TOPOLOGY OPTIMIZATION OF STRUCTURES AND MATERIALS

TOMAS ZEGARD

GT MAP

GEORGIA TECH MATHEMATICS AND APPLICATIONS PORTAL

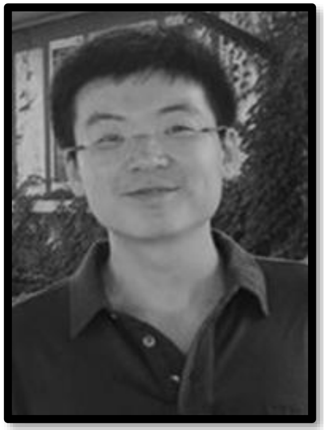
Georgia Tech

G. H. PAULINO'S GROUP

HENG CHI

XIAOJIA ZHANG

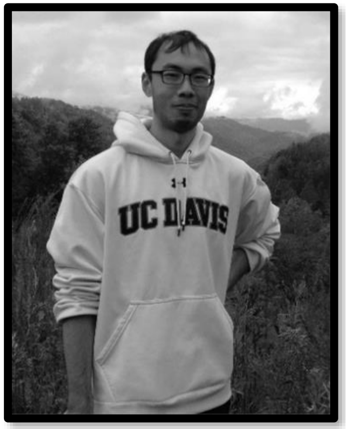
KE LIU



TUO ZHAO

EMILY DANIELS

OLIVER GIRALDO–LONDOÑO

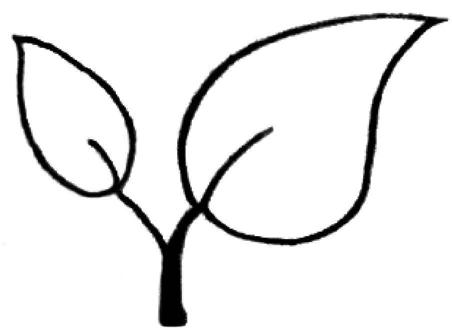


YANG JIANG

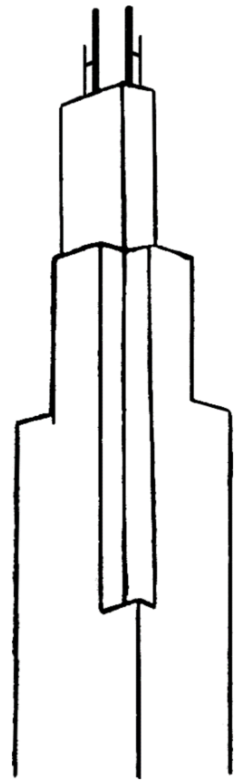
LARISSA NOVELINO

1) INTRO & MOTIVATION

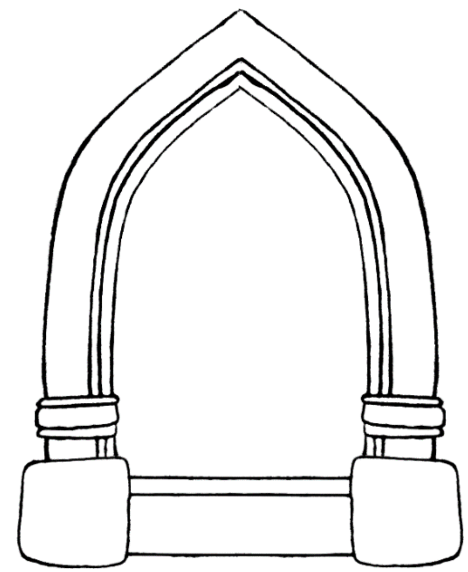
- WHY USE TOPOLOGY OPTIMIZATION?



