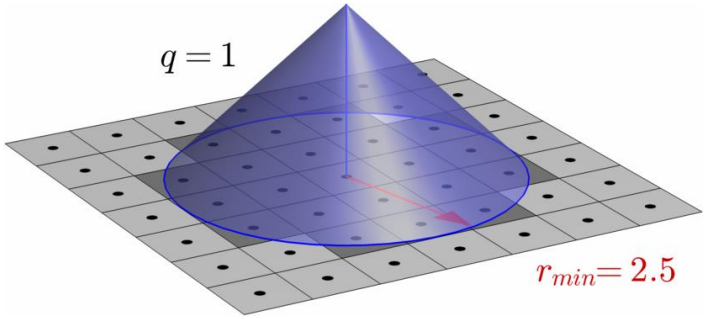
6) ADDITIVE MANUF. OF OPT. STRUCTS.

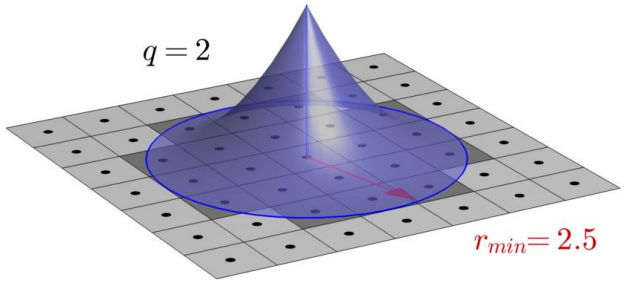
- CONVOLUTION (BLURRING) OF THE DENSITY FIELD

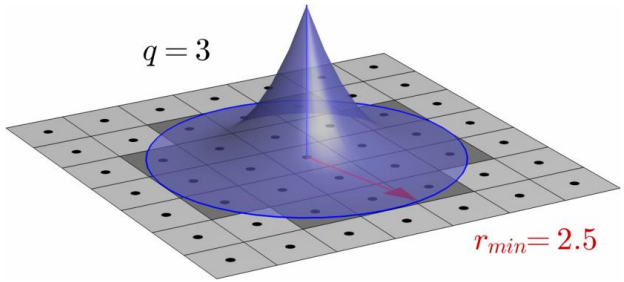
$$\overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho}$$

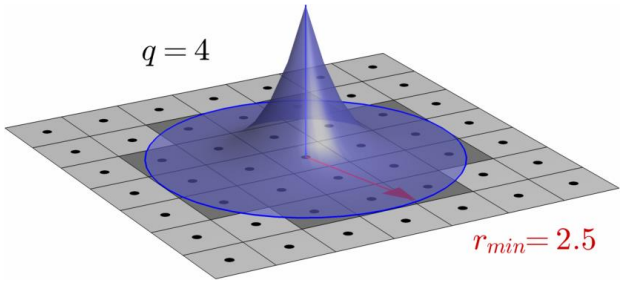
$$\text{with} \quad \mathbf{H}_{ij} = \frac{h\left(i,j\right) v_j}{\sum_k^{N_e} h\left(i,k\right) v_k}$$

$$h\left(i,j\right) = \begin{cases} \left[r_{min} - \text{dist}\left(i,j\right)\right]^q & \text{for} \quad r_{min} - \text{dist}\left(i,j\right) > 0 \\ 0 & \text{otherwise} \end{cases}$$



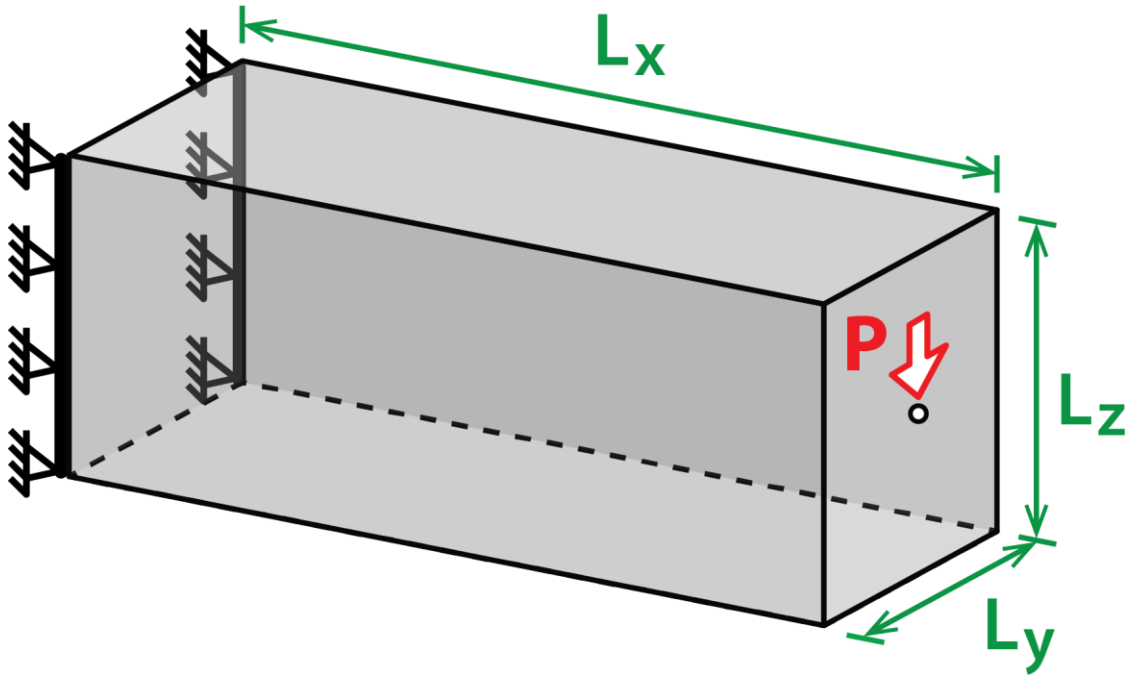






6) ADDITIVE MANUF. OF OPT. STRUCTS.

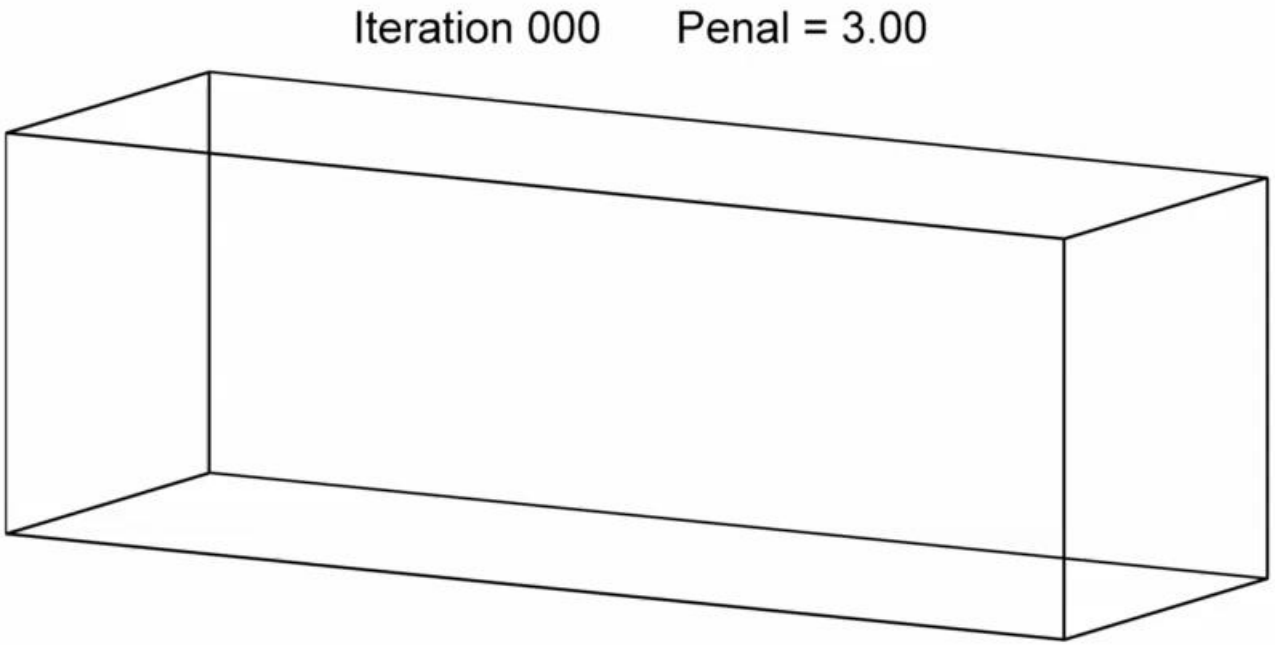
- EDGE-SUPPORTED CANTILEVER BEAM
  Lx=3, Ly=Lz=1, Q=1, R=5 AND VOLFRAC=10%



559,872 DVs FOR ½
(1,119,744 TOTAL)

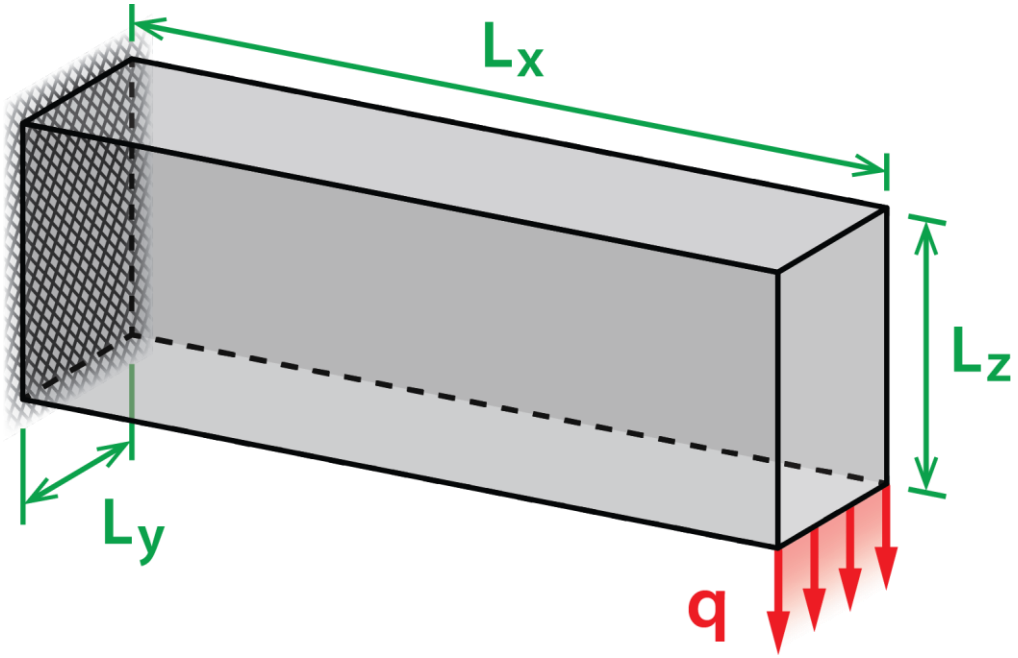
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-SUPPORTED CANTILEVER BEAM
  Lx=3, Ly=Lz=1, Q=1, R=5 AND VOLFRAC=10%



6) ADDITIVE MANUF. OF OPT. STRUCTS.

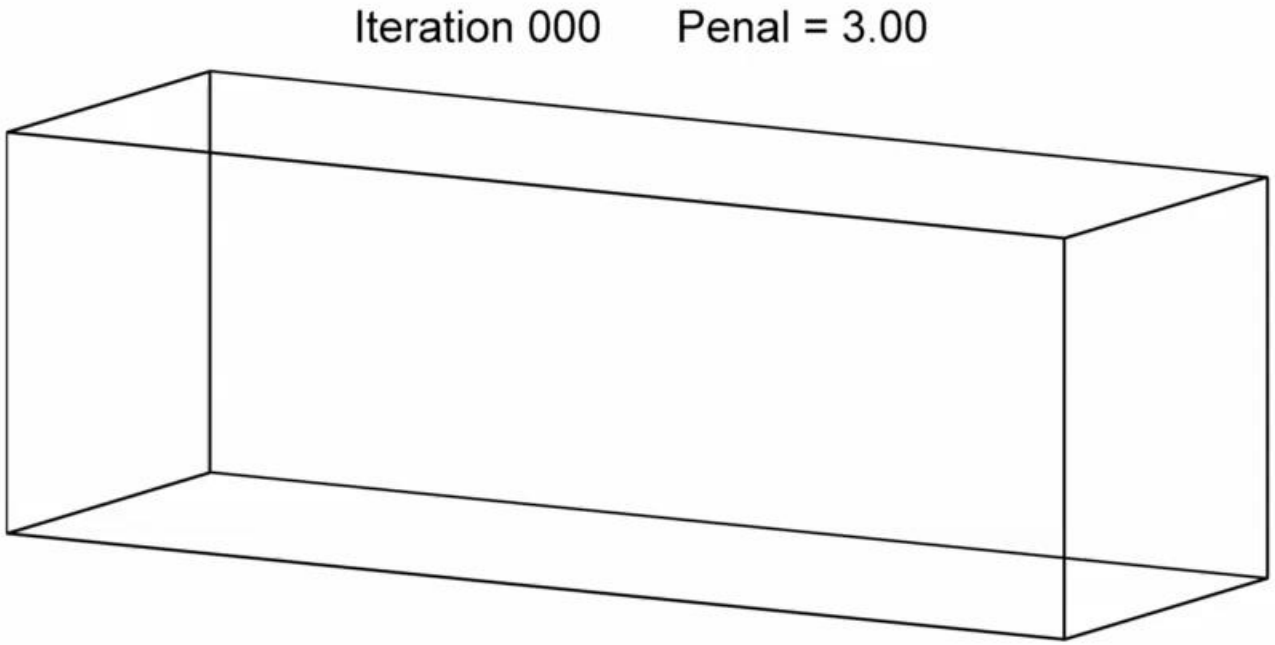
- EDGE-LOADED CANTILEVER BEAM
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=1 AND P=3



559,872 DVS FOR ½
(1,119,744 TOTAL)

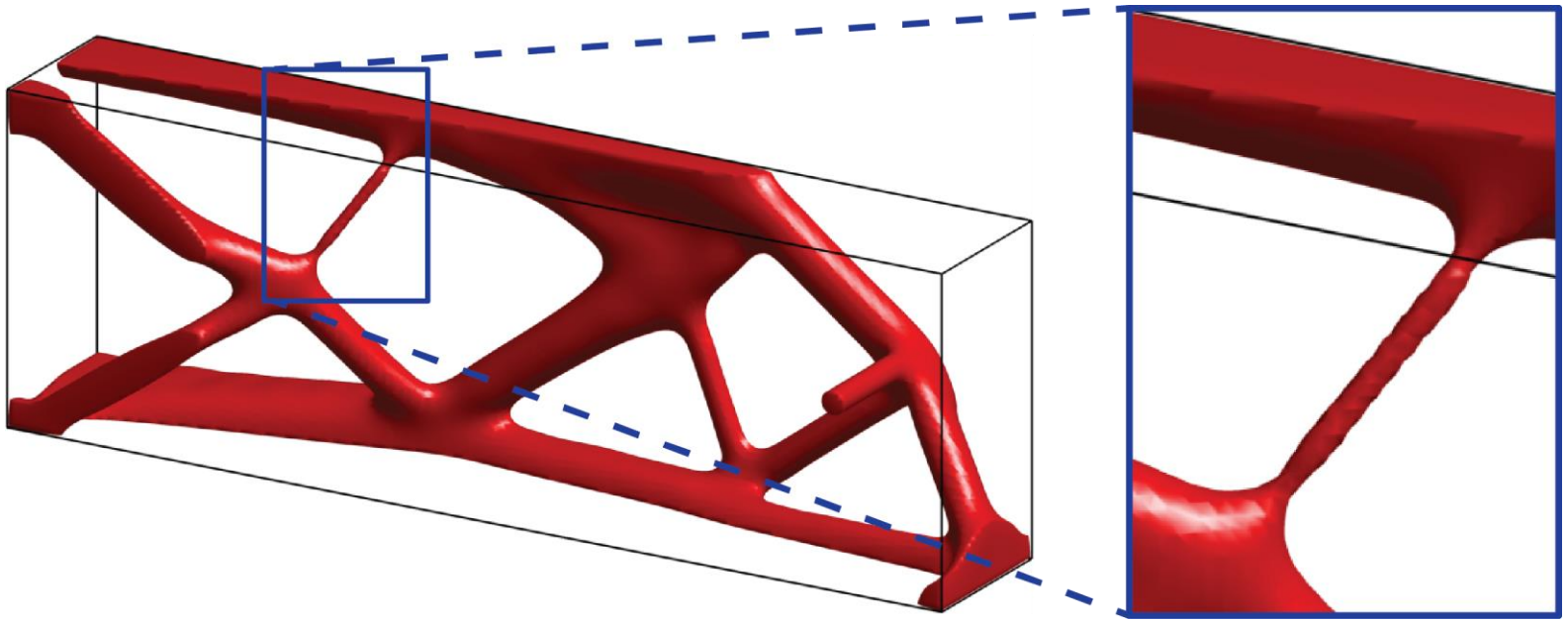
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER BEAM
  LX=3, LY=LZ=1, VOLFRAC=10%, R=6, Q=1 AND P=3



6) ADDITIVE MANUF. OF OPT. STRUCTS.

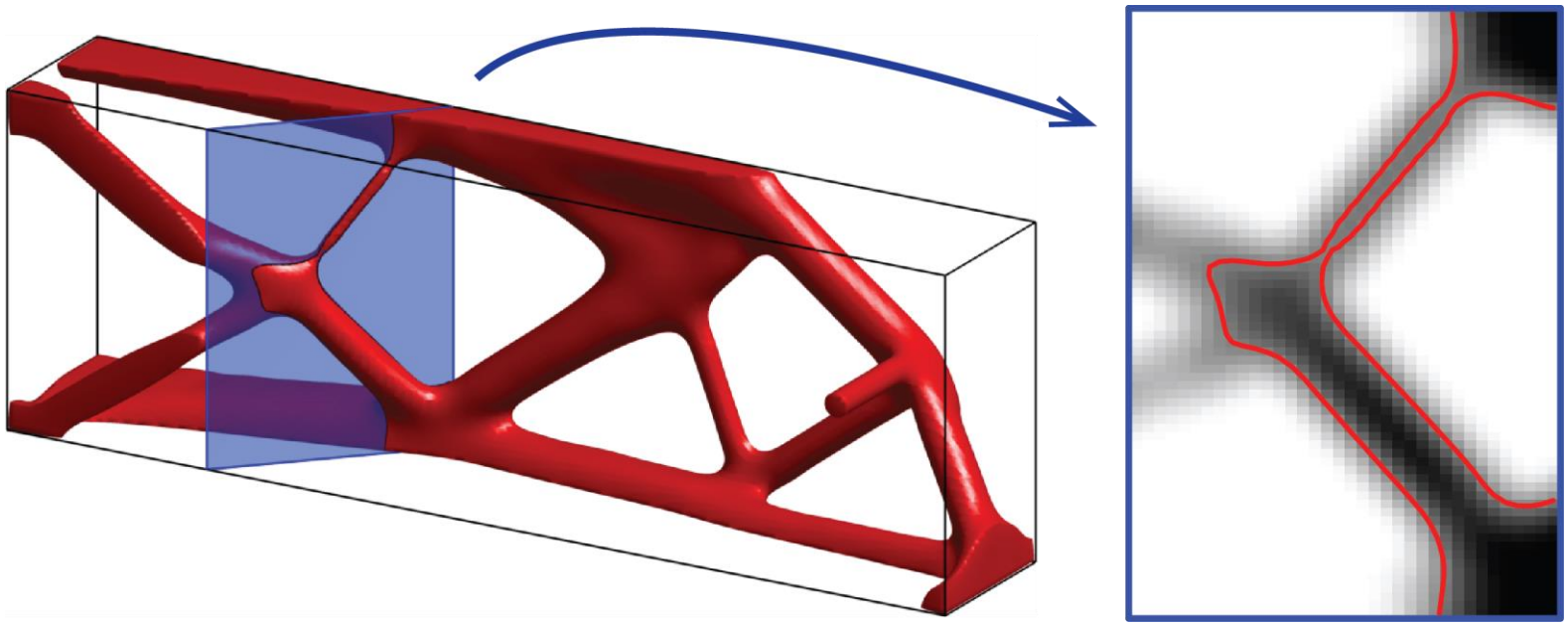
- EDGE-LOADED CANTILEVER
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=1 AND P=3



559,872 DVS FOR ½
(1,119,744 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

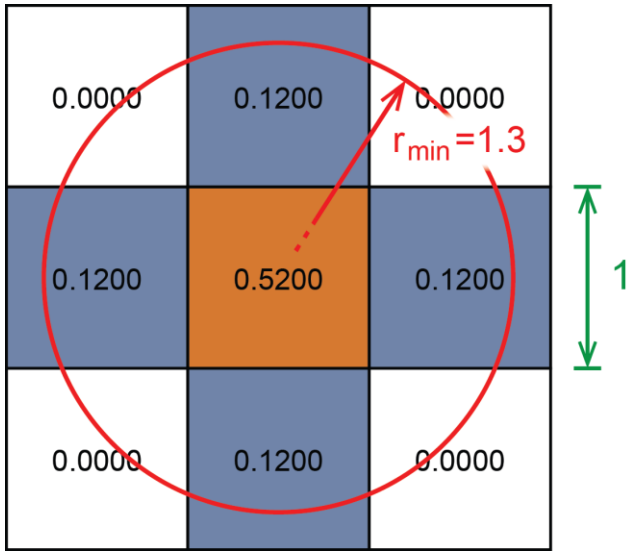
- EDGE-LOADED CANTILEVER
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=1 AND P=3