LIMITED RESOURCES



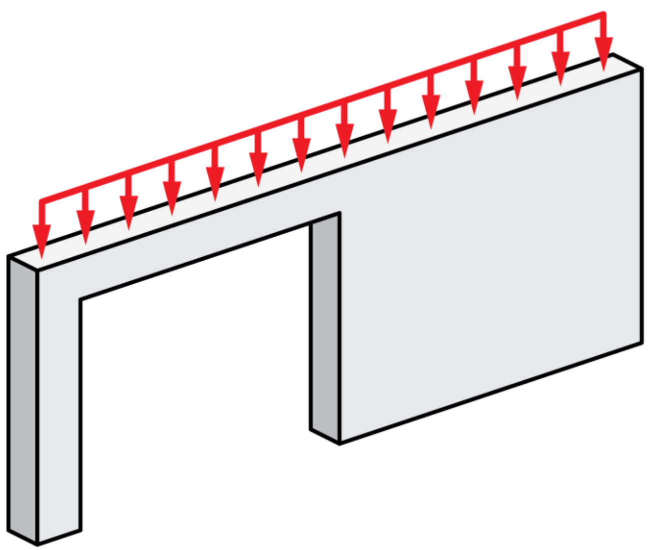
EXTREME STRUCTURES AND MATERIALS



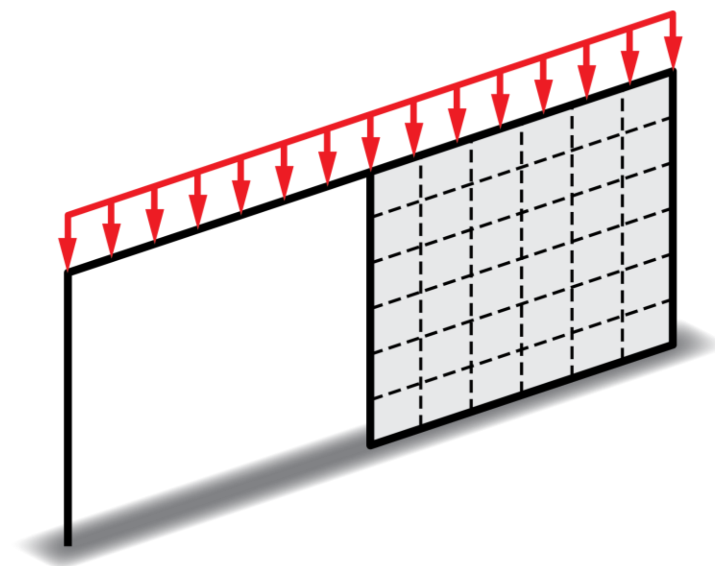
FUNCTIONAL

1) INTRO & MOTIVATION

- DISCRETE? CONTINUUM?
  - LIMITED MODELING CAPABILITY
  - REASONABLE SIMPLIFICATIONS OF REALITY
  - STEER TOWARDS A SOLUTION TYPE



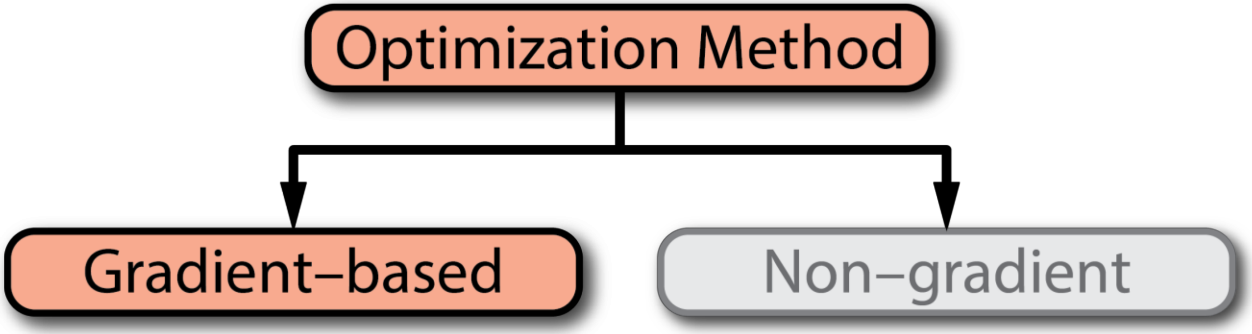
REAL FRAME

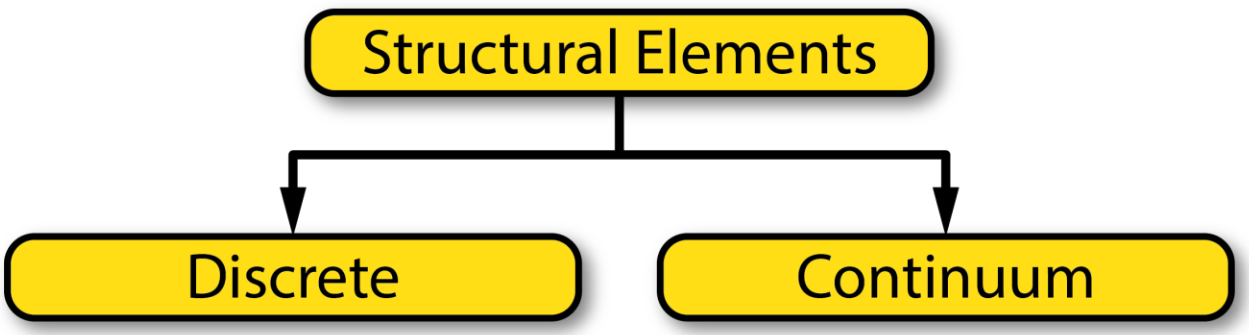


SIMPLIFIED FRAME MODEL

1) Intro & Motivation

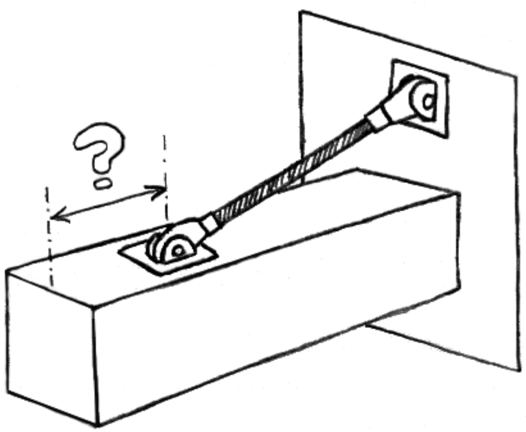
- Topology optimization methods



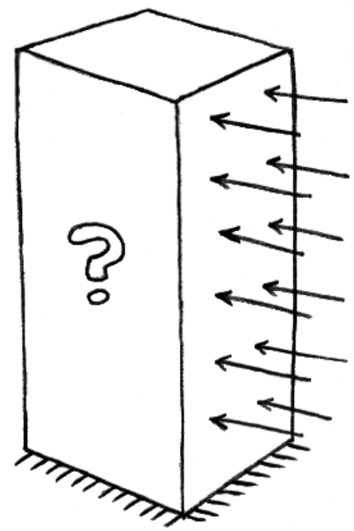


1) INTRO & MOTIVATION

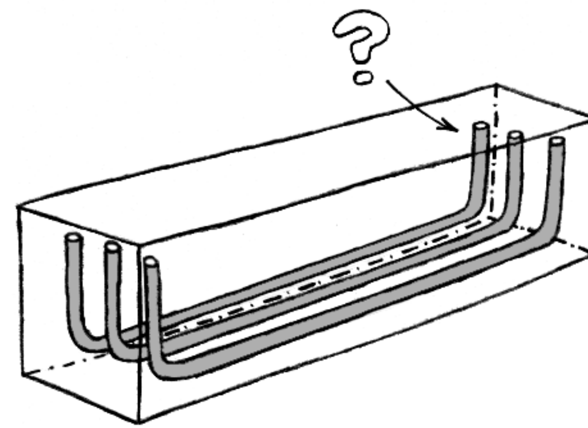
- SIMPLE PROBLEMS WITH NO SOLUTION



ANCHOR POINT LOCATION



LATERAL BRACING SYSTEM



REINFORCEMENT LAYOUT

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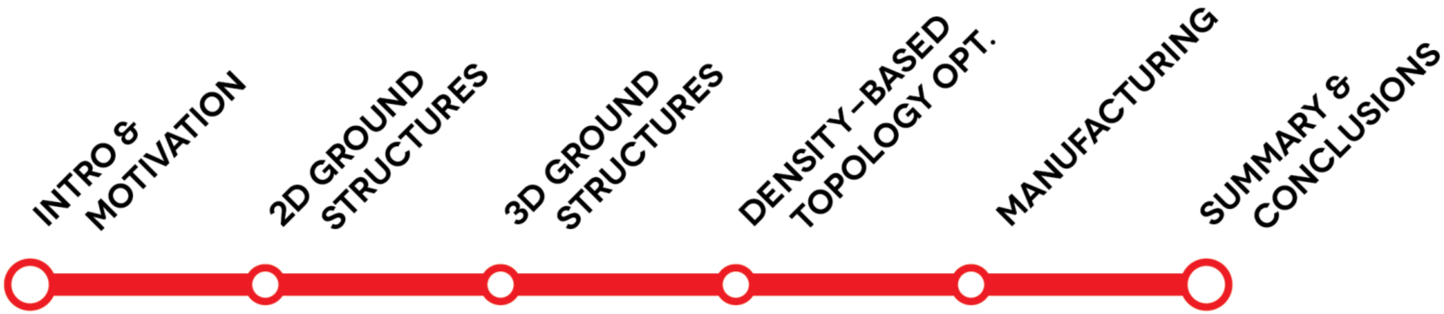
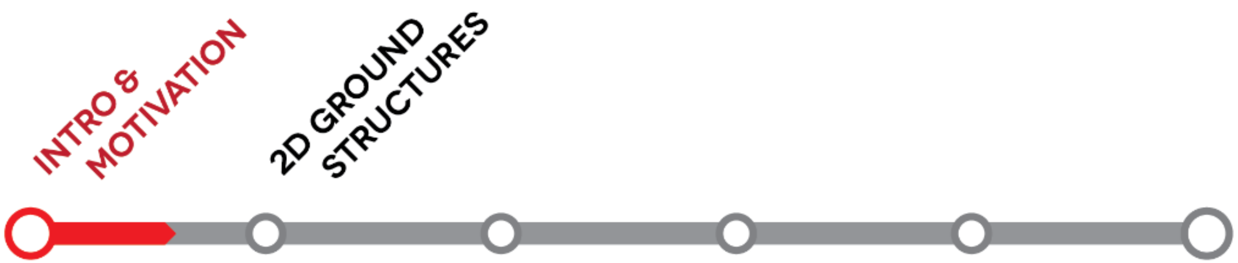
2. 2D GROUND STRUCTURES
3. 3D GROUND STRUCTURES
4. DENSITY-BASED TOPOLOGY OPTIMIZATION
5. MANUFACTURING