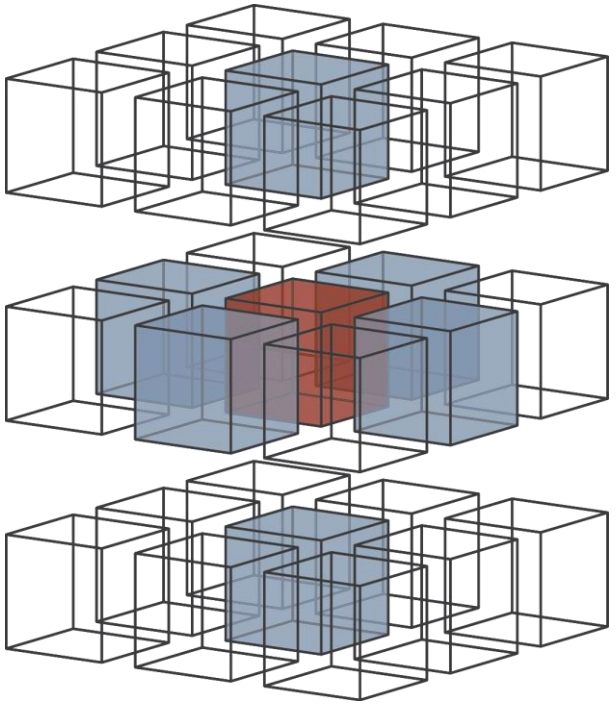
559,872 DVS FOR ½
(1,119,744 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- FILTER'S WEIGHTS FOR A REGULAR MESH
  RMIN=1.3, Q=1 AND ELEM SIZE IS L=1



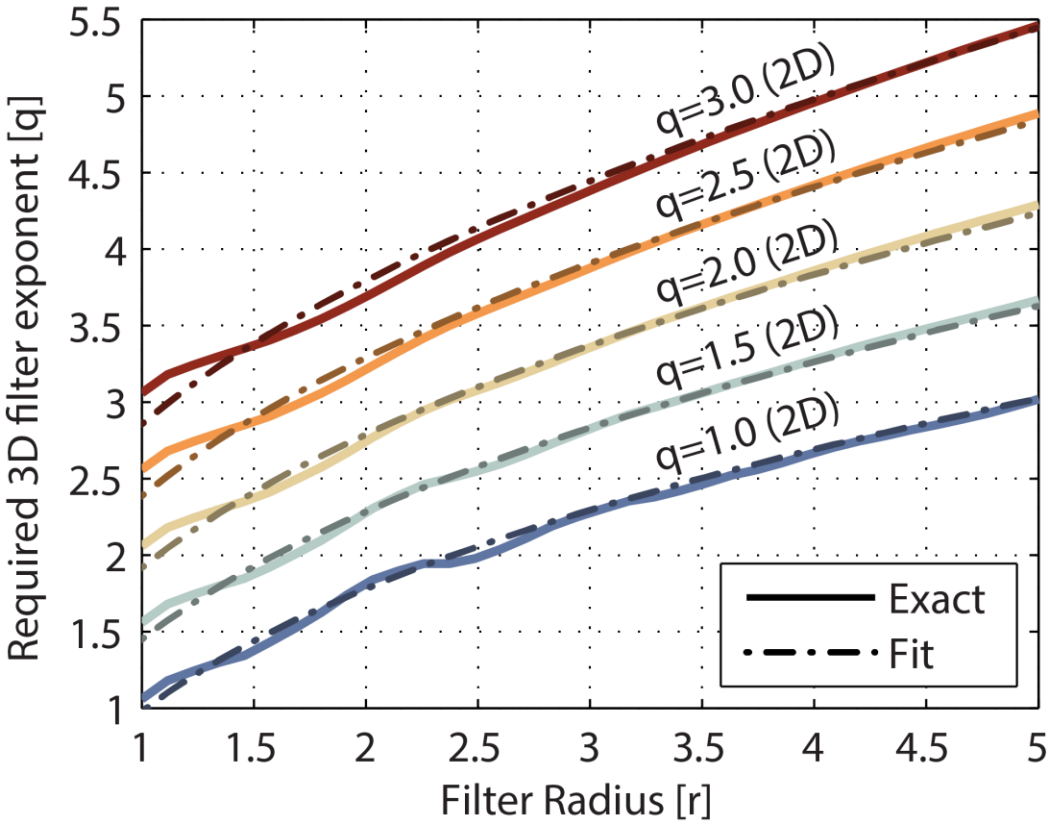
TWO–DIMENSIONS



THREE–DIMENSIONS
(Hii = 0.4194)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

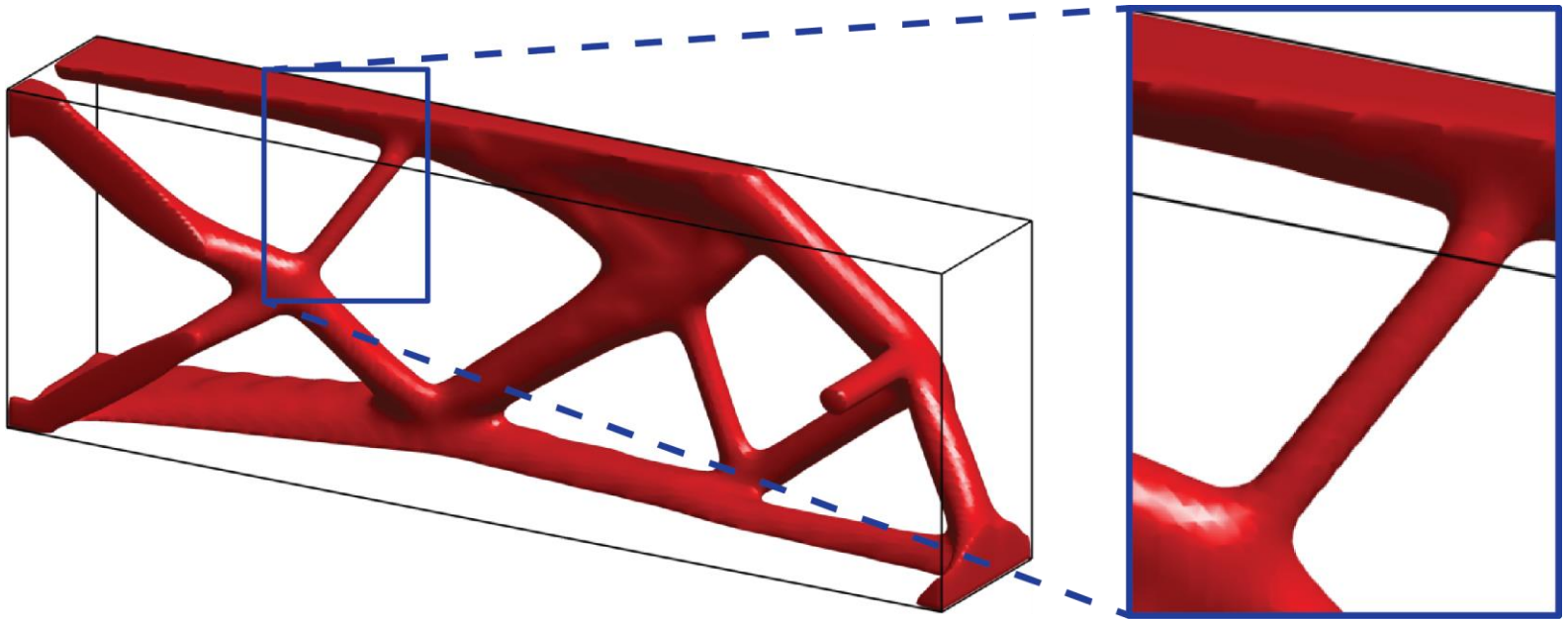
IDEA: WHAT EXPONENT Q MAKES $H_{ii}^{(2D)} = H_{ii}^{(3D)}$?



$$q^{(3D)} = \log\left(r_{min}\right) + \frac{17}{20}q^{(2D)} + \frac{4}{57}q^{(2D)}r_{min} + \frac{4}{87}r_{min}$$

6) ADDITIVE MANUF. OF OPT. STRUCTS.

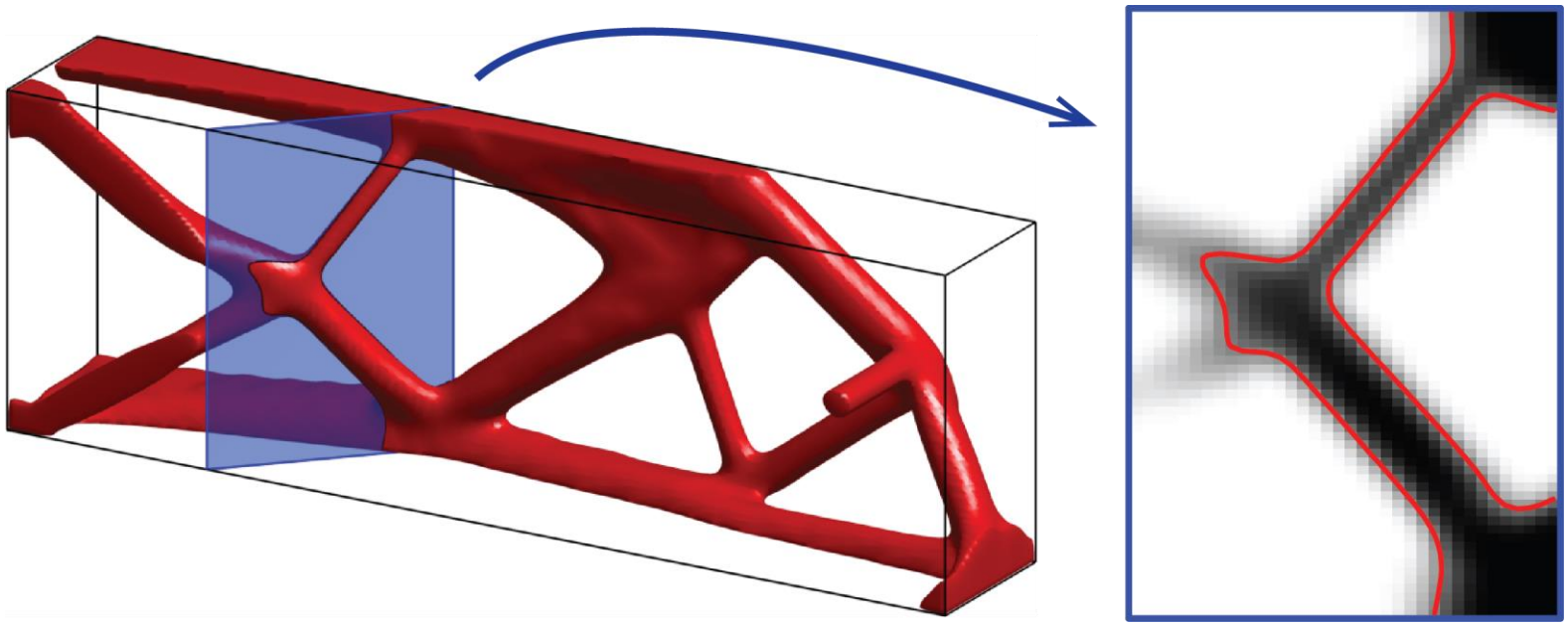
- EDGE-LOADED CANTILEVER
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=3 AND P=3



559,872 DVS FOR ½
(1,119,744 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

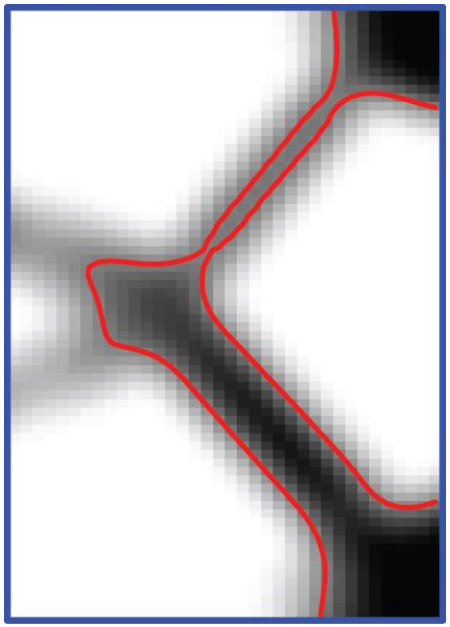
- EDGE-LOADED CANTILEVER
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=3 AND P=3



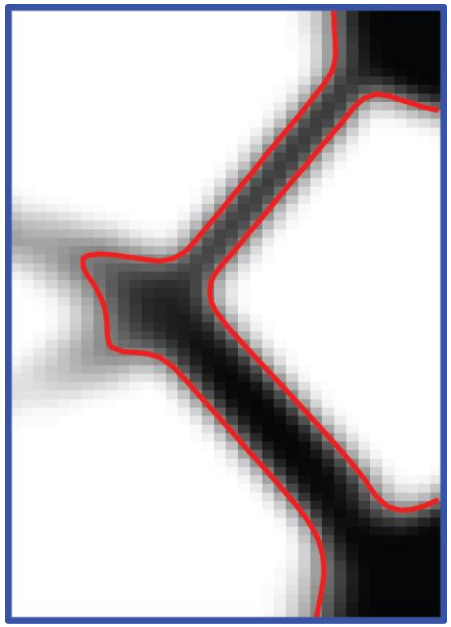
559,872 DVS FOR ½
(1,119,744 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- EDGE-LOADED CANTILEVER
  DENSITY FILTER: **R=6**



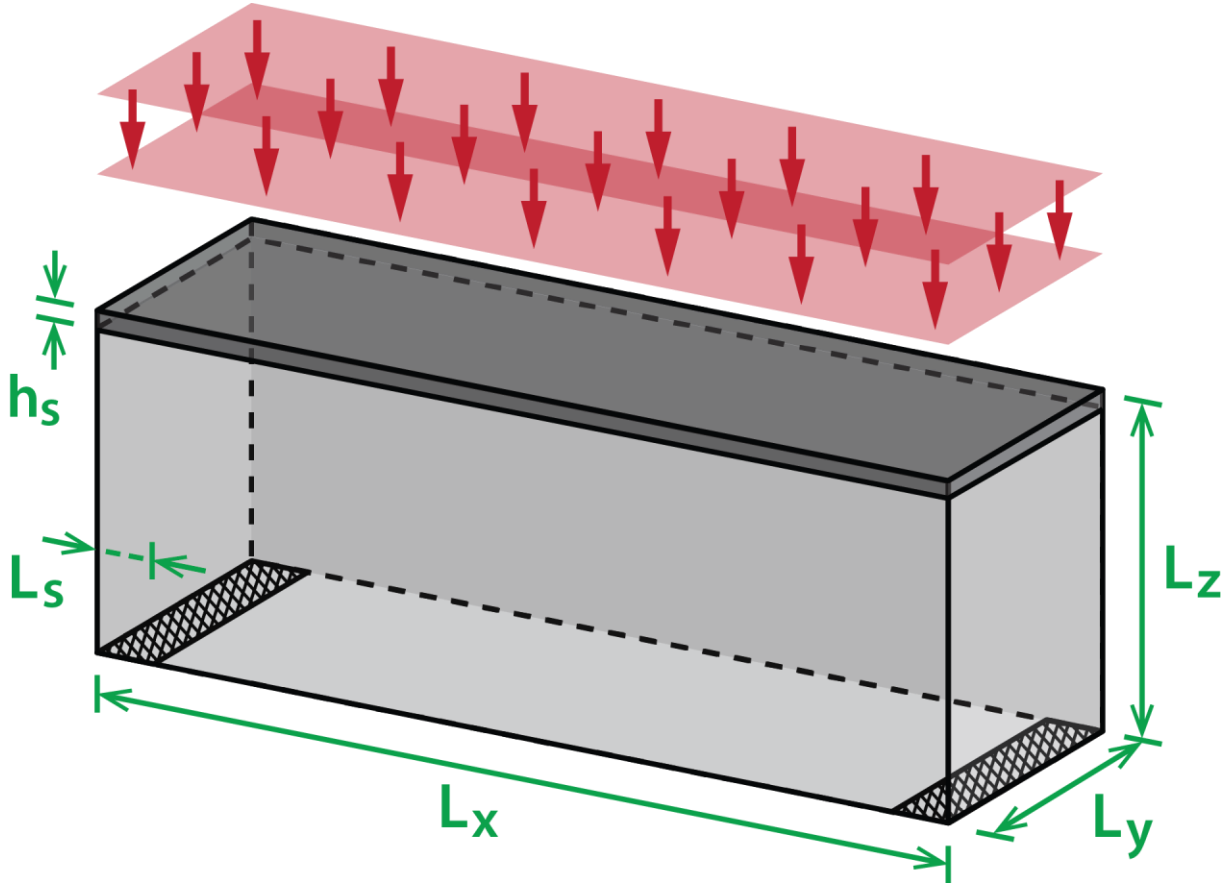
LINEAR DENSITY FILTER



CUBIC DENSITY FILTER

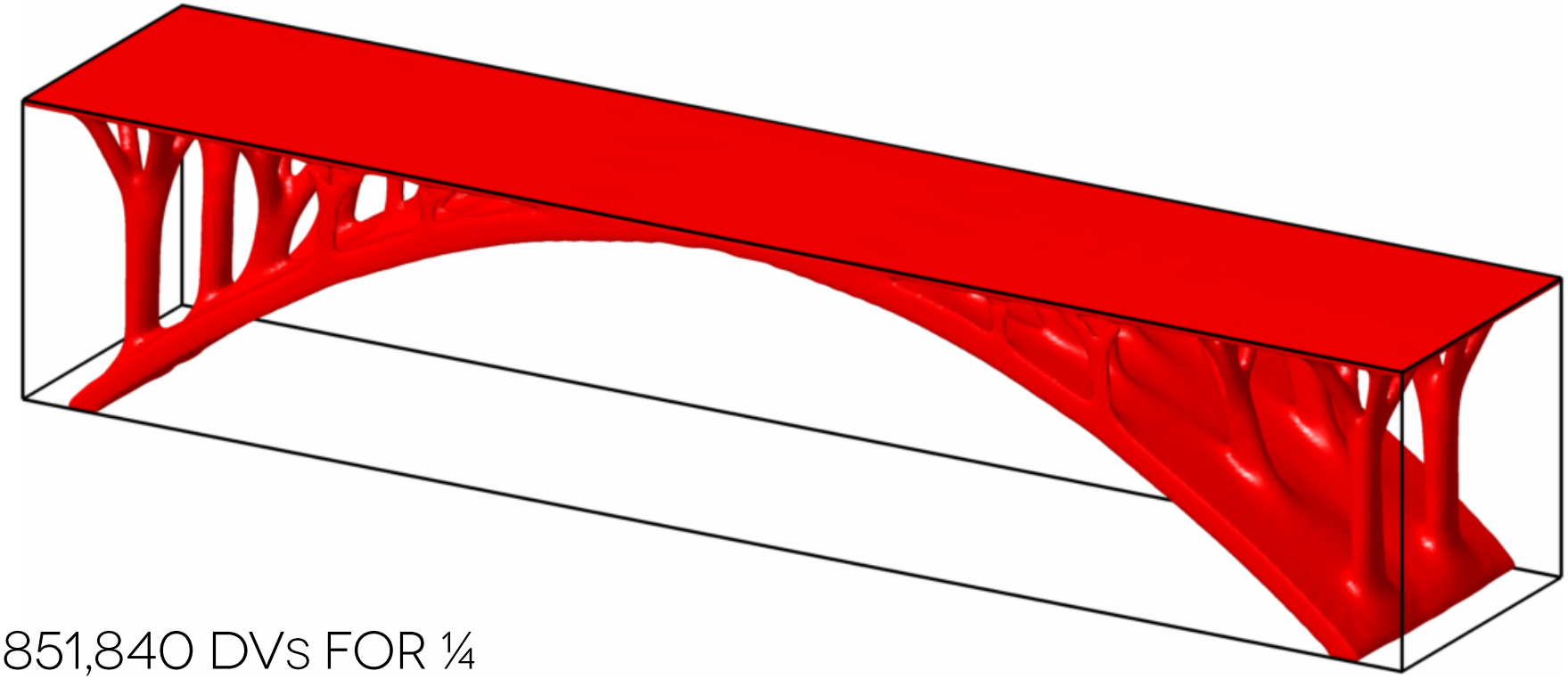
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  LX=XXX, LY=LZ=YYY, Q=ZZZ, R=5 AND VOLFRAC=10%



6) ADDITIVE MANUF. OF OPT. STRUCTS.

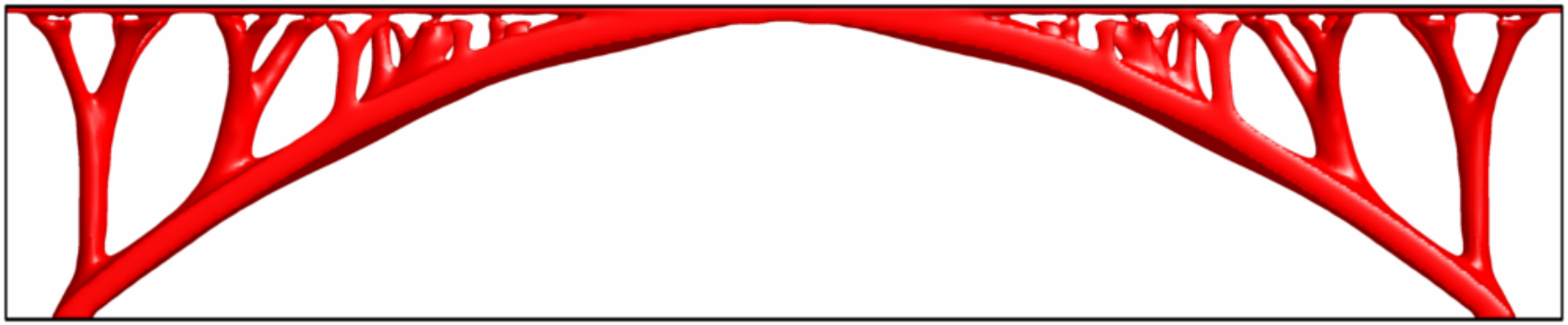
- BRIDGE PROBLEM
  LX=25, LY=LZ=5
  VOLFRAC=10%, R=5, Q=3 AND P=3



851,840 DVS FOR ¼
(3,407,360 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

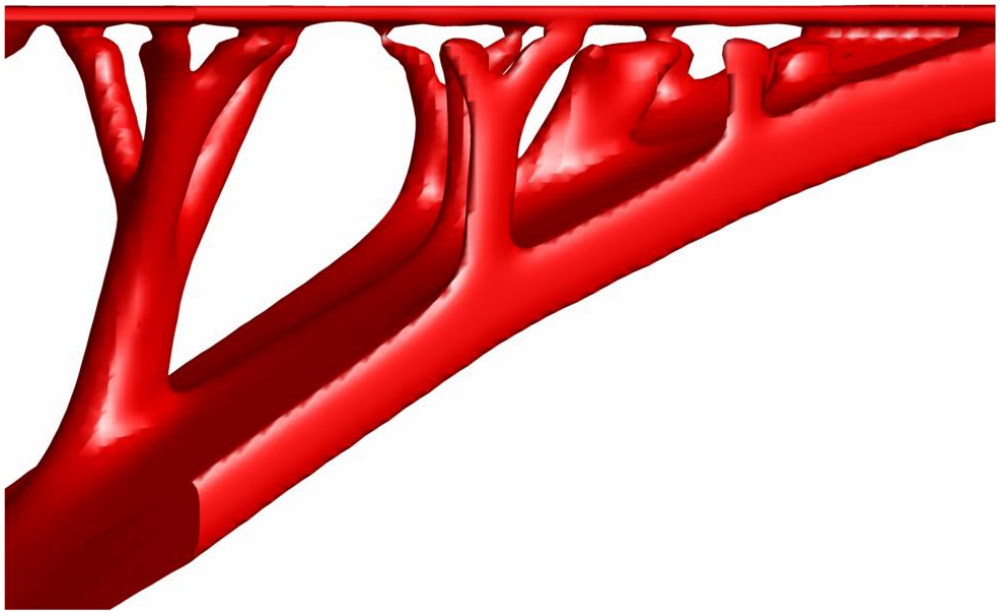
- BRIDGE PROBLEM
  LX=25, LY=LZ=5
  VOLFRAC=10%, R=5, Q=3 AND P=3



851,840 DVS FOR ¼
(3,407,360 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

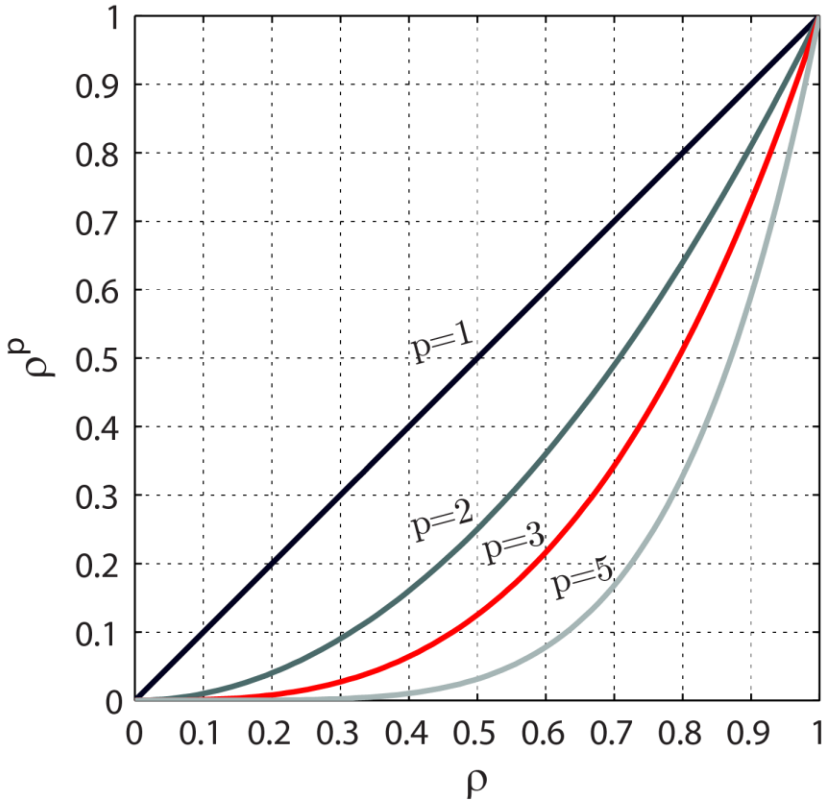
- BRIDGE PROBLEM
  Lx=25, Ly=Lz=5
  VolFrac=10%, R=5, Q=3 AND P=3



851,840 DVs FOR ¼
(3,407,360 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SIMP'S POWER-LAW:



SIMP:

$$E_i(\rho_i) = \rho_i^p E_0$$

$$0 \lesssim \rho_{min} \leq \rho_j \leq 1$$

MODIFIED SIMP:

$$E_k(\rho_k) = E_{min} + \rho_k^p (E_0 - E_{min})$$

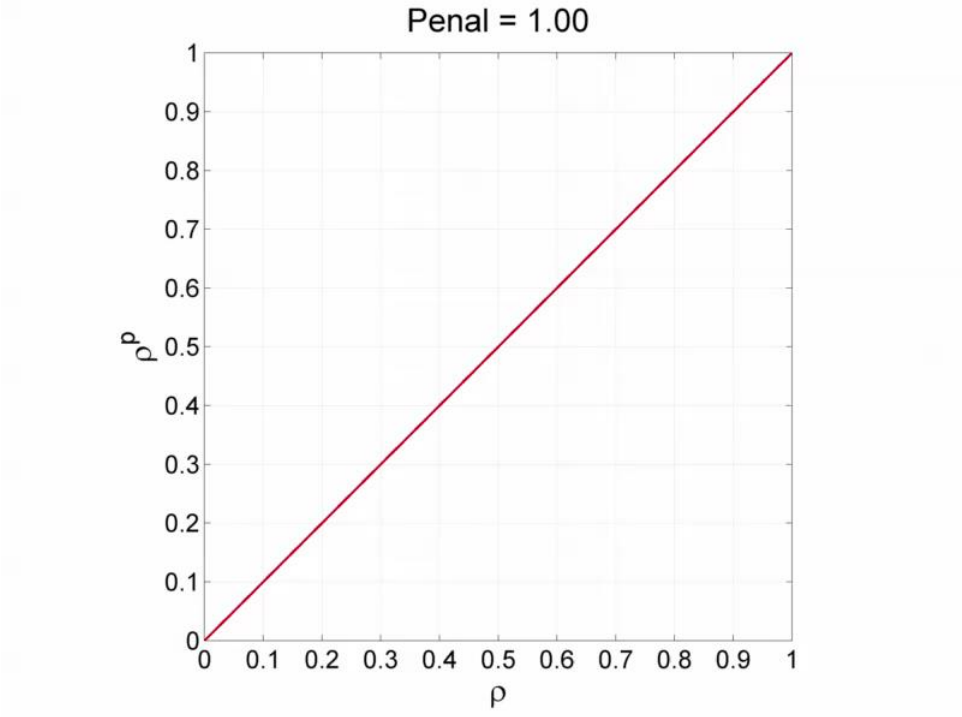
$$0 \leq \rho_j \leq 1$$

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193–202

3) ZHOU M, ROZVANY G (1991) "THE COC ALGORITHM, PART II: TOPOLOGICAL, GEOMETRICAL AND GENERALIZED SHAPE OPTIMIZATION. "COMP METH APPL MECH ENGRG 89:309–336

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- CONTINUATION OF “P” PARAMETER



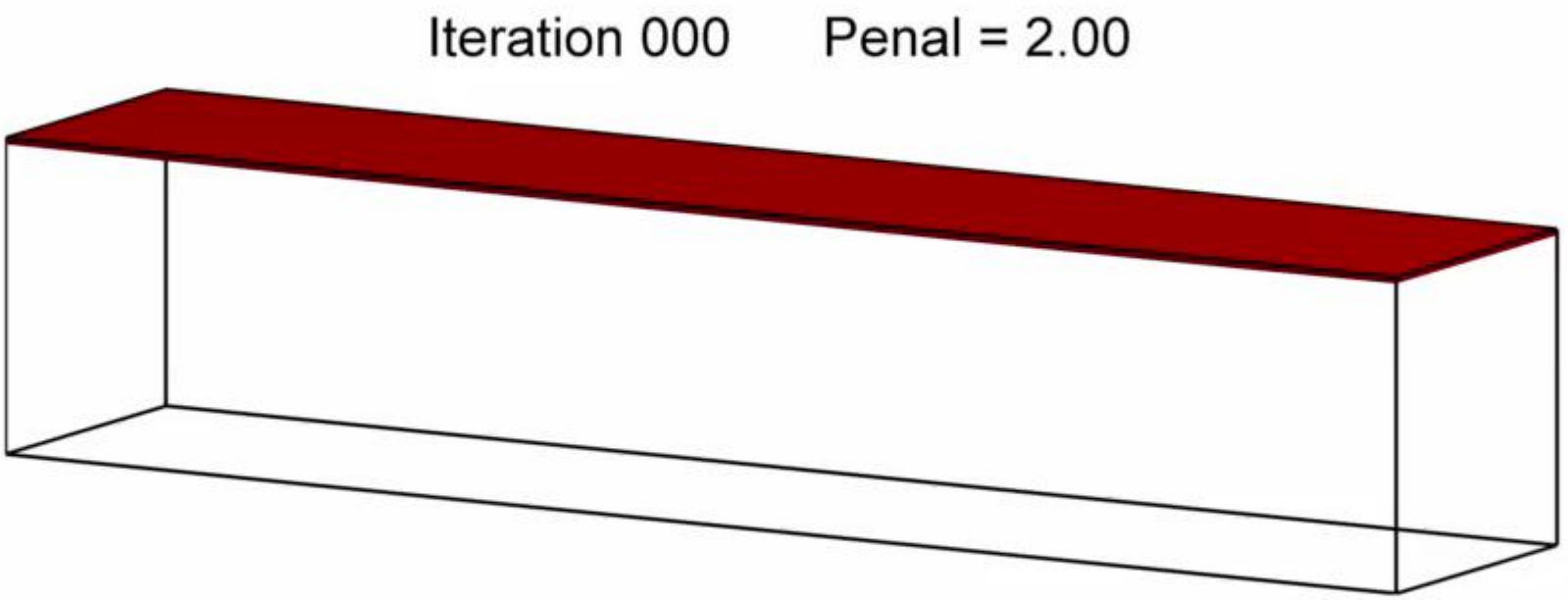
ALLAIRE G, FRANCFORT G (1993) "A NUMERICAL ALGORITHM FOR TOPOLOGY AND SHAPE OPTIMIZATION." IN TOPOLOGY DESIGN OF STRUCTURES, SPRINGER

ALLAIRE G, KOHN R (1993) "TOPOLOGY OPTIMIZATION AND OPTIMAL SHAPE DESIGN USING HOMOGENIZATION." IN TOPOLOGY DESIGN OF STRUCTURES, SPRINGER

SIGMUND O, PETERSSON J (1998) "NUMERICAL INSTABILITIES IN TOPOLOGY OPTIMIZATION" STRUCTURAL OPTIMIZATION, 16(1):68–75

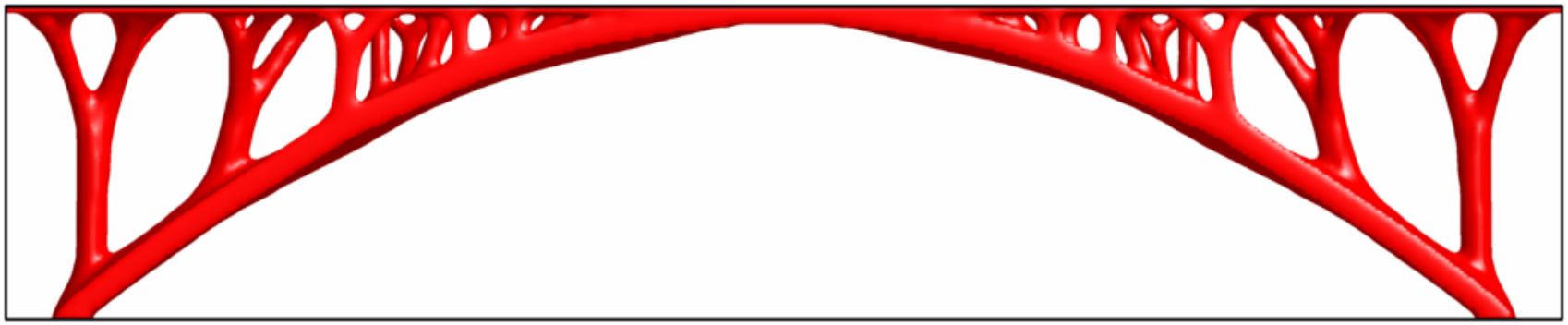
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- BRIDGE PROBLEM
  Lx=25, Ly=Lz=5, VolFrac=10%, R=5, Q=3 and P=[cont]



6) ADDITIVE MANUF. OF OPT. STRUCTS.

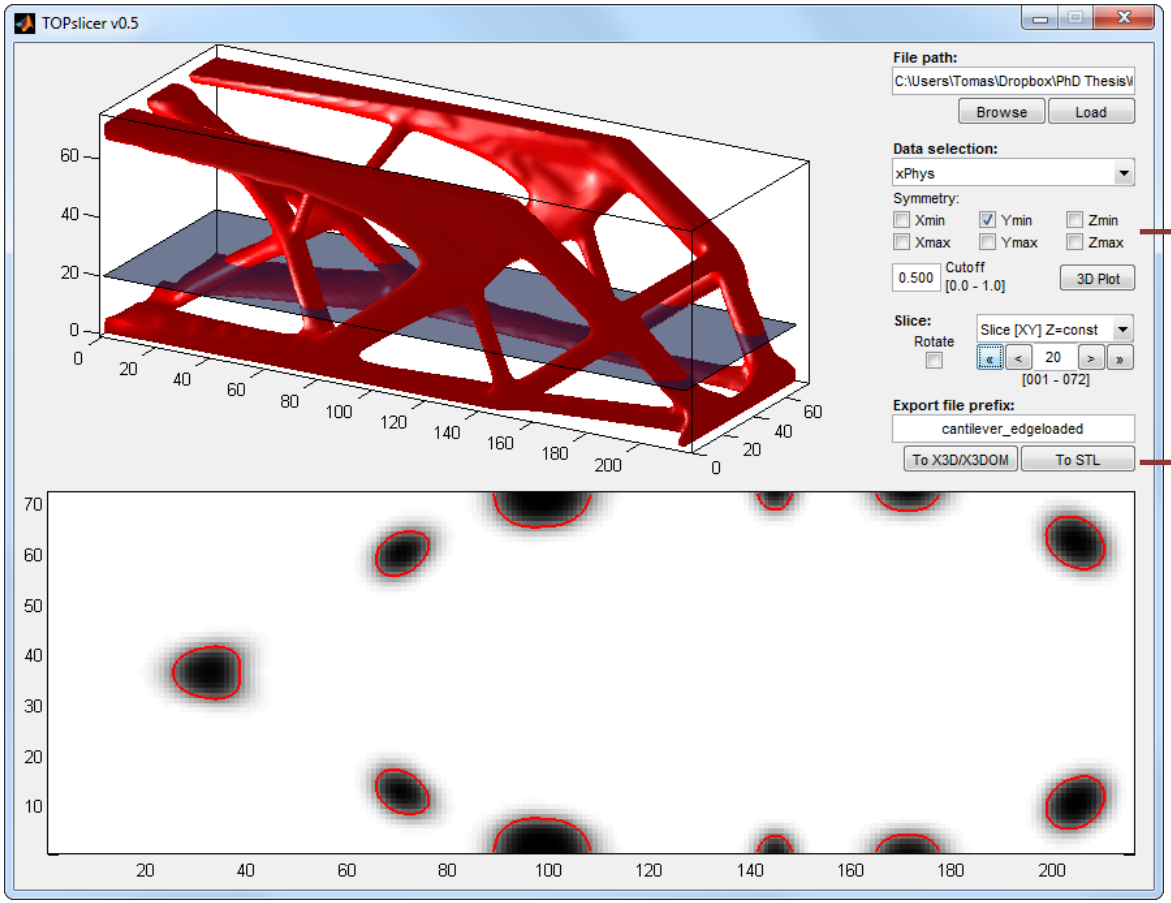
- BRIDGE PROBLEM
  Lx=25, Ly=Lz=5, VolFrac=10%, R=5, Q=3 and P=[cont]



851,840 DVs FOR ¼
(3,407,360 TOTAL)

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- TOPSLICER: AN INSPECTOR/EXPORTER OF 3D DENSITY-BASED TOPOLOGIES

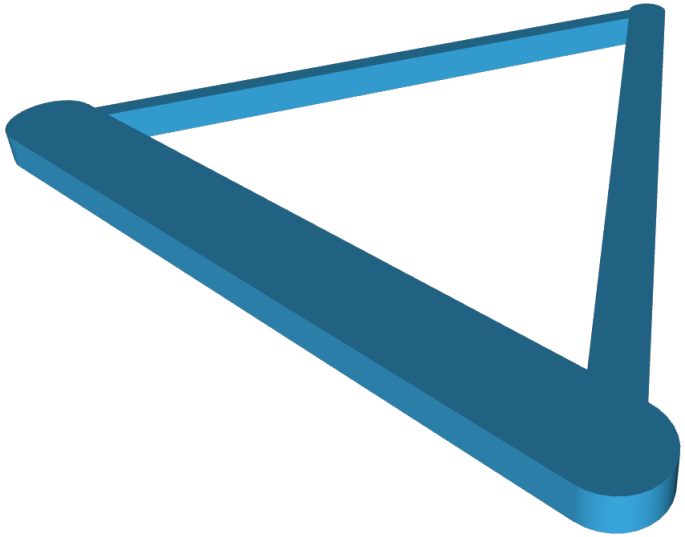


RESTORE SYMMETRY

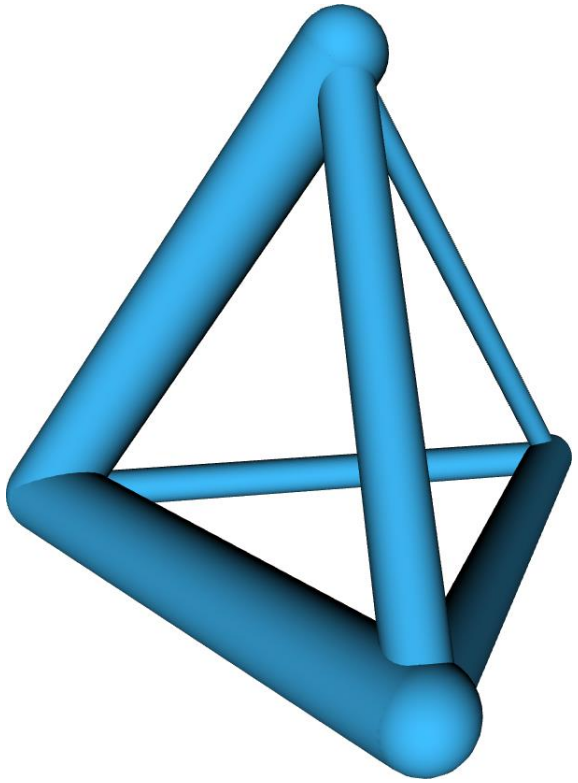
EXPORT TO X3D OR STL

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- MANUFACTURING OF GROUND STRUCTURES