6. SUMMARY & CONCLUSIONS

ROADMAP

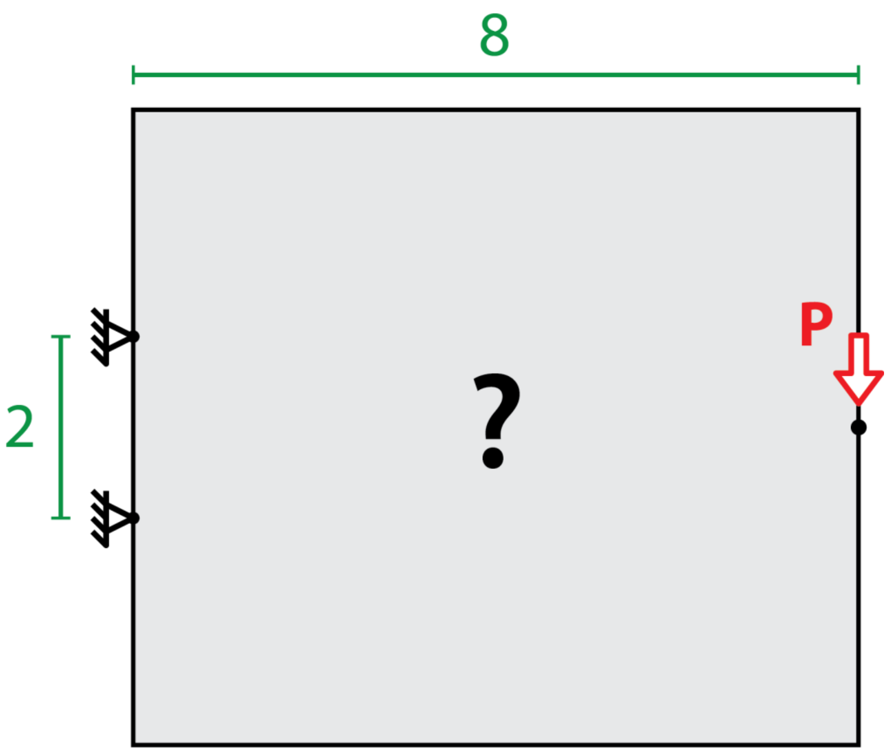

INTRO &
MOTIVATION
2D GROUND
STRUCTURES
3D GROUND
STRUCTURES
DENSITY-BASED
TOPOLOGY OPT.
MANUFACTURING
SUMMARY &
CONCLUSIONS

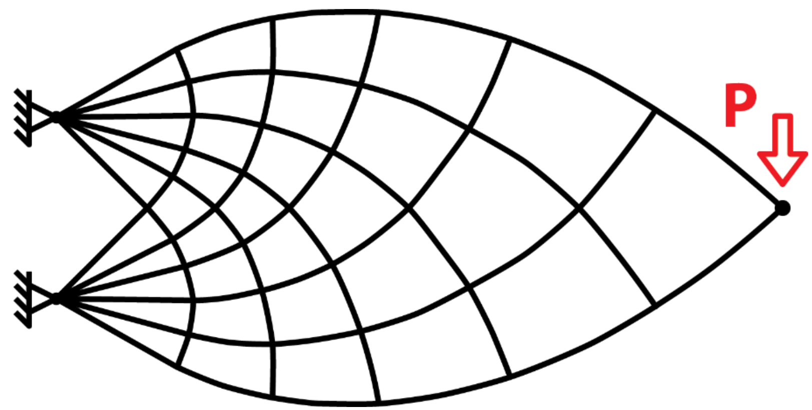
ROADMAP



2) GROUND STRUCTURES IN 2D

- TRUSS LAYOUT OPTIMIZATION IS HIGHLY NONLINEAR





MAZUREK A (2012) – "GEOMETRICAL ASPECTS OF OPTIMUM TRUSS LIKE STRUCTURES FOR THREE-FORCE PROBLEM", STRUCT AND MULTIDISCIPLINARY OPT 45(1), PP 21–32