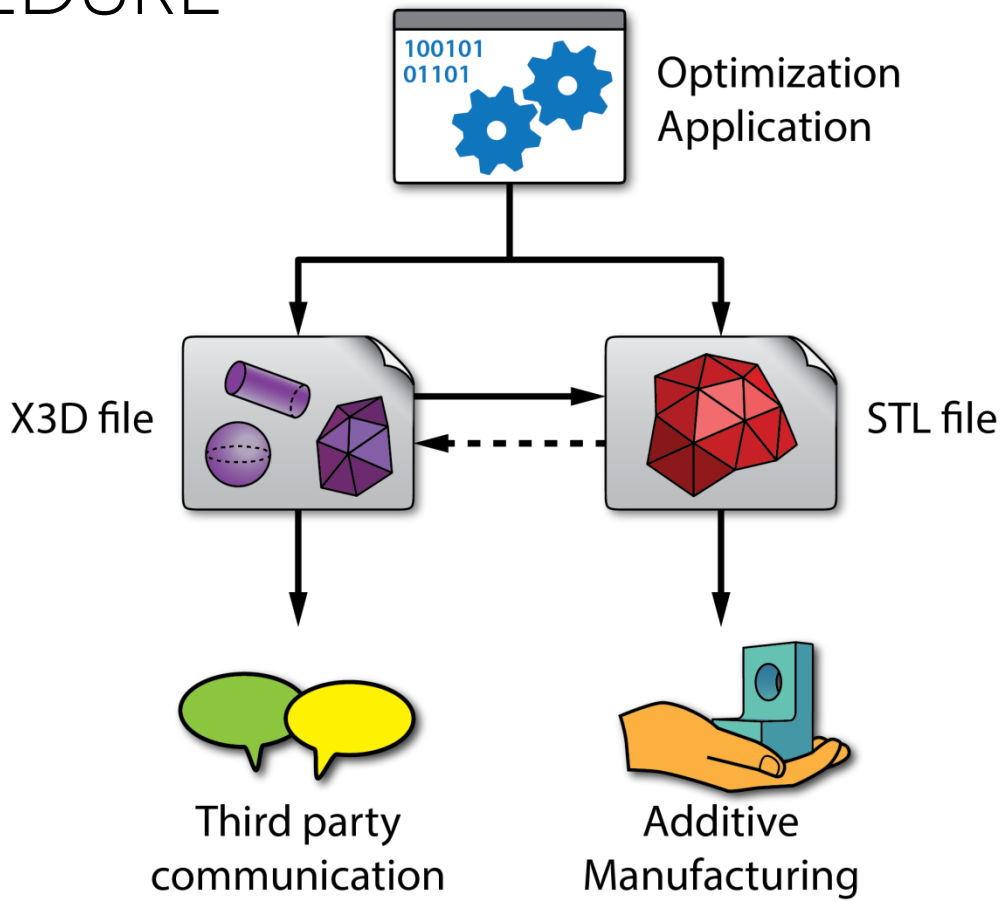
2D GROUND STRUCTURES



3D GROUND STRUCTURES

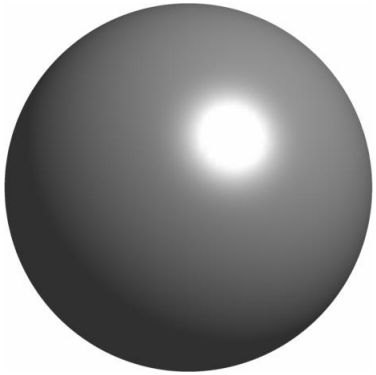
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- PROCEDURE

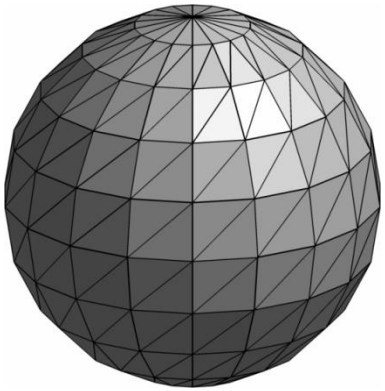


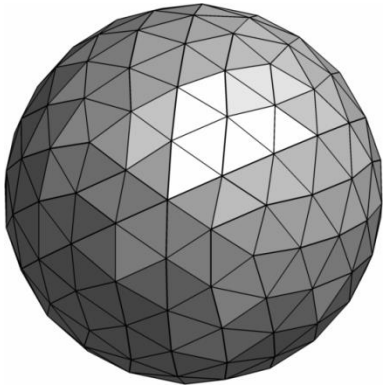
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- X3D SUPPORTS IMPLICIT 3D PRIMITIVES



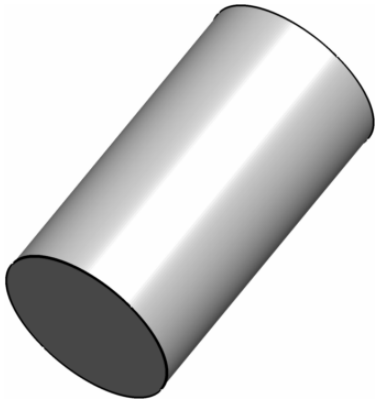
- STL ONLY SUPPORTS DISCRETIZED (TESSELLATED) SOLIDS



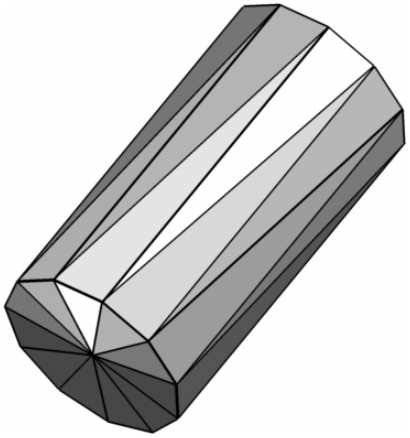


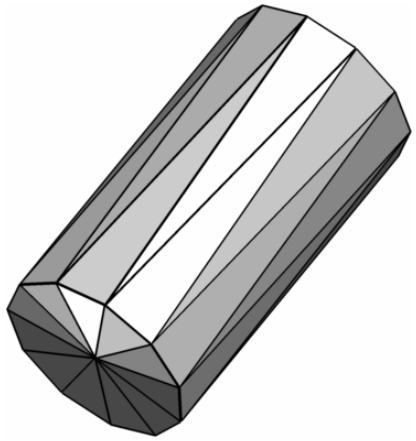
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- X3D SUPPORTS IMPLICIT 3D PRIMITIVES



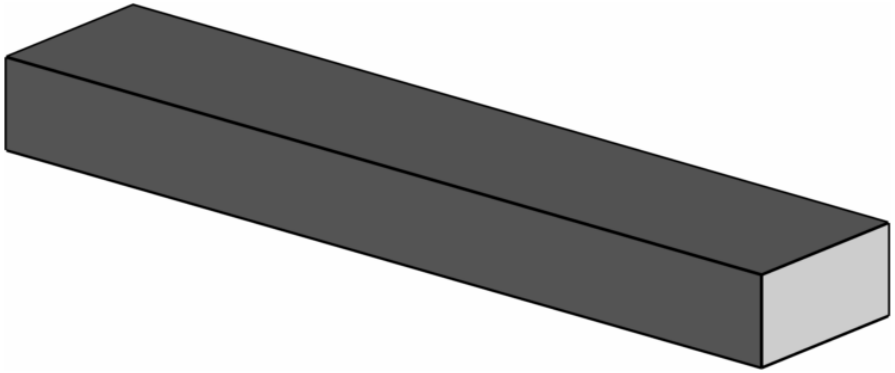
- STL ONLY SUPPORTS DISCRETIZED (TESSELLATED) SOLIDS



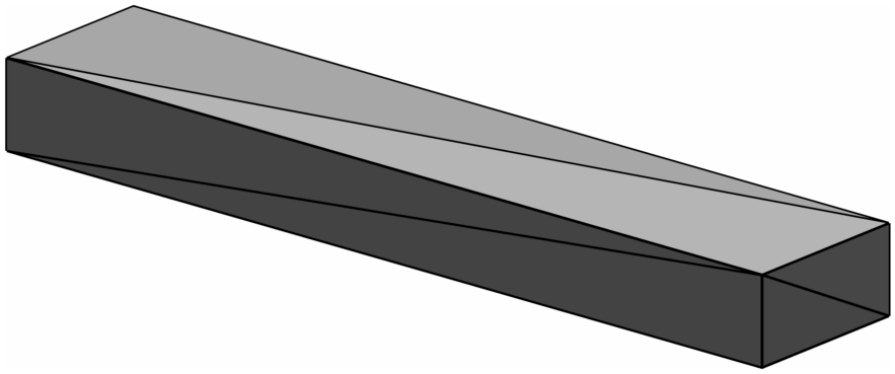


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- X3D SUPPORTS IMPLICIT 3D PRIMITIVES

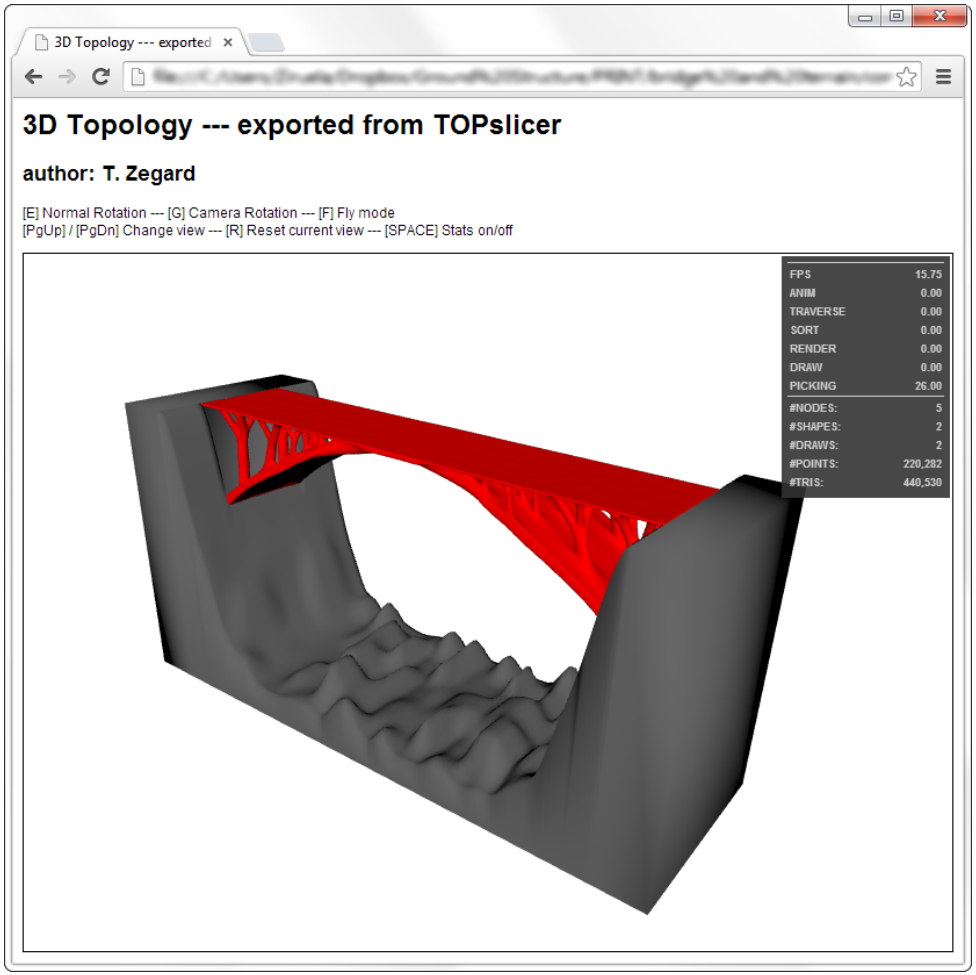


- STL ONLY SUPPORTS DISCRETIZED (TESSELLATED) SOLIDS



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- X3D: ROYALTY-FREE FORMAT FOR REPRESENTING 3D COMPUTER GRAPHICS. MANAGED BY THE WEB3D CONSORTIUM.



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL:

- COLOR CODE

  WHITE — 3D GROUND STRUCTURES

  BLUE — 2D GROUND STRUCTURES

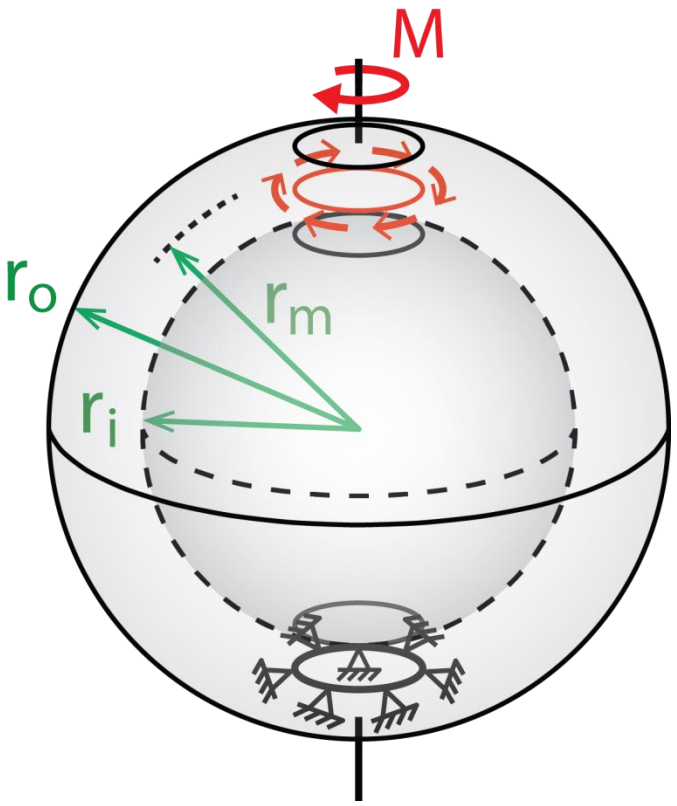
  RED — 3D DENSITY METHOD

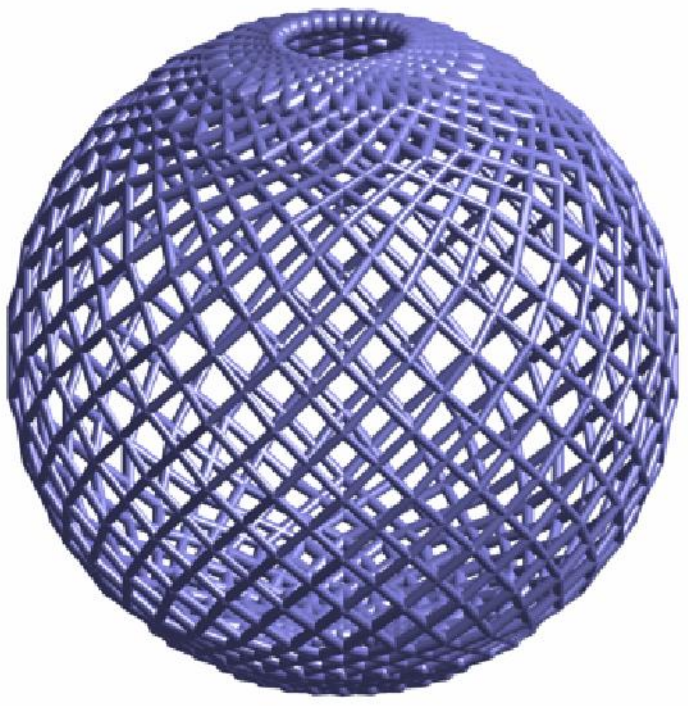
  BLACK — APPLICATION-ORIENTED

- MANUFACTURED USING:
  - FDM: FUSED DEPOSITION MODELING
  - SLS: SELECTIVE LASER SINTERING

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION BALL



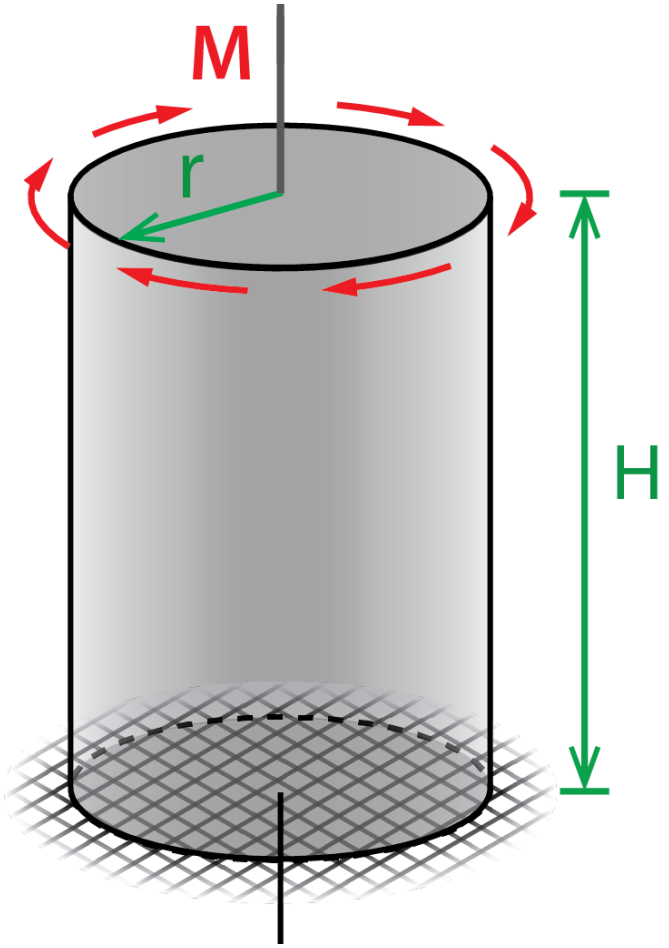


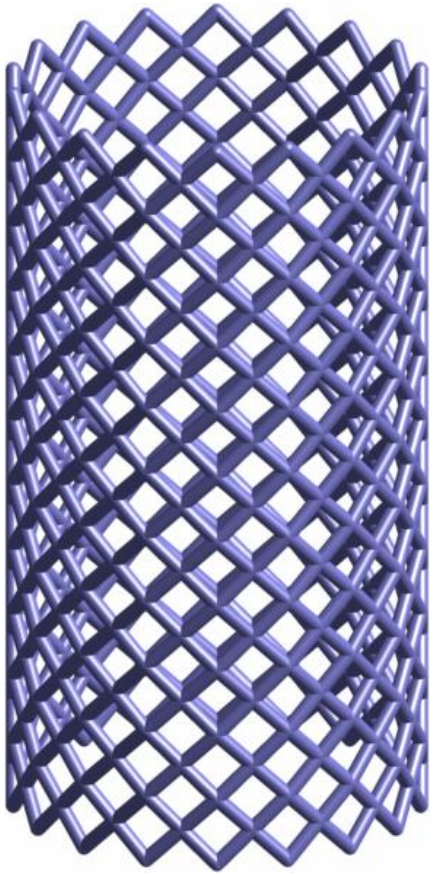
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION BALL

6) ADDITIVE MANUF. OF OPT. STRUCTS.

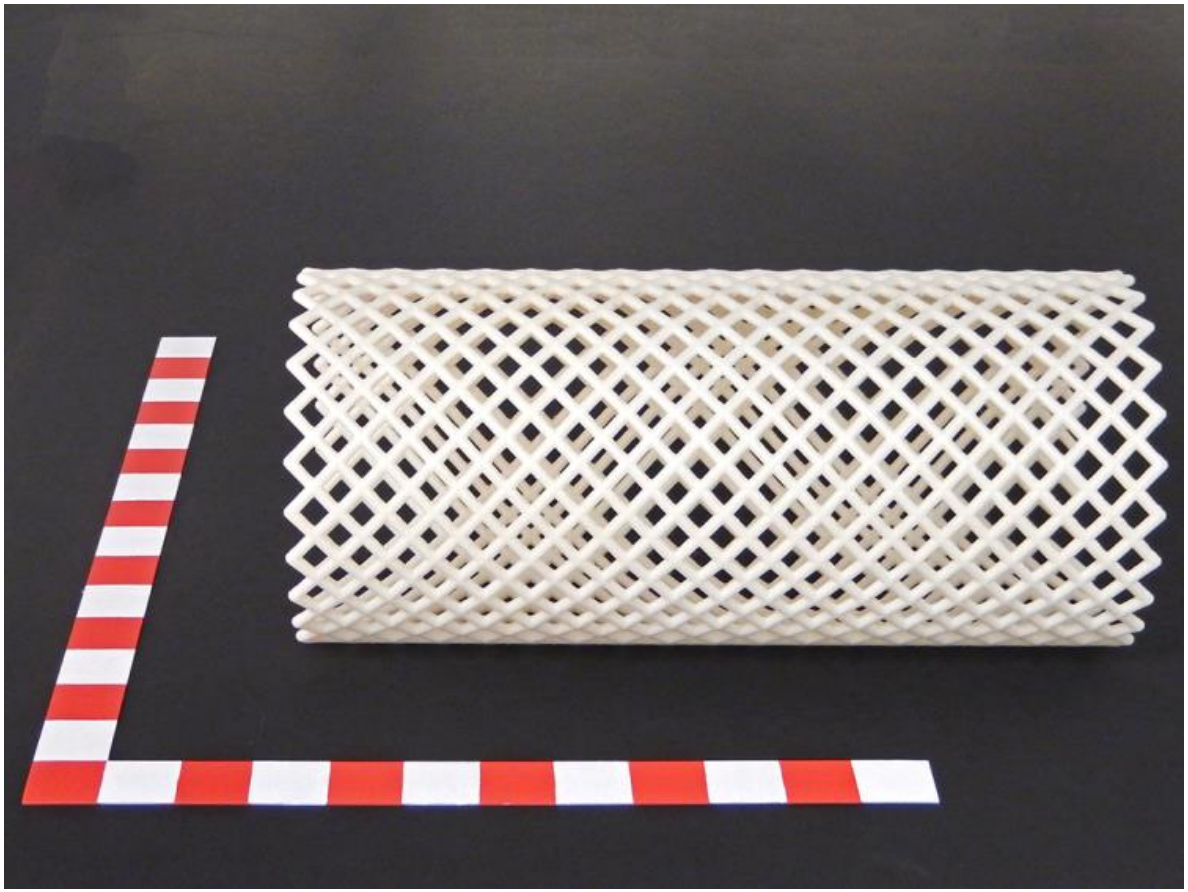
- SHOW AND TELL: TORSION CYLINDER





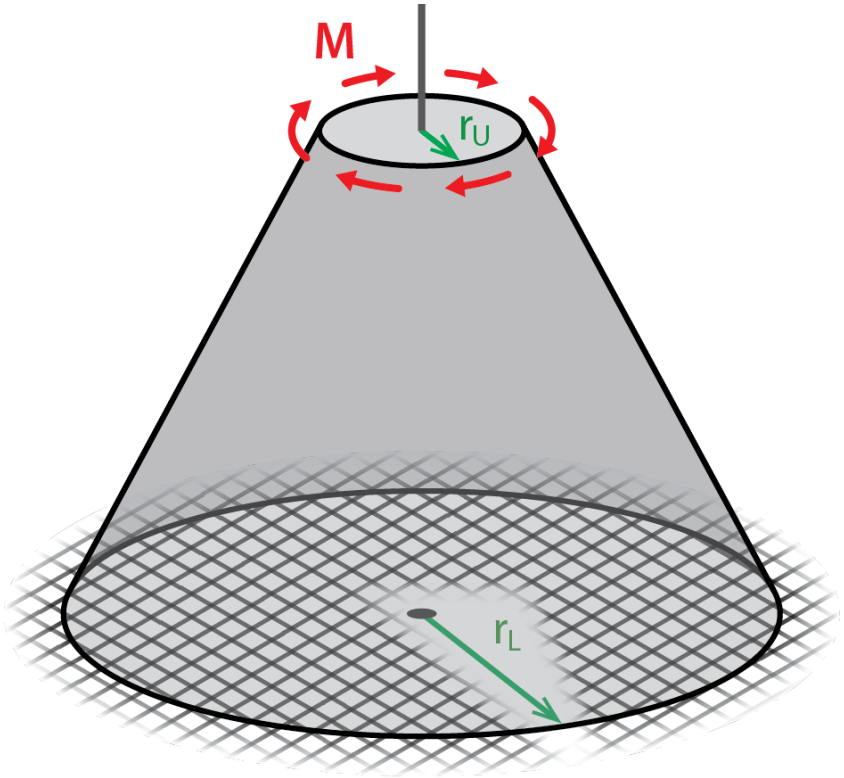
6) ADDITIVE MANUF. OF OPT. STRUCTS.

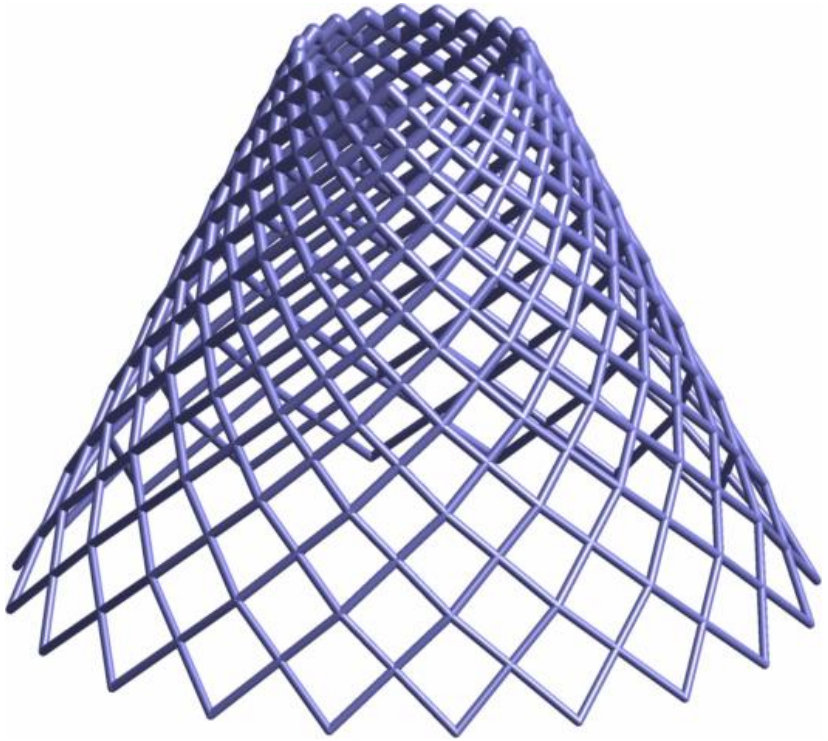
- SHOW AND TELL: TORSION CYLINDER



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: TORSION CONE





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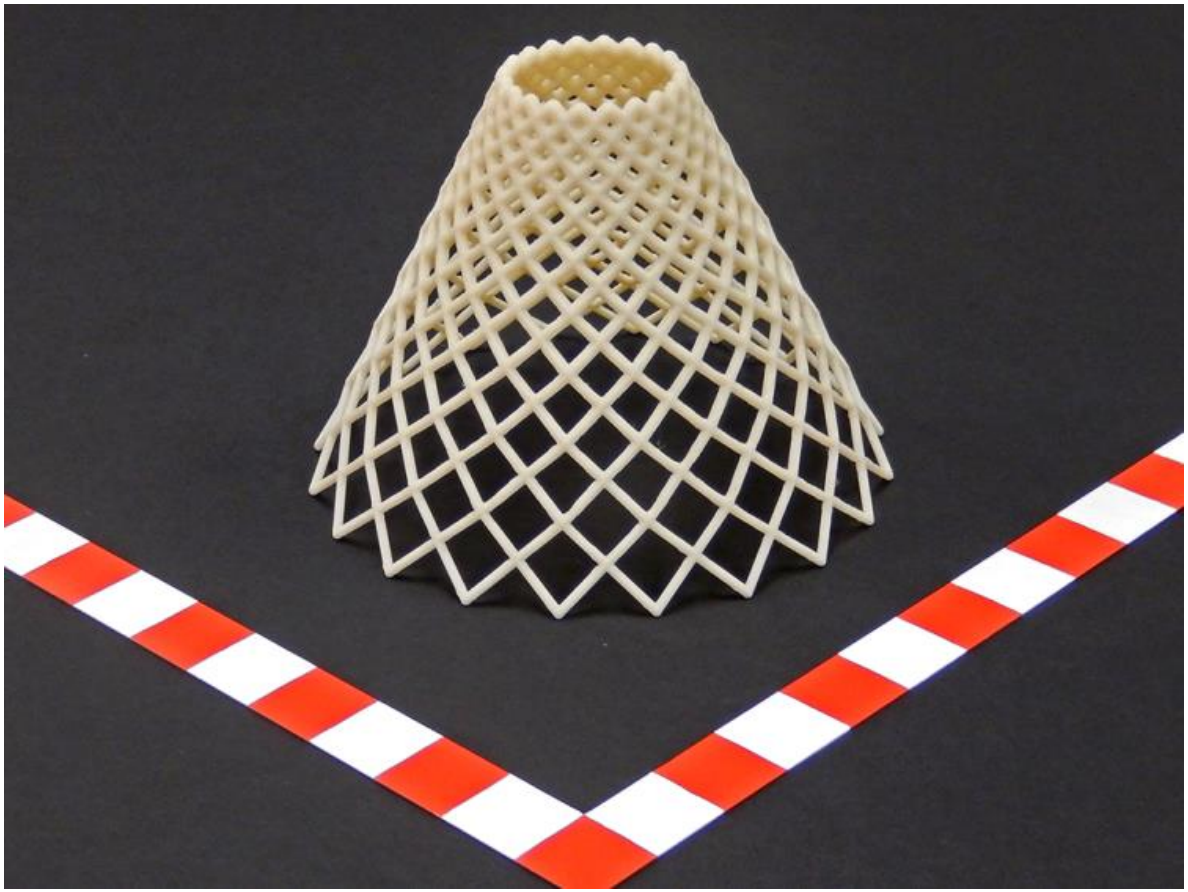
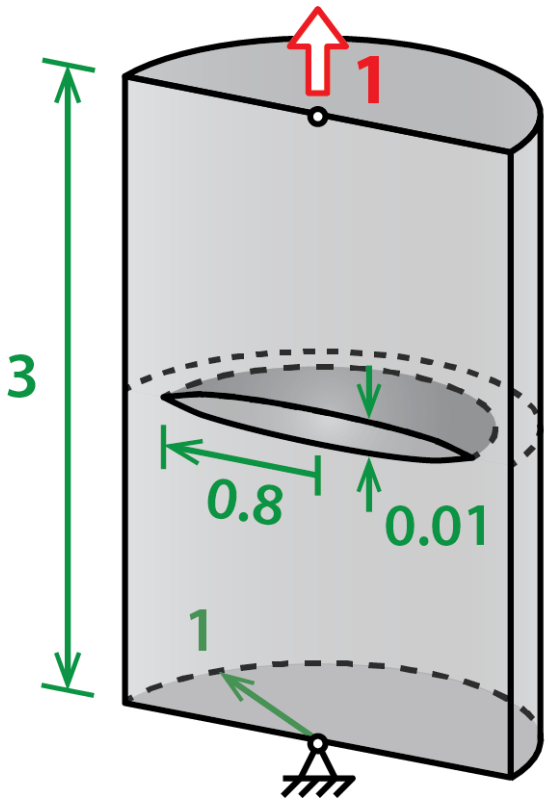
- SHOW AND TELL: TORSION CONE

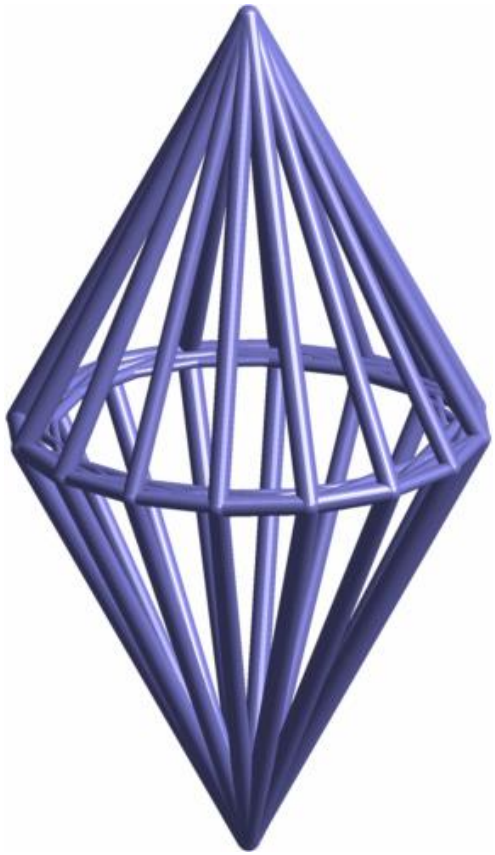


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: DIAMOND



NOTE: HALF-DOMAIN



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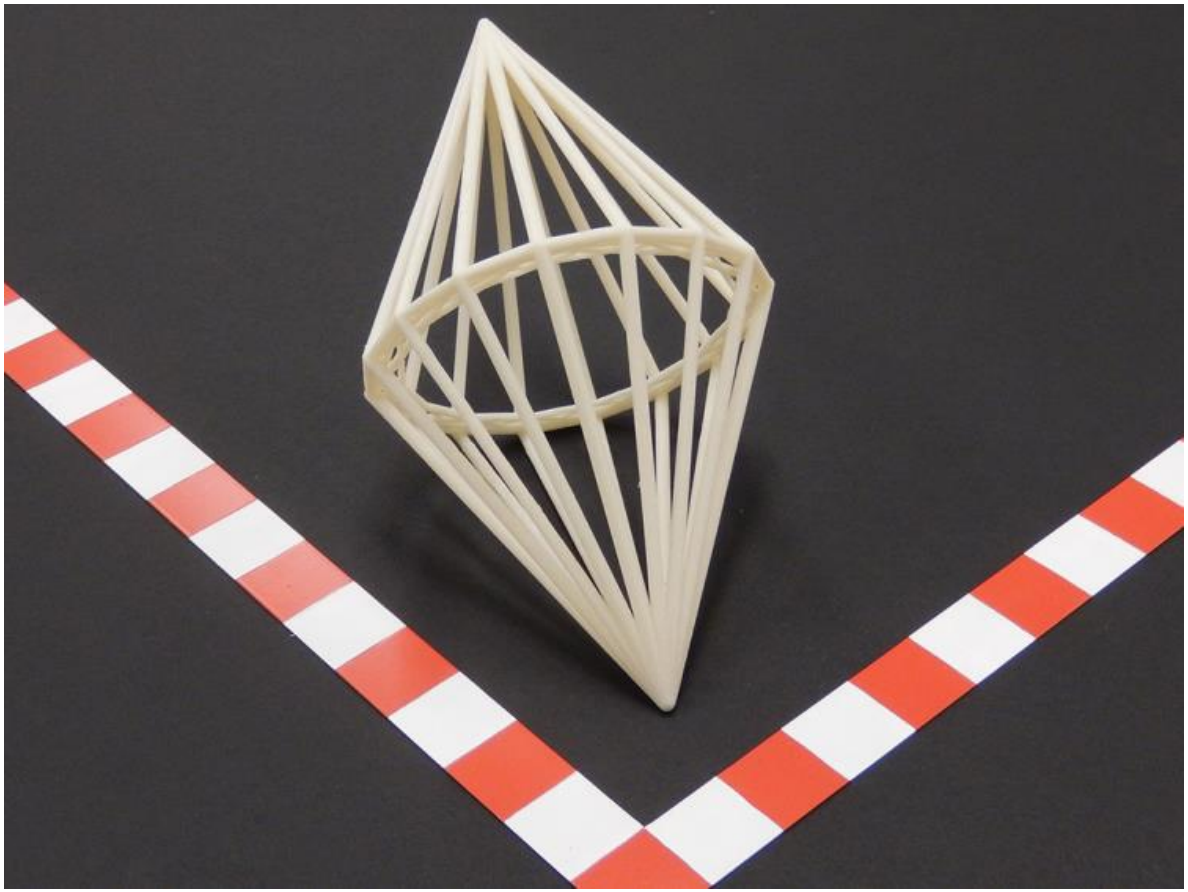
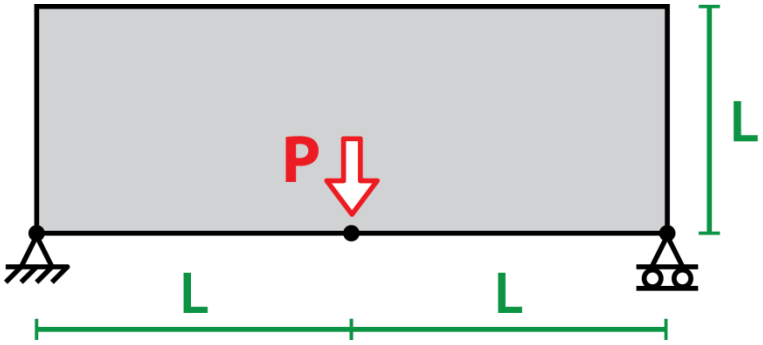
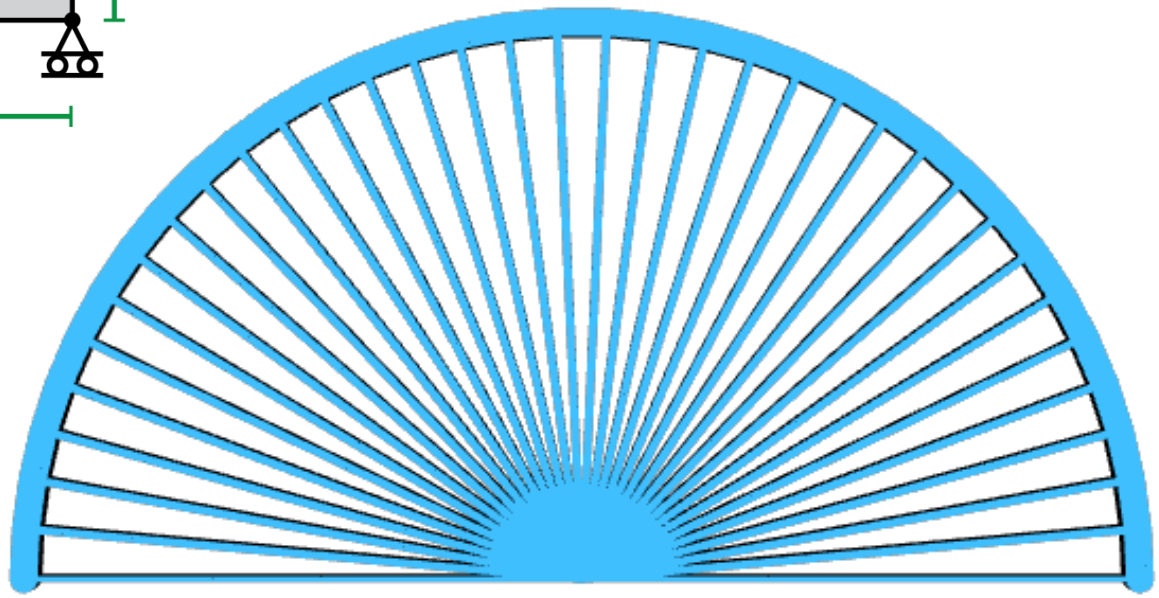
- SHOW AND TELL: DIAMOND



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: PINWHEEL





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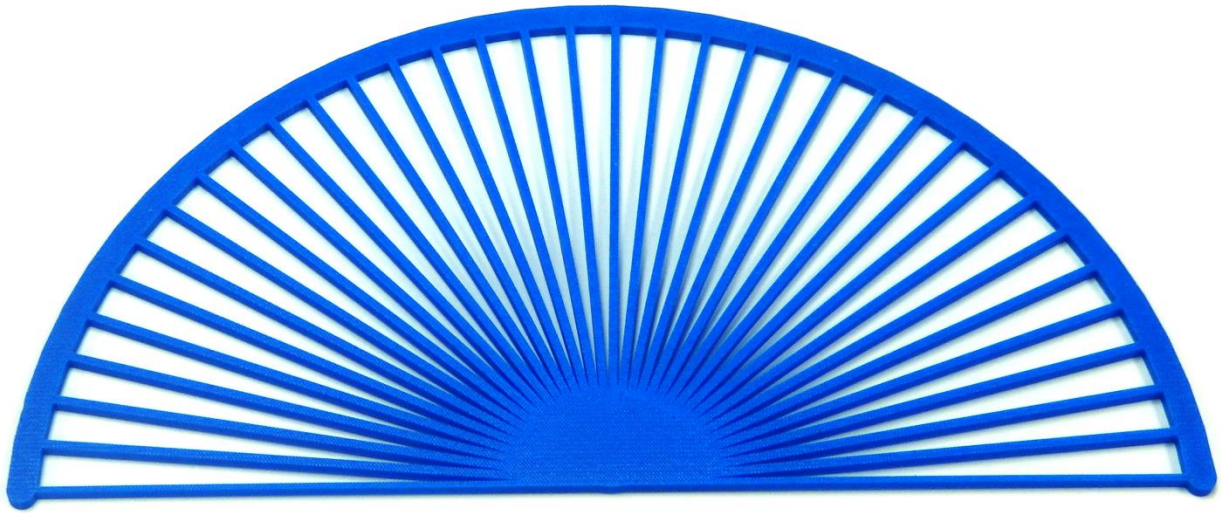
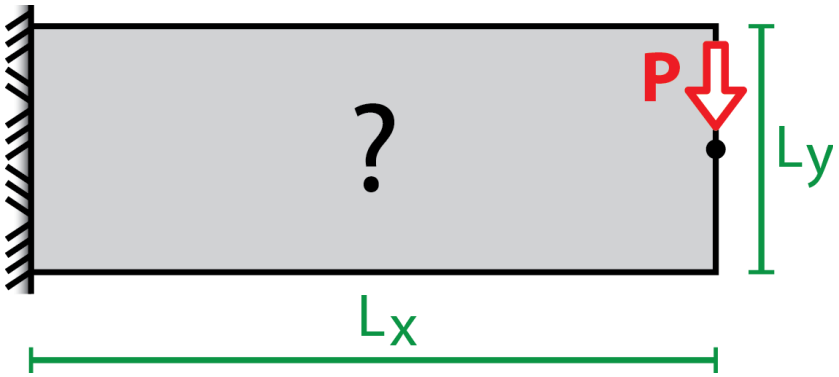
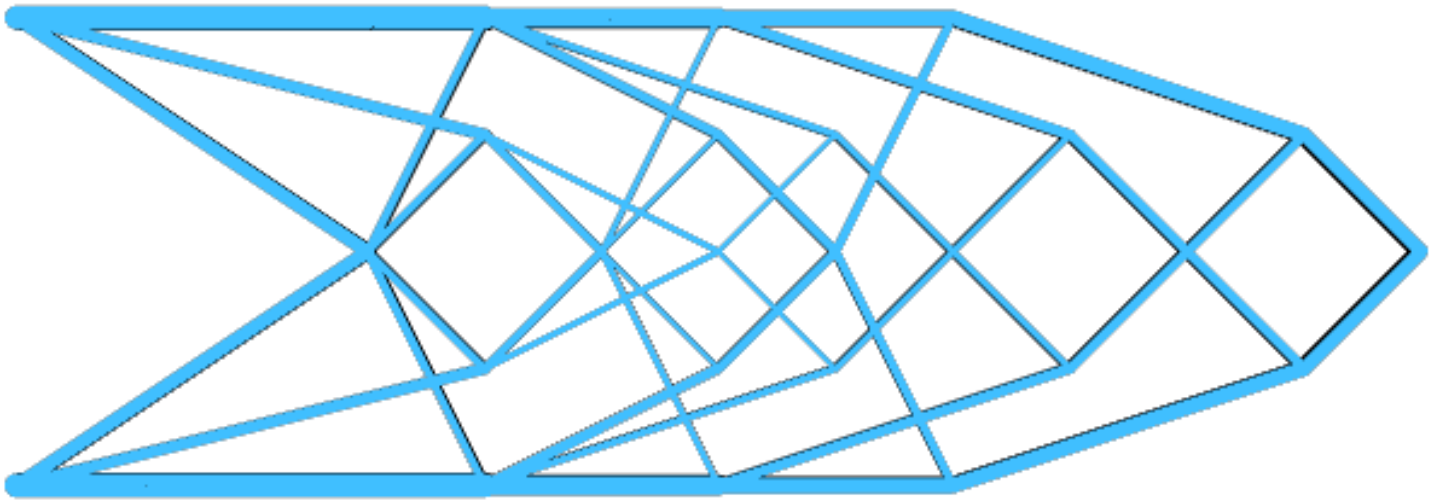
- SHOW AND TELL: PINWHEEL



6) ADDITIVE MANUF. OF OPT. STRUCTS.