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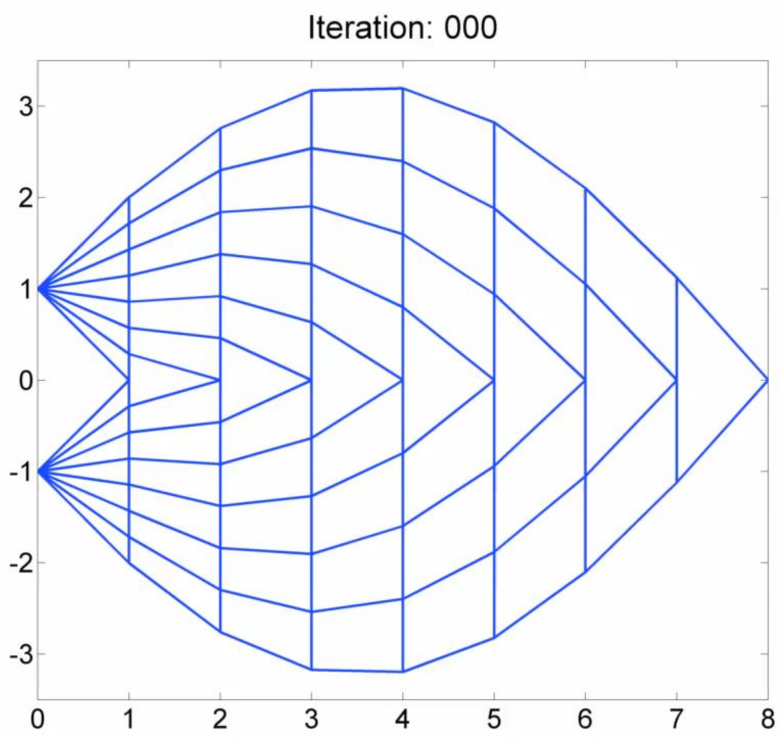

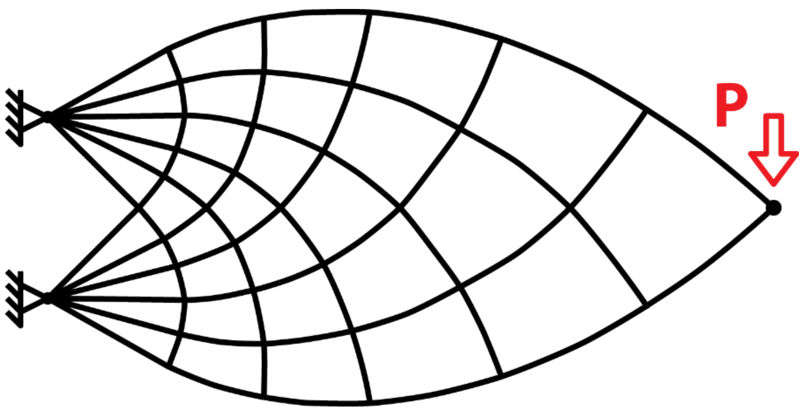

MAZUREK A (2012) – "GEOMETRICAL ASPECTS OF OPTIMUM TRUSS LIKE STRUCTURES FOR THREE-FORCE PROBLEM", STRUCT AND MULTIDISCIPLINARY OPT 45(1), PP 21–32

2) GROUND STRUCTURES IN 2D

- MAIN IDEA
  CONVERT A GEOMETRY AND SIZE OPTIMIZATION TO A SIZING-ONLY PROBLEM

- PLASTIC FORMULATION

$$
\begin{array}{ll}
\min\limits_{\mathbf{a}} & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
 & -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \\
 & a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{array}
$$

DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES"
JOURNAL DE MECANIQUE 3(1), PP 25–52

2) GROUND STRUCTURES IN 2D

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C \leq \sigma_i \leq \sigma_T \quad \text{if} \quad a_i > 0 \\
& a_i \geq 0 \quad i = 1, 2 \ldots N_b
\end{aligned}
$$

VANISHING CONSTRAINT

- MULTIPLYING THE INEQUALITY BY CROSS-SECTIONAL AREA

$$
\begin{aligned}
\min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\
\text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\
& -\sigma_C a_i \leq n_i \leq \sigma_T a_i
\end{aligned}
$$