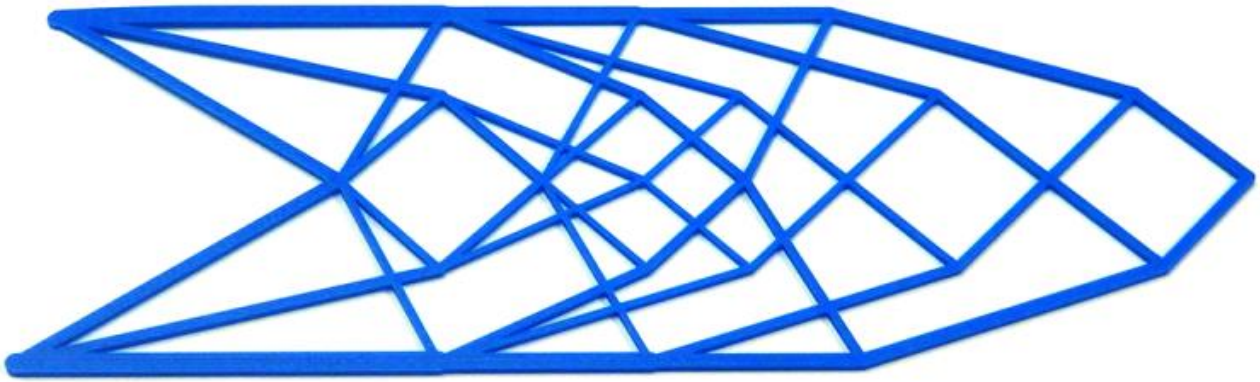
- SHOW AND TELL: CANTILEVER





6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: CANTILEVER



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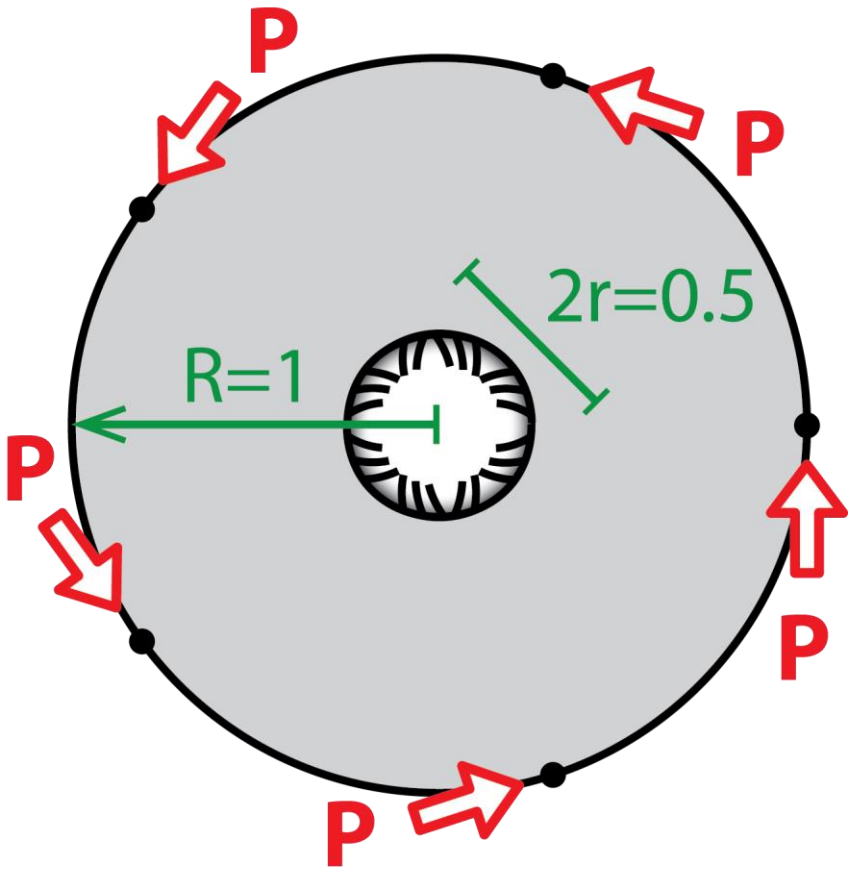
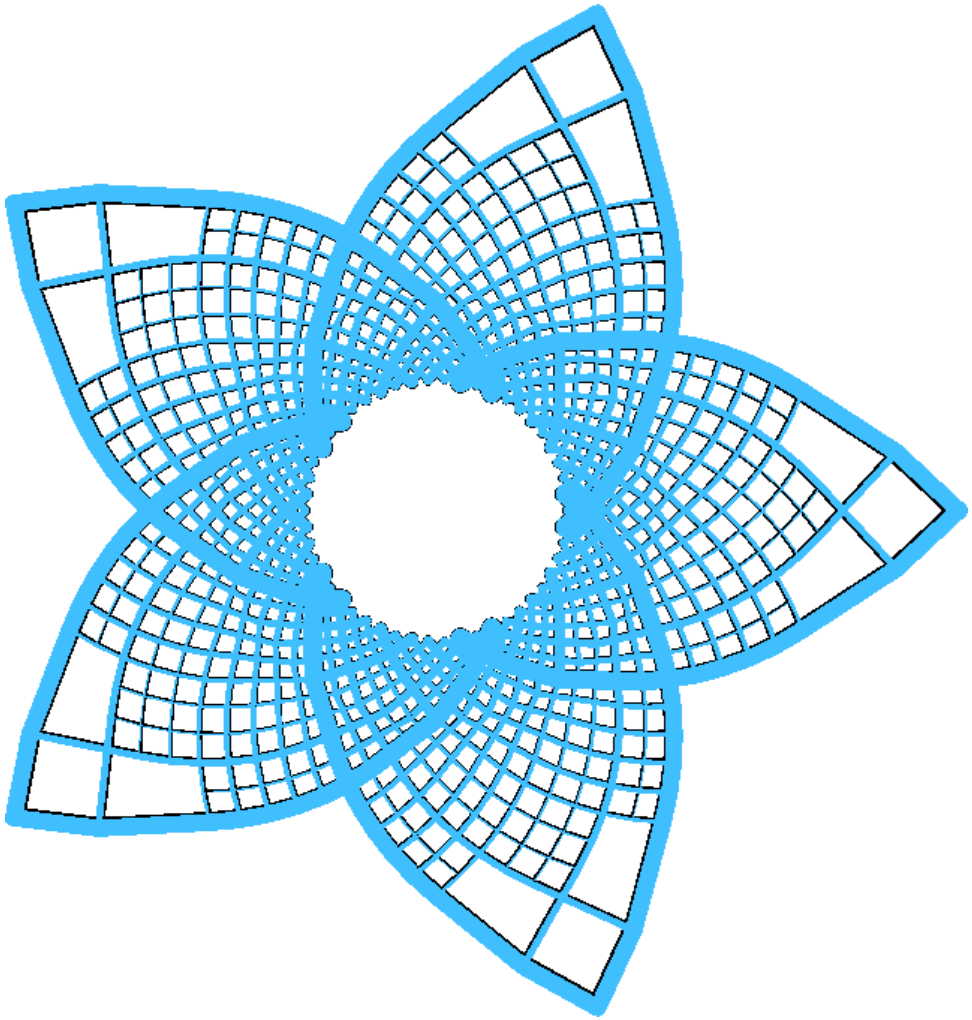
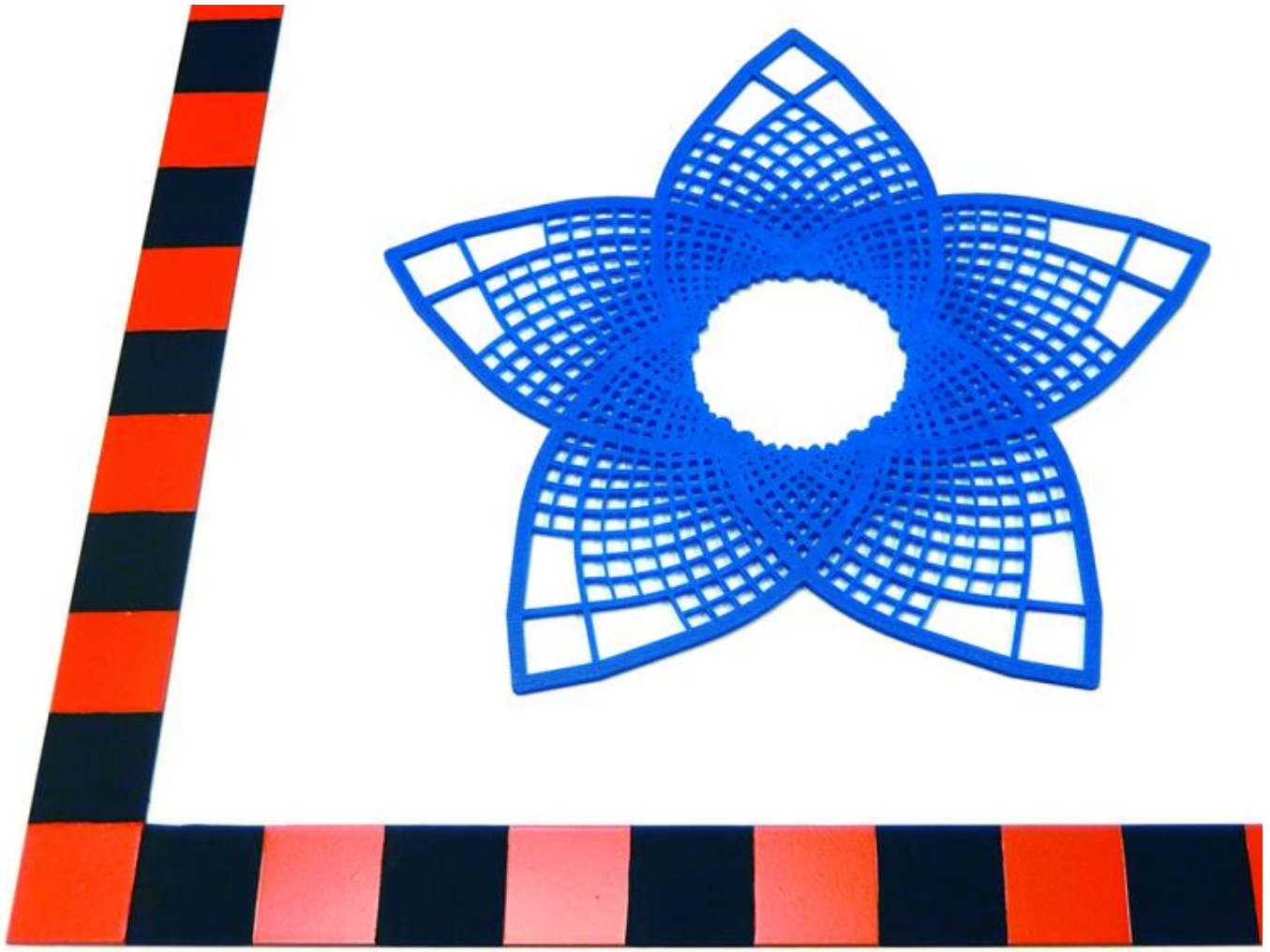
- SHOW AND TELL: FLOWER





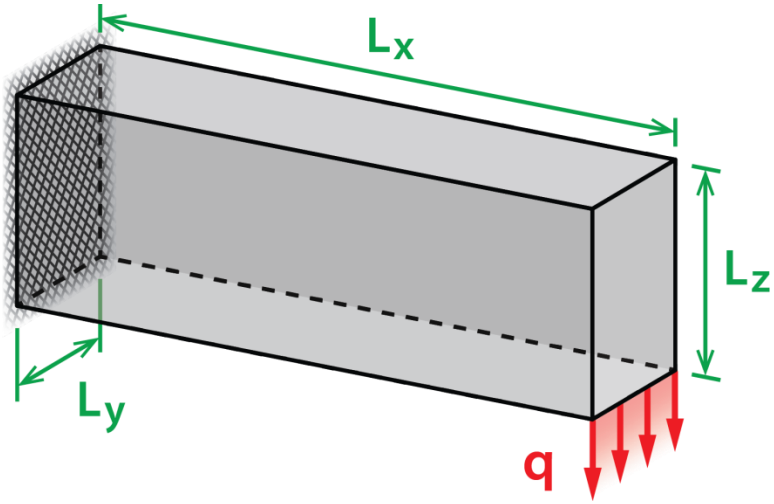
6) ADDITIVE MANUF. OF OPT. STRUCTS.

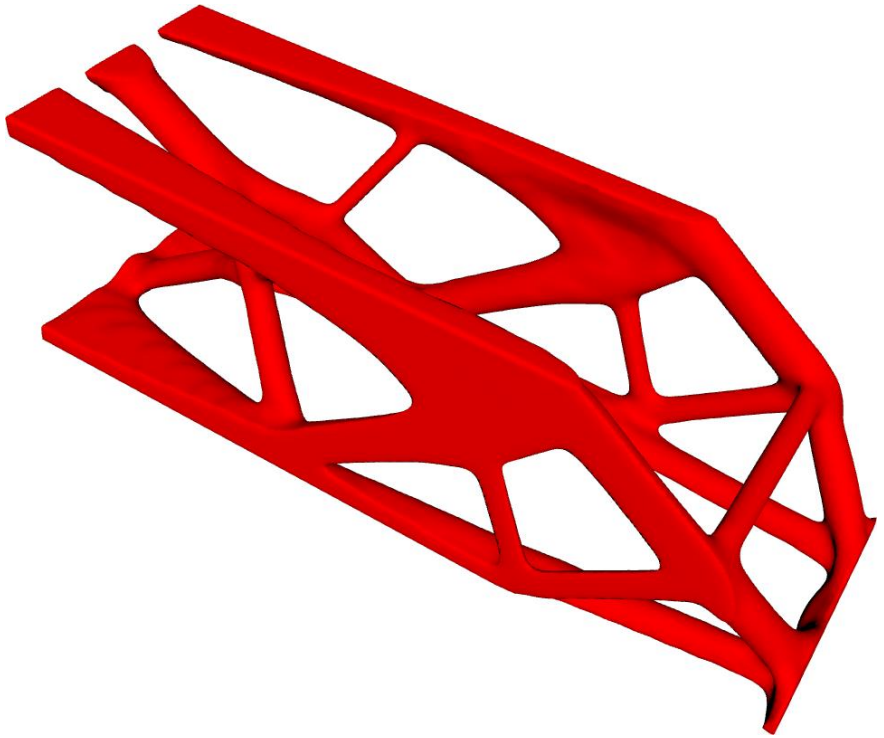
- SHOW AND TELL: FLOWER



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL:
  EDGE-LOADED 3D CANTILEVER (NO FIX)





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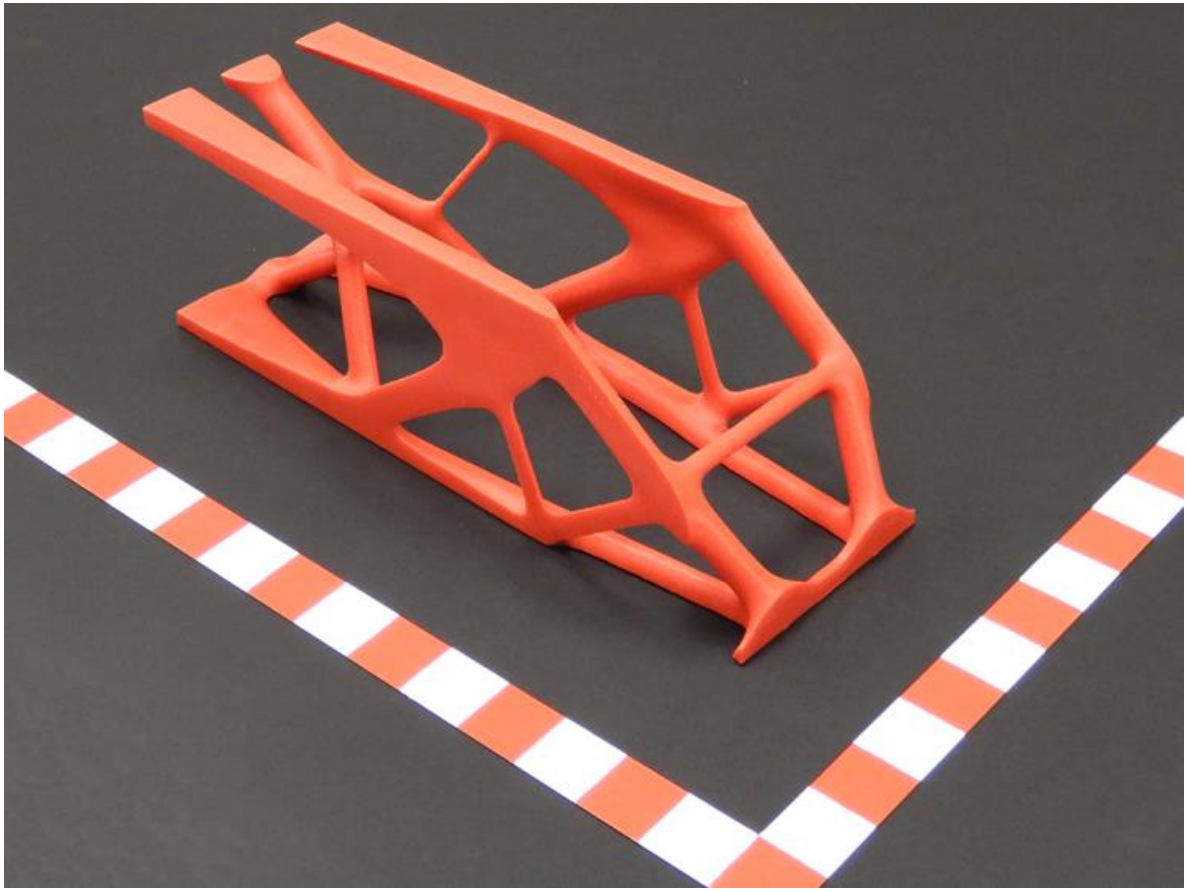
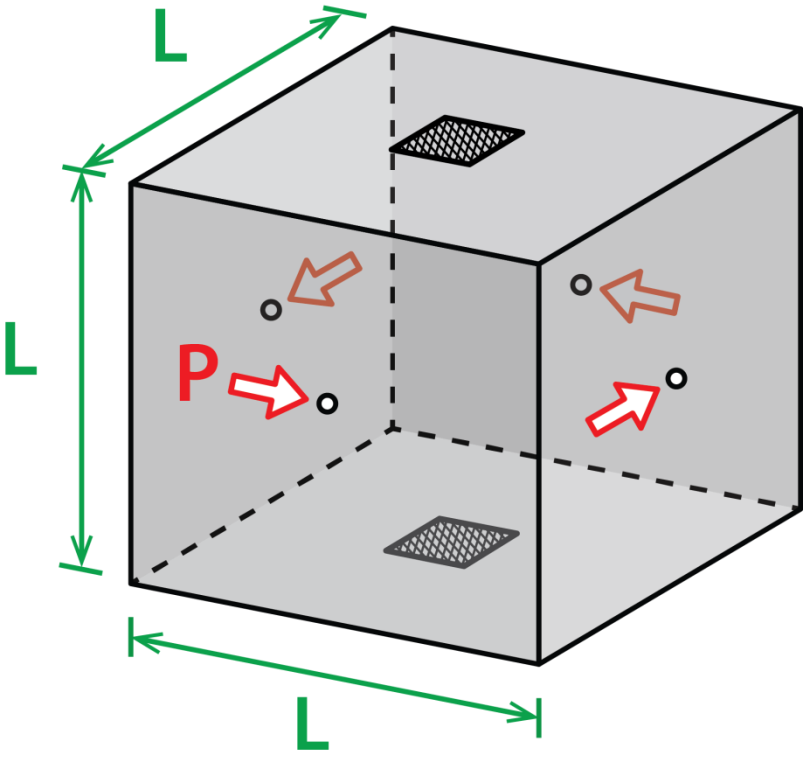
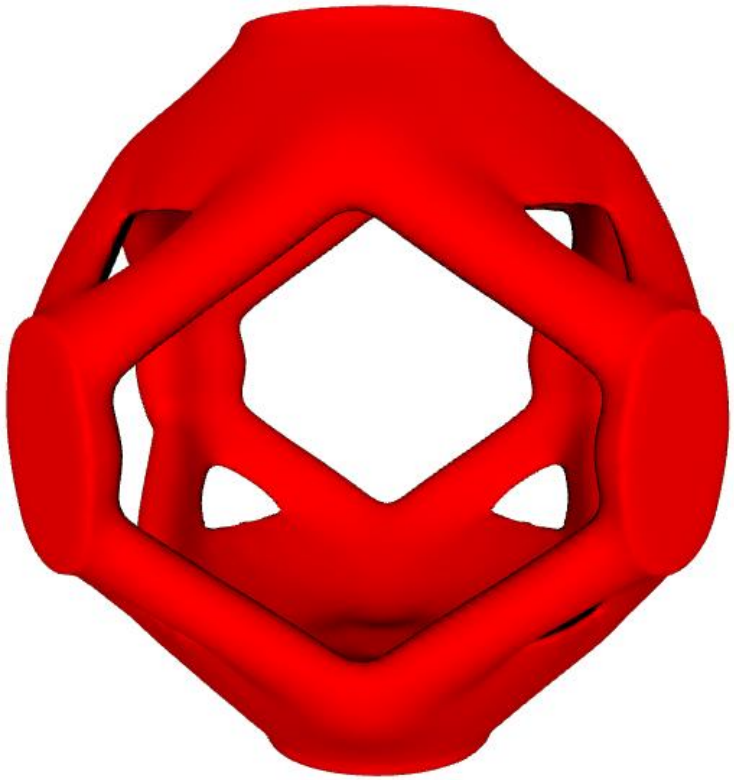


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- SHOW AND TELL: SHEAR BOX

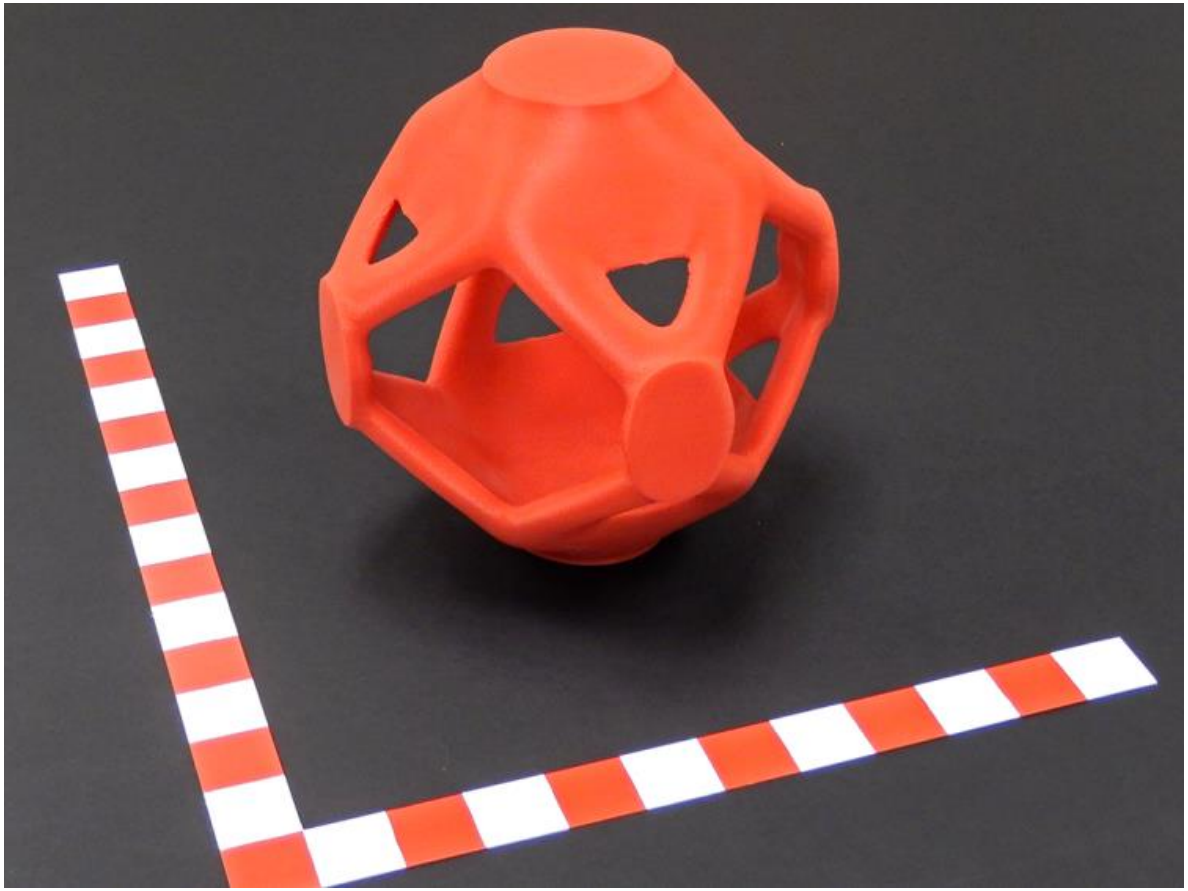




NGUYEN TH, PAULINO GH, SONG J, LE CH (2009) "A COMPUTATIONAL PARADIGM FOR MULTIRESOLUTION TOPOLOGY OPTIMIZATION (MTOP)." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 41(4):525–39.

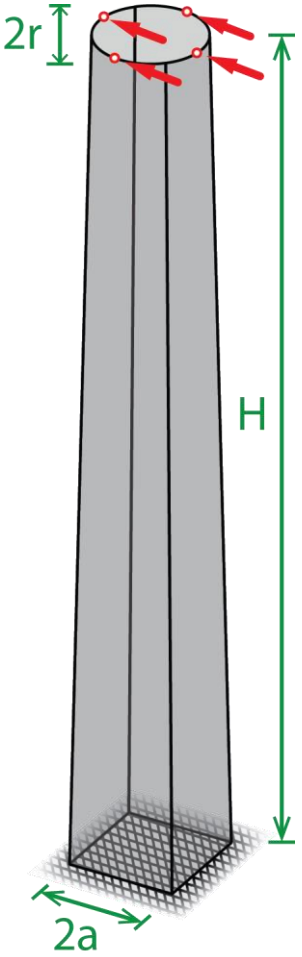
6) ADDITIVE MANUF. OF OPT. STRUCTS.

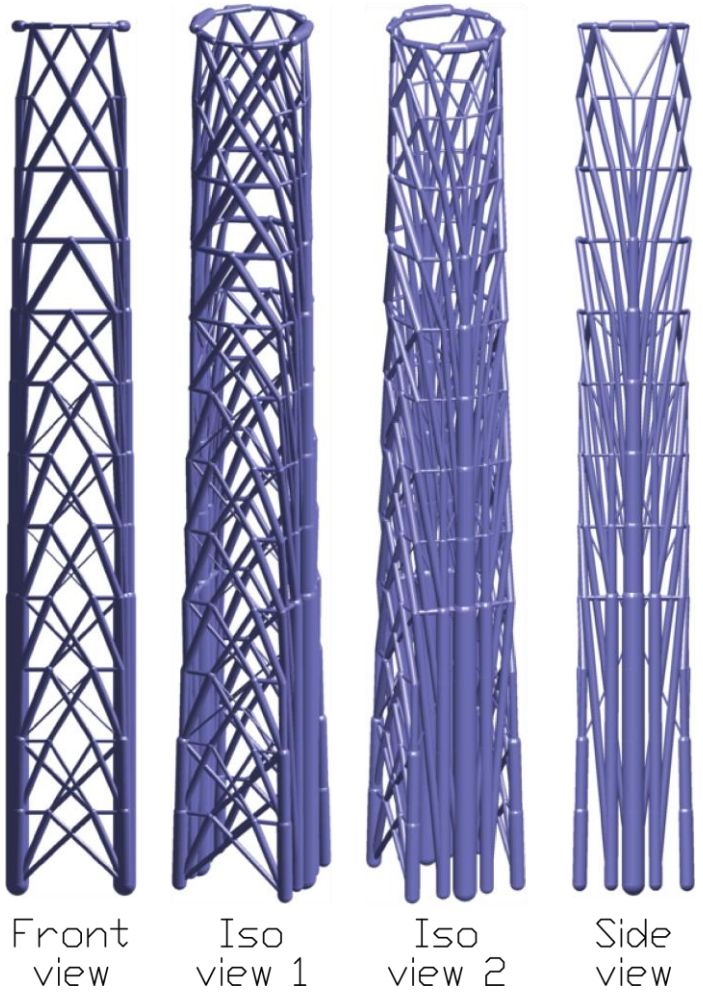
- SHOW AND TELL: SHEAR BOX



6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: LOTTE TOWER





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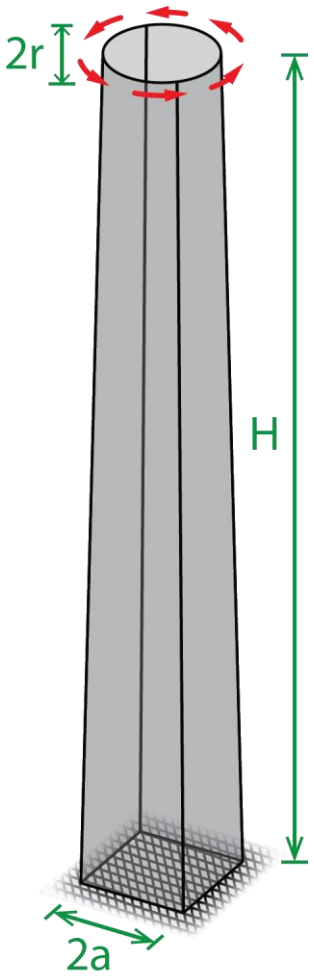
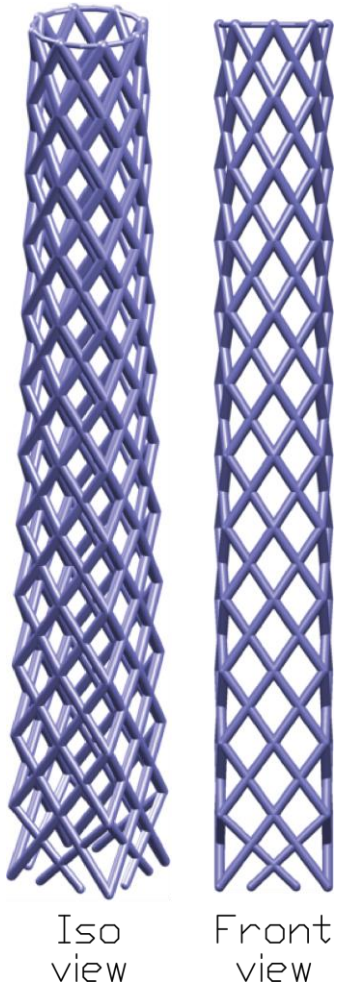


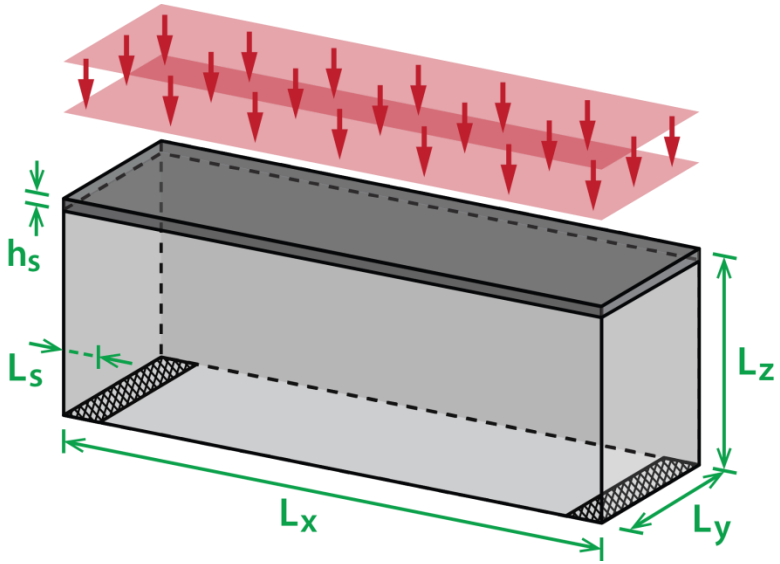


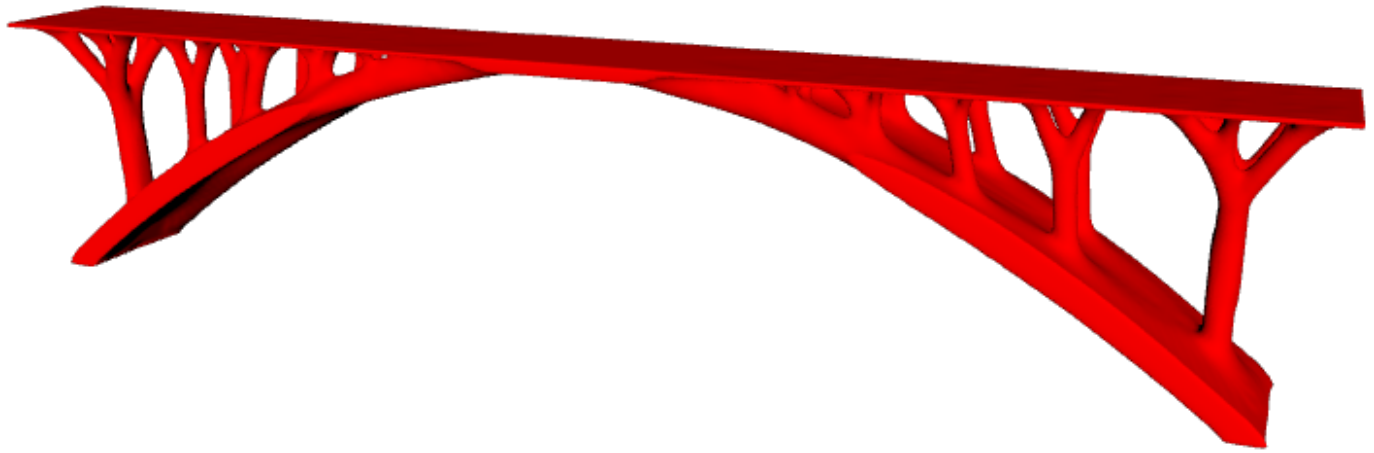
6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: LOTTE TOWER

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE





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- APPLICATION-ORIENTED: BRIDGE

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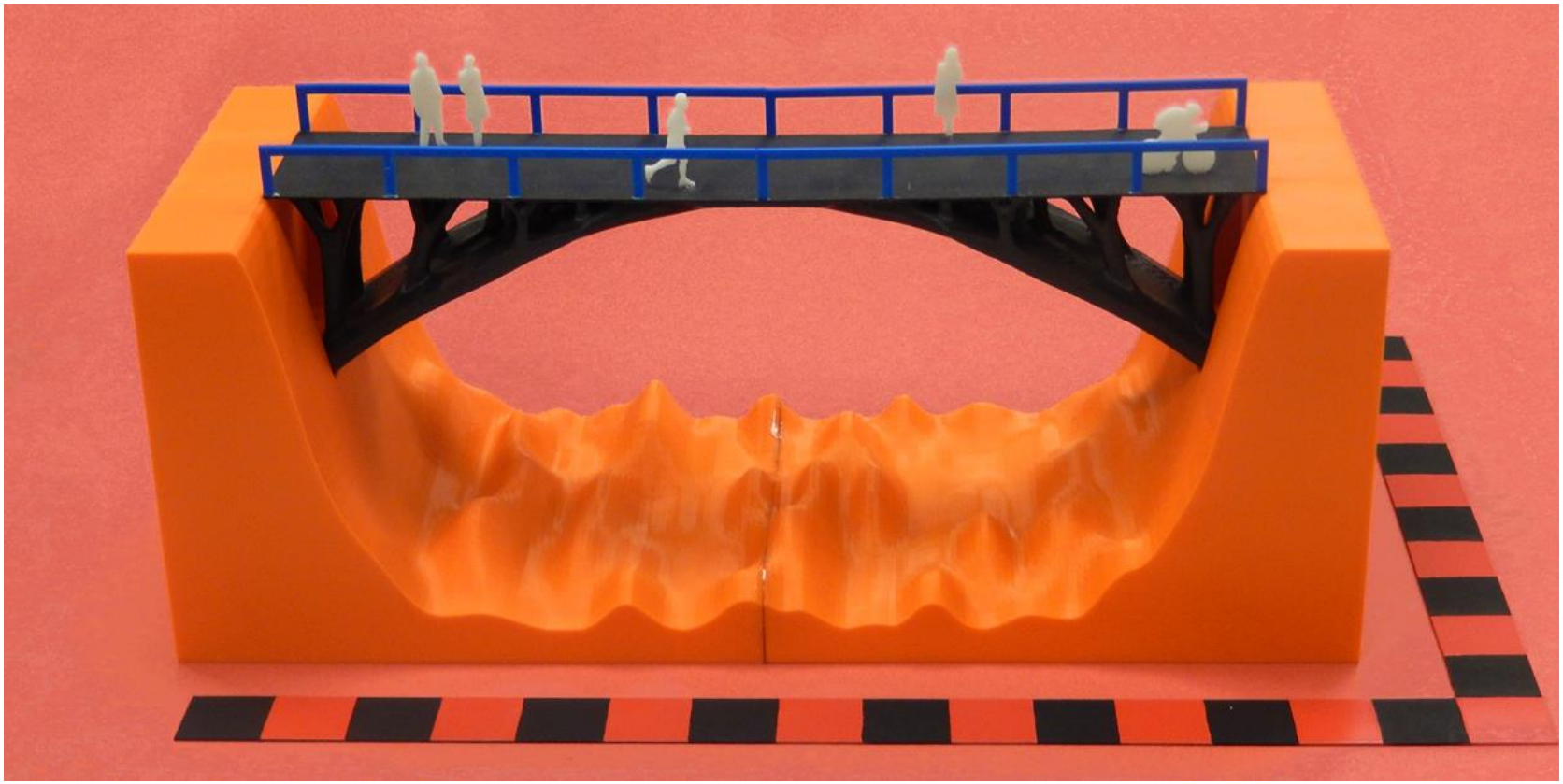
- APPLICATION-ORIENTED: BRIDGE

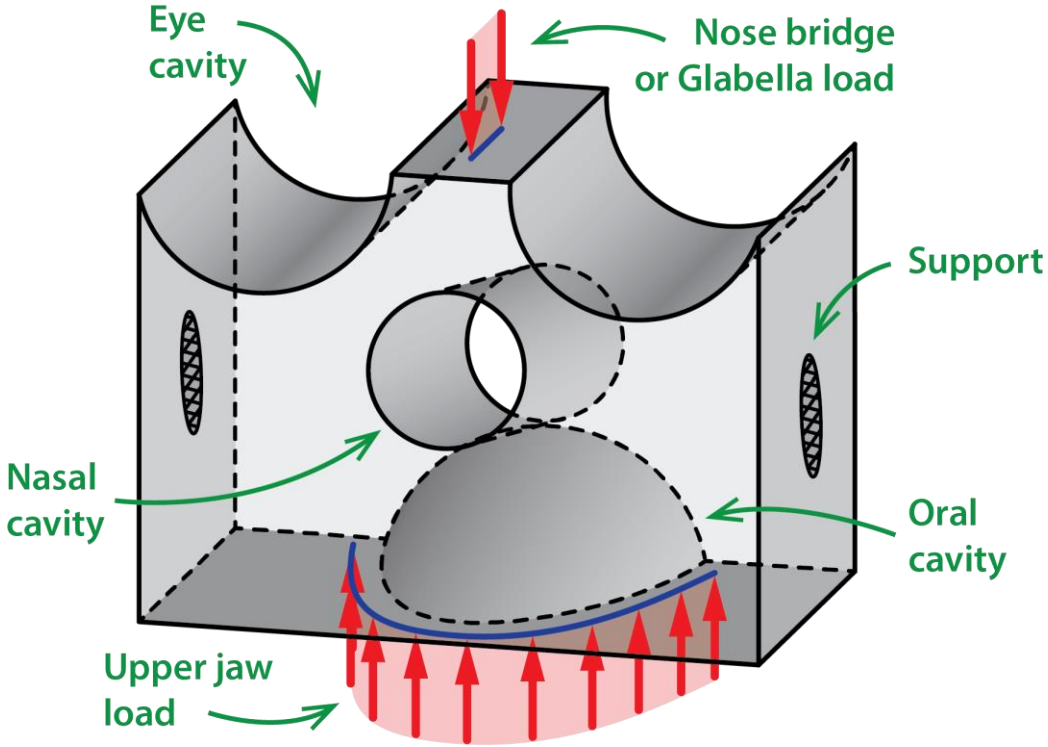


6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED: BRIDGE
  ACHIEVING LARGE SCALES

6) ADDITIVE MANUF. OF OPT. STRUCTS.