DORN W S, GOMORY R E, GREENBERG H J (1964) – "AUTOMATIC DESIGN OF OPTIMAL STRUCTURES"
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2) GROUND STRUCTURES IN 2D

$$\begin{aligned} \min_{\mathbf{a}} \quad & V = \mathbf{l}^T \mathbf{a} \\ \text{s.t.} \quad & \mathbf{B}^T \mathbf{n} = \mathbf{f} \\ & -\sigma_C a_i \leq n_i \leq \sigma_T a_i \end{aligned}$$

- INTRODUCING SLACK VARIABLES

$$\left.\begin{aligned} n_i + 2\frac{\sigma_0}{\sigma_C} s_i^- &= \sigma_T a_i \\ -n_i + 2\frac{\sigma_0}{\sigma_T} s_i^+ &= \sigma_C a_i \\ \sigma_0 = (\sigma_T + \sigma_C)/2 & \end{aligned}\right\} \qquad \begin{aligned} a_i &= \frac{s_i^+}{\sigma_T} + \frac{s_i^-}{\sigma_C} \\ n_i &= s_i^+ - s_i^- \end{aligned}$$

$$\begin{aligned} \min_{\mathbf{s}^+,\mathbf{s}^-} \quad & V = \mathbf{l}^T \left(\frac{\mathbf{s}^+}{\sigma_T} + \frac{\mathbf{s}^-}{\sigma_C} \right) \\ \text{s.t.} \quad & \mathbf{B}^\mathbf{T} \left(\mathbf{s}^+ - \mathbf{s}^- \right) = \mathbf{f} \\ & s_i^+, s_i^- \geq 0 \end{aligned}$$

2) GROUND STRUCTURES IN 2D

- REMARKS
  - DESIGN VARIABLES DOUBLED: S^+ AND S^-
  - NO MORE VANISHING CONSTRAINT
  - DIFFERENT LIMITS IN TENSION AND COMPRESSION
  - LINEAR PROGRAM

    KARMARKAR N (1984) "A NEW POLYNOMIAL-TIME ALGORITHM FOR LINEAR PROGRAMMING." COMBINATORICA, 4(4):373-395.

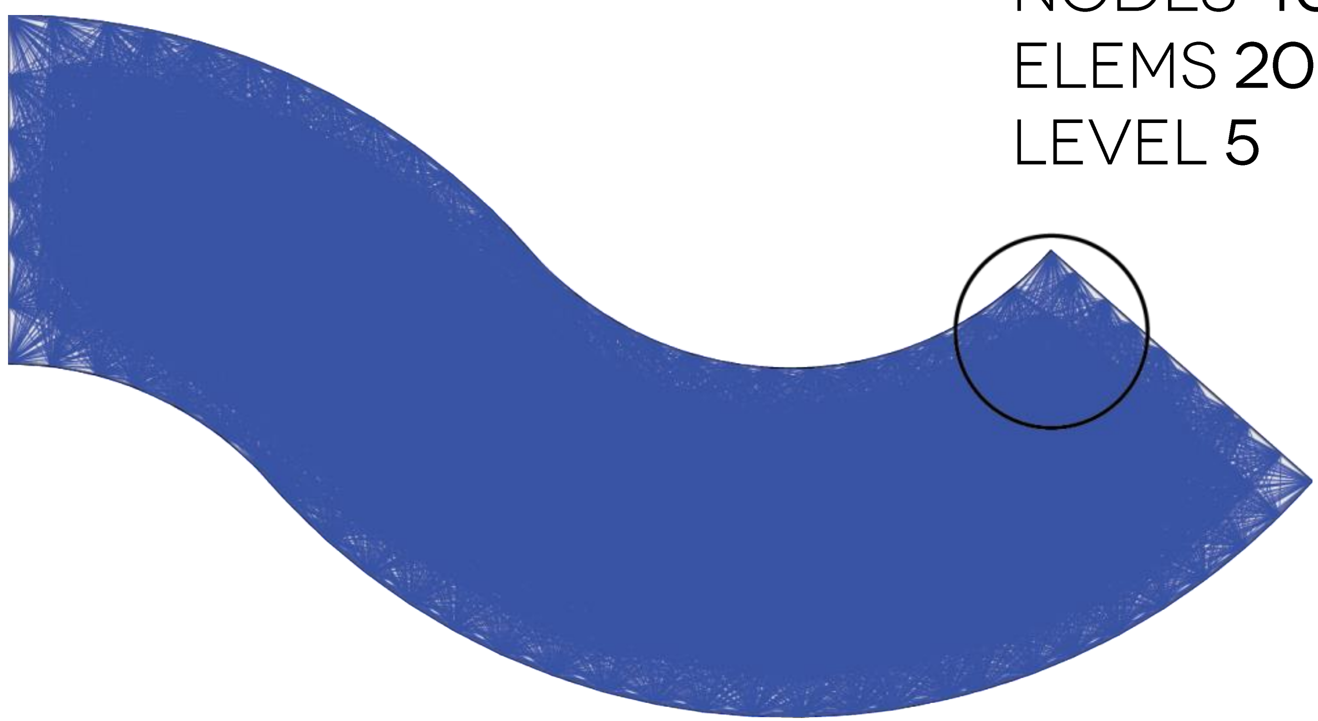
    WRIGHT MH (2004) “THE INTERIOR-POINT REVOLUTION IN OPTIMIZATION: HISTORY, RECENT DEVELOPMENTS, AND LASTING CONSEQUENCES.” BULLETIN OF THE AMERICAN MATHEMATICAL SOCIETY, 42(1):39-56.