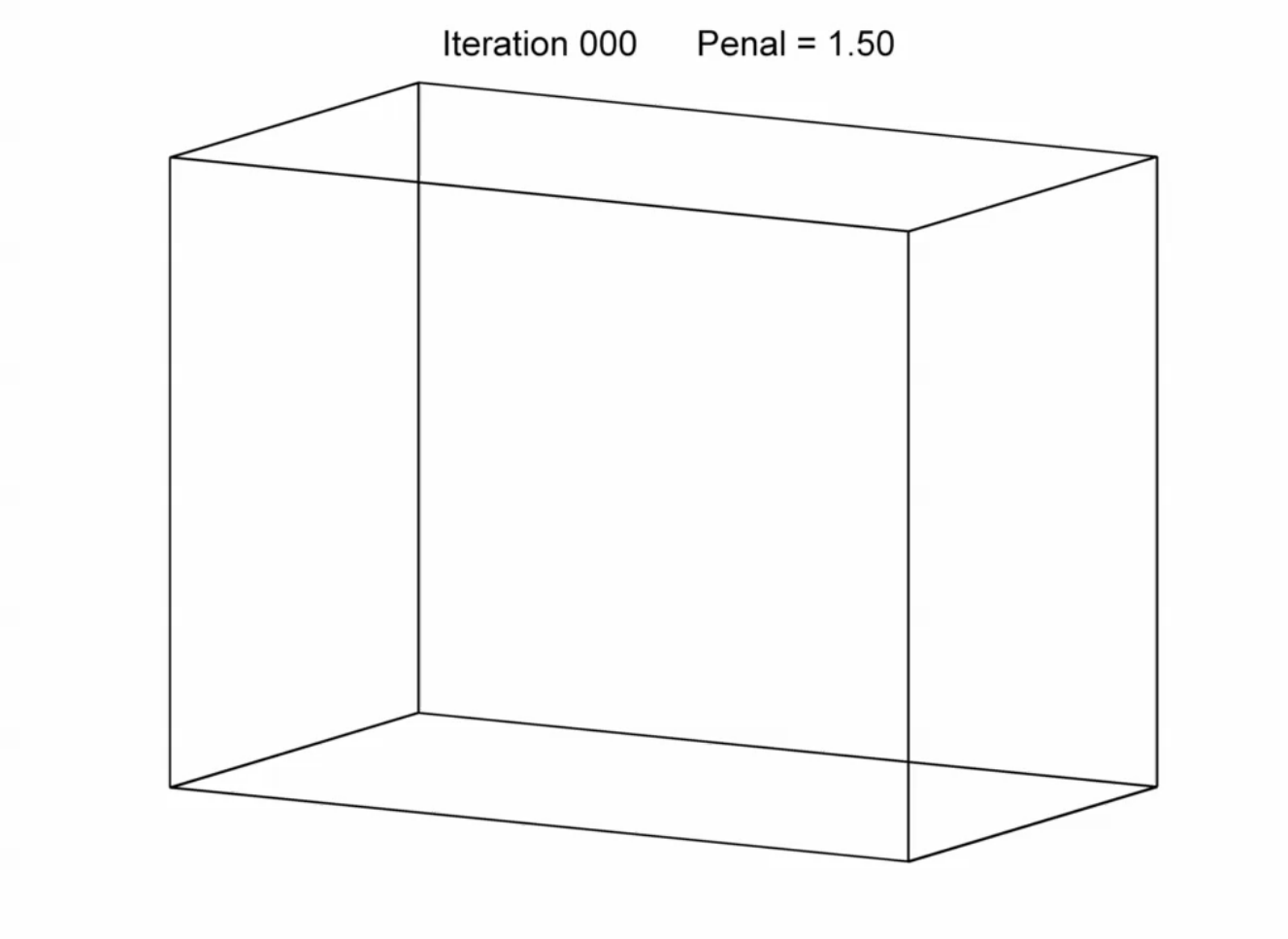
- APPLICATION-ORIENTED:
  CRANIOFACIAL RECONSTRUCTION



SUTRADHAR A, PAULINO GH, MILLER MJ, NGUYEN TH (2010) "TOPOLOGICAL OPTIMIZATION FOR DESIGNING PATIENT-SPECIFIC LARGE CRANIOFACIAL SEGMENTAL BONE REPLACEMENTS." PNAS, 107(30):13222-13227

6) ADDITIVE MANUF. OF OPT. STRUCTS.

- APPLICATION-ORIENTED:
  CRANIOFACIAL RECONSTRUCTION



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- APPLICATION-ORIENTED:
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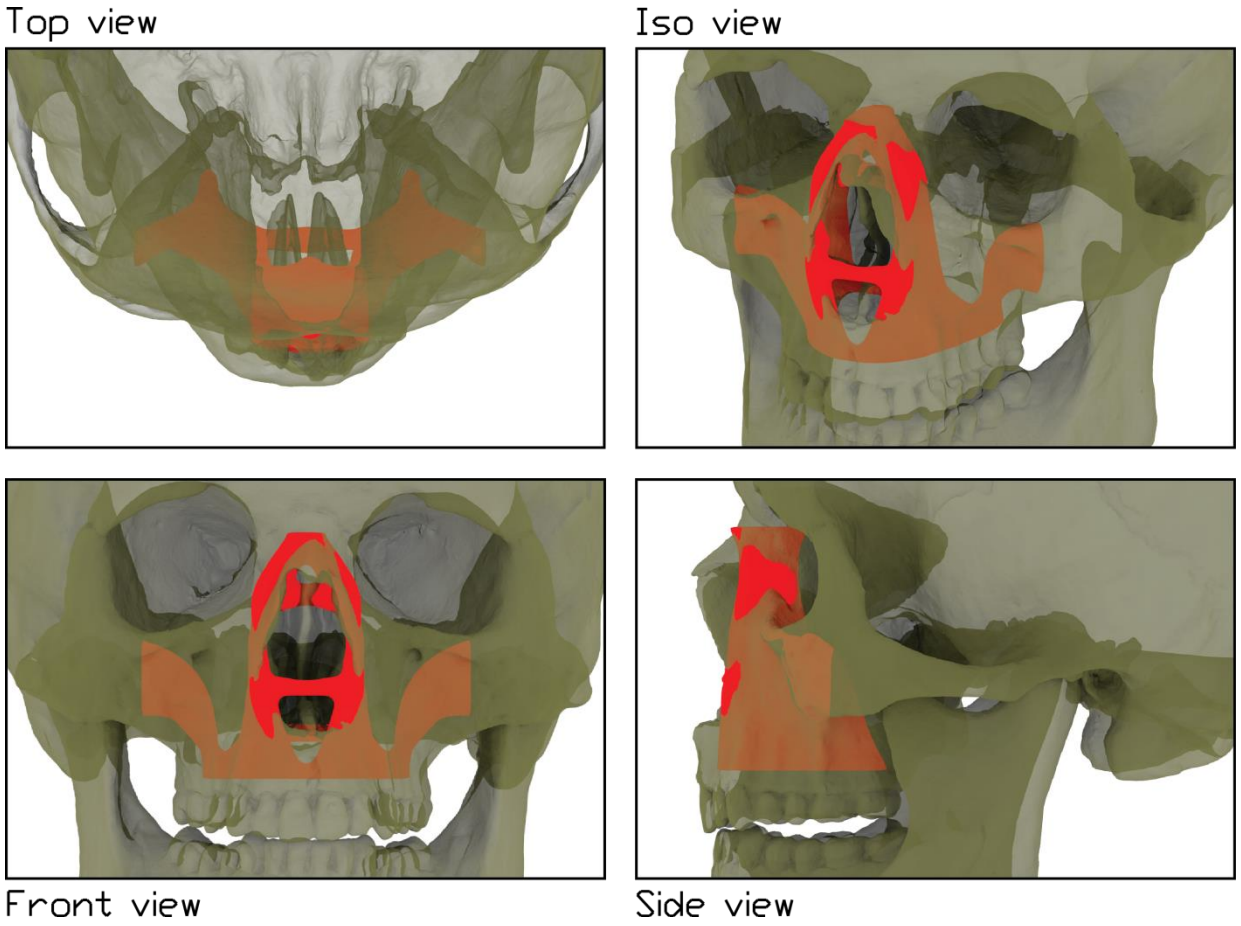

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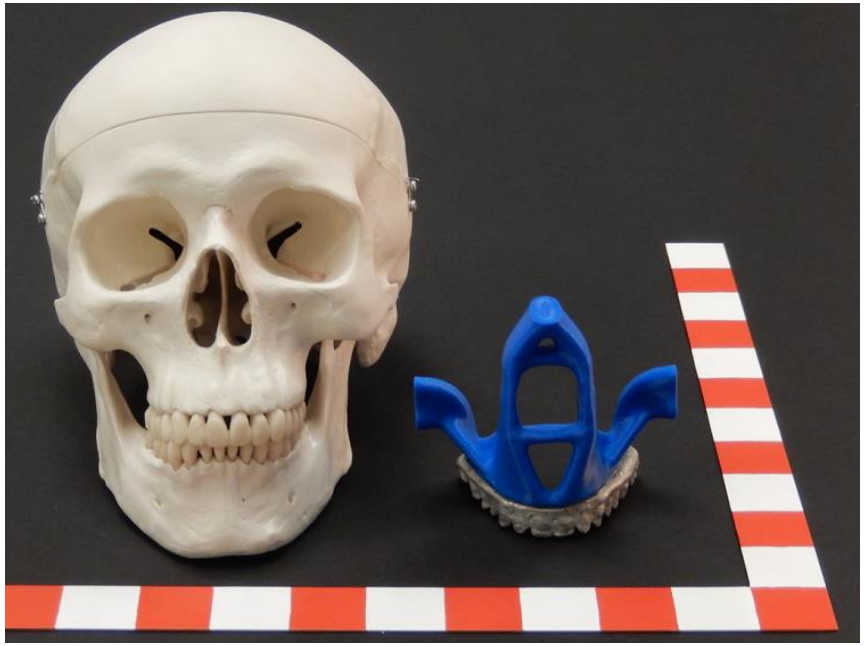
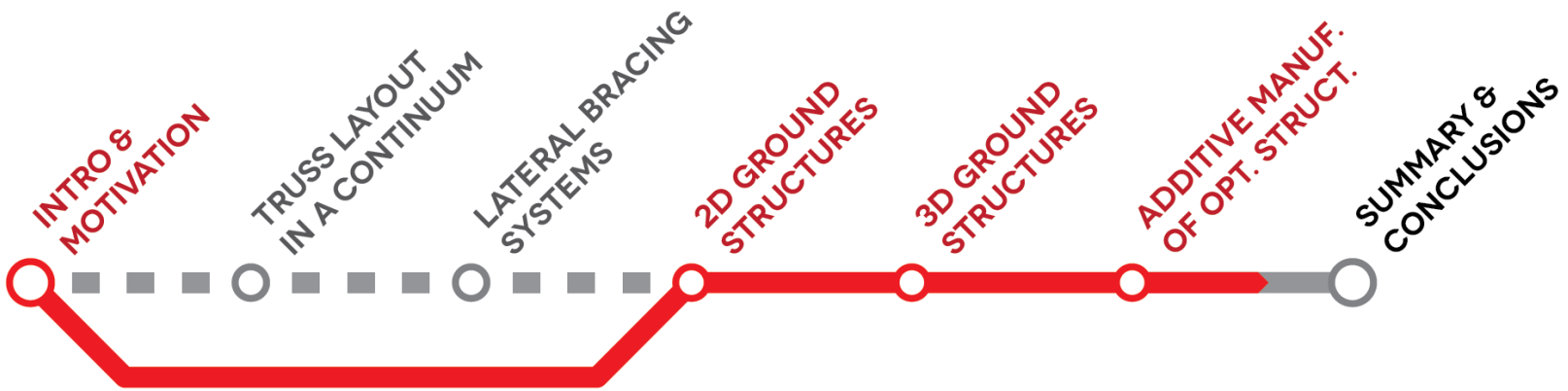
- APPLICATION–ORIENTED:
  CRANIOFACIAL RECONSTRUCTION



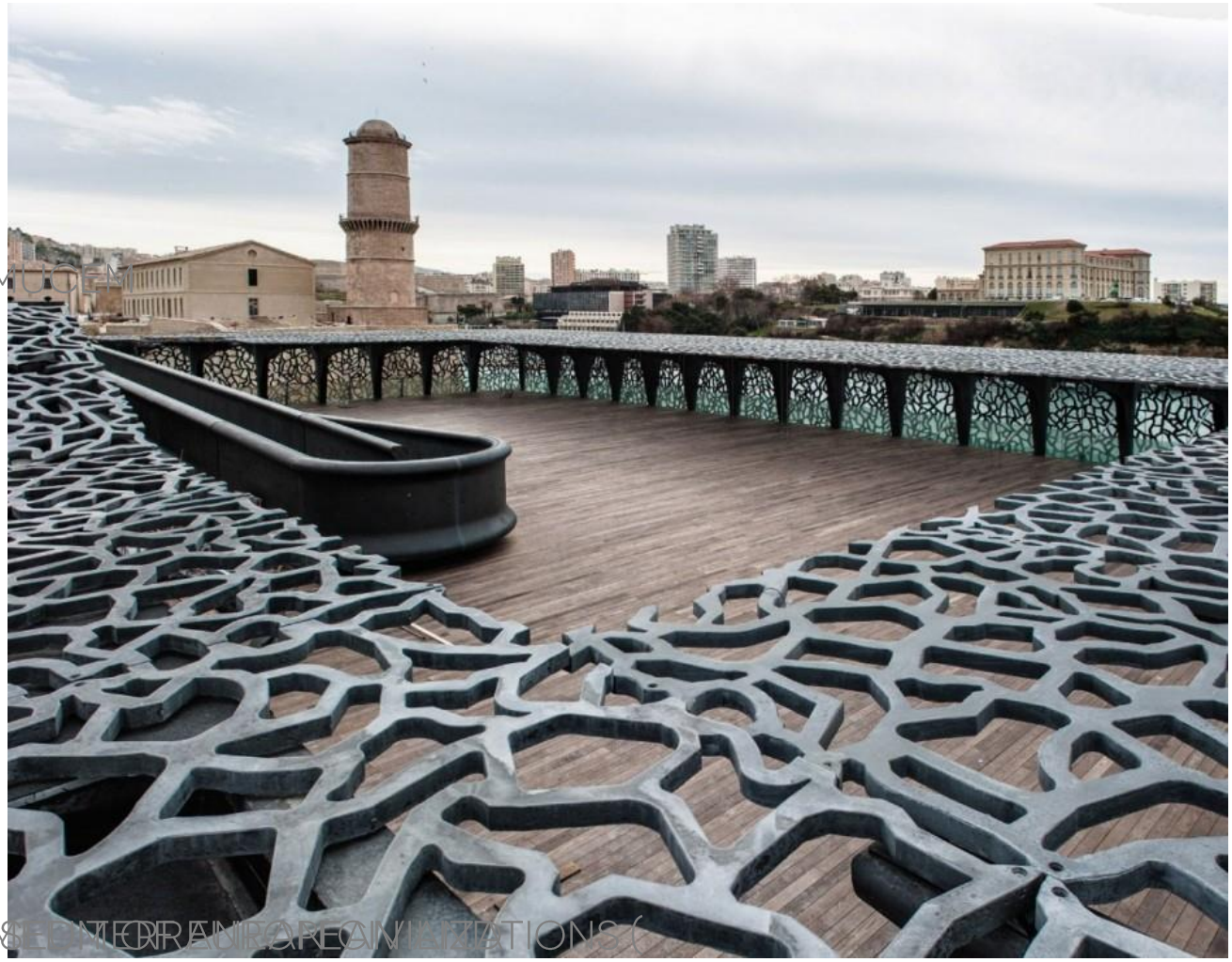
ROADMAP


INTRO & MOTIVATION
TRUSS LAYOUT IN A CONTINUUM
LATERAL BRACING SYSTEMS
2D GROUND STRUCTURES
3D GROUND STRUCTURES
ADDITIVE MANUF. OF OPT. STRUCT.
SUMMARY & CONCLUSIONS

7) SUMMARY AND CONCLUSIONS

- OPTIMIZATION:
  1. ESSENTIAL FOR SUSTAINABILITY
  2. CAN BE INCORPORATED INTO DESIGN TODAY
  3. GIVE A DESIGN, AND I WILL TRY TO MAKE IT BETTER
  4. DIFFERENT METHODS FOR DIFFERENT PROBLEMS
  5. YES, WE CAN MANUFACTURE THIS
  6. DESIGN GUIDED BY FUNCTIONALITY AND NOT JUST BEAUTY

7) SUMMARY AND CONCLUSIONS



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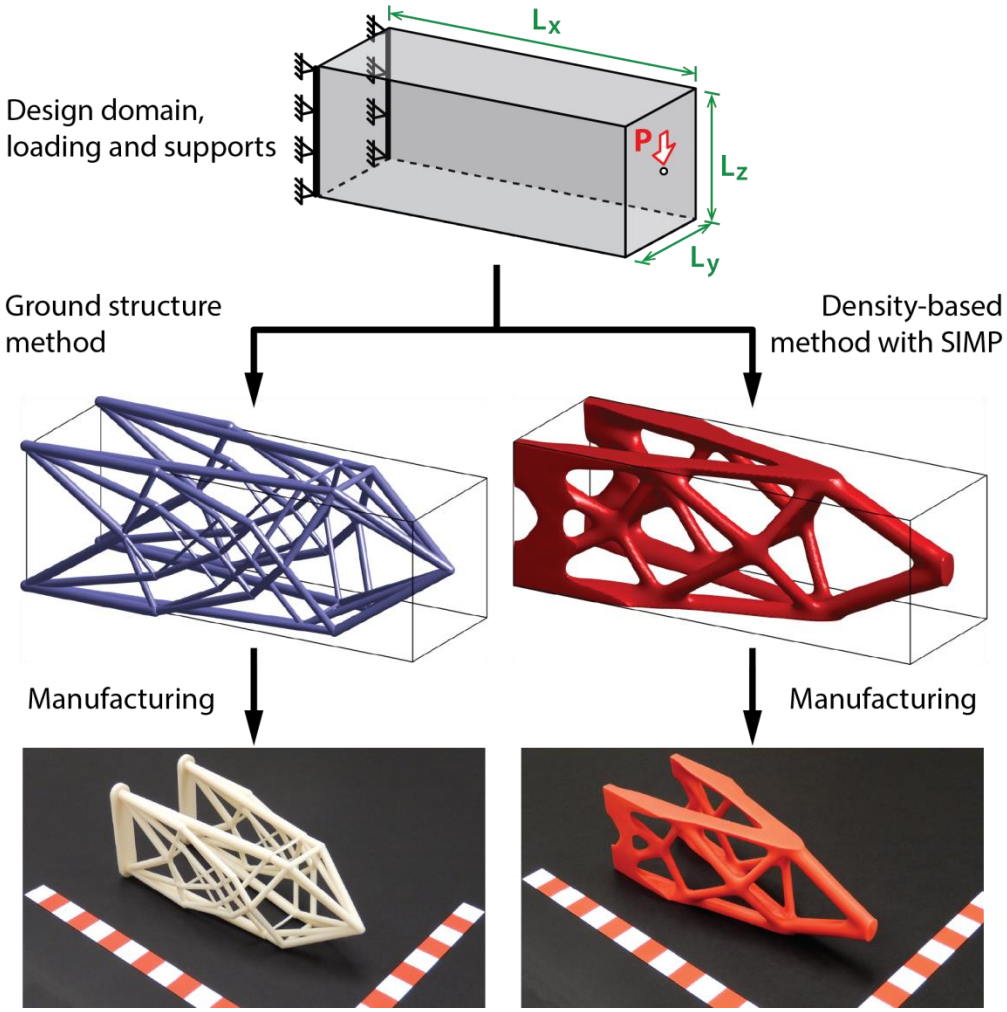
7) SUMMARY AND CONCLUSIONS

- INTEGRATED DESIGN PROCESS: START TO FINISH



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- LCCV @ UFAL
- CEE @ ILLINOIS

THE END