$$\begin{aligned} \min_{\mathbf{s}^+,\mathbf{s}^-} \quad & V = \mathbf{l}^T \left(\frac{\mathbf{s}^+}{\sigma_T} + \frac{\mathbf{s}^-}{\sigma_C} \right) \\ \text{s.t.} \quad & \mathbf{B}^\mathbf{T} \left(\mathbf{s}^+ - \mathbf{s}^- \right) = \mathbf{f} \\ & s_i^+, s_i^- \geq 0 \end{aligned}$$

2) Ground structures in 2D

- Sizing of a highly **interconnected** and **redundant** truss

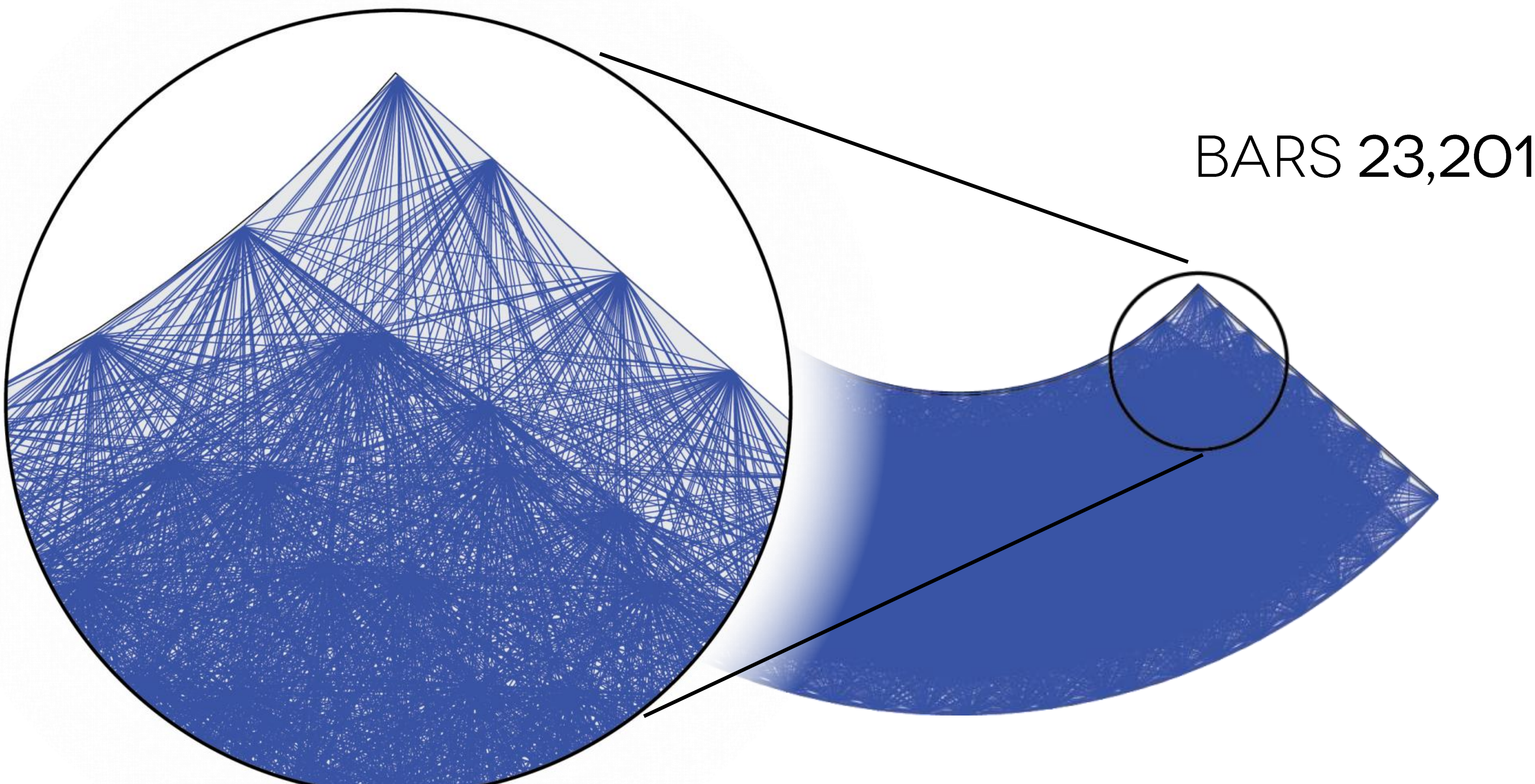
Nodes **404**
Elems **200**
Level **5**



Dorn W S, Gomory R E, Greenberg H J (1964) – "Automatic design of optimal structures"
Journal de Mecanique 3(1), pp 25–52

2) GROUND STRUCTURES IN 2D

- SIZING OF A HIGHLY **INTERCONNECTED** AND **REDUNDANT** TRUSS



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4) GROUND STRUCTURES IN 2D

- UNIQUE SOLUTION — NO COLLINEAR BARS

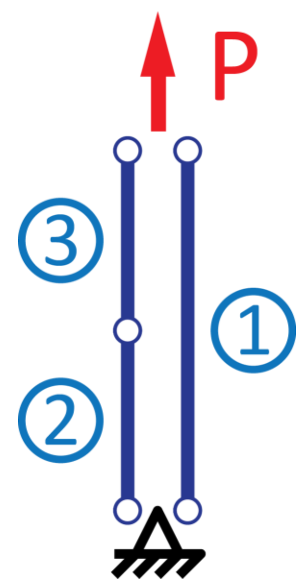
GIVEN $\sigma_T = 1$ AND $P = 1$

$a_1 = 1.0$

$a_1 = 1.0 \quad a_2 = a_3 = 0.0$

$a_1 = 0.0 \quad a_2 = a_3 = 1.0$

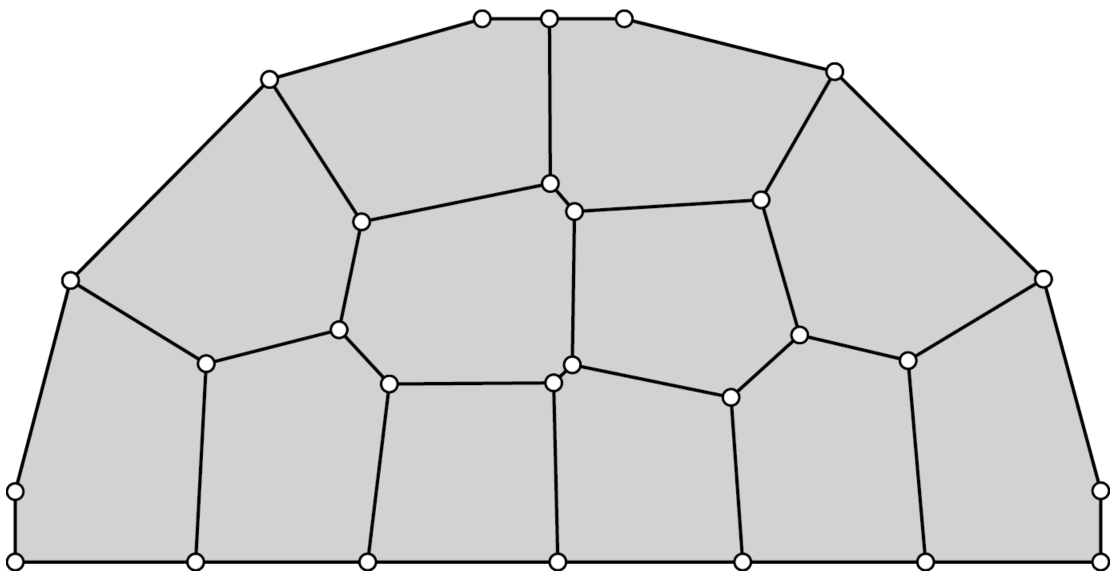
$a_1 = 0.5 \quad a_2 = a_3 = 0.5$



2) GROUND STRUCTURES IN 2D

- EXAMPLE
  - BASE MESH

$$\mathbf{L}_{ij} = \begin{cases} 1 & \text{if element } e \ni i, j \\ 0 & \text{otherwise} \end{cases}$$



2) GROUND STRUCTURES IN 2D

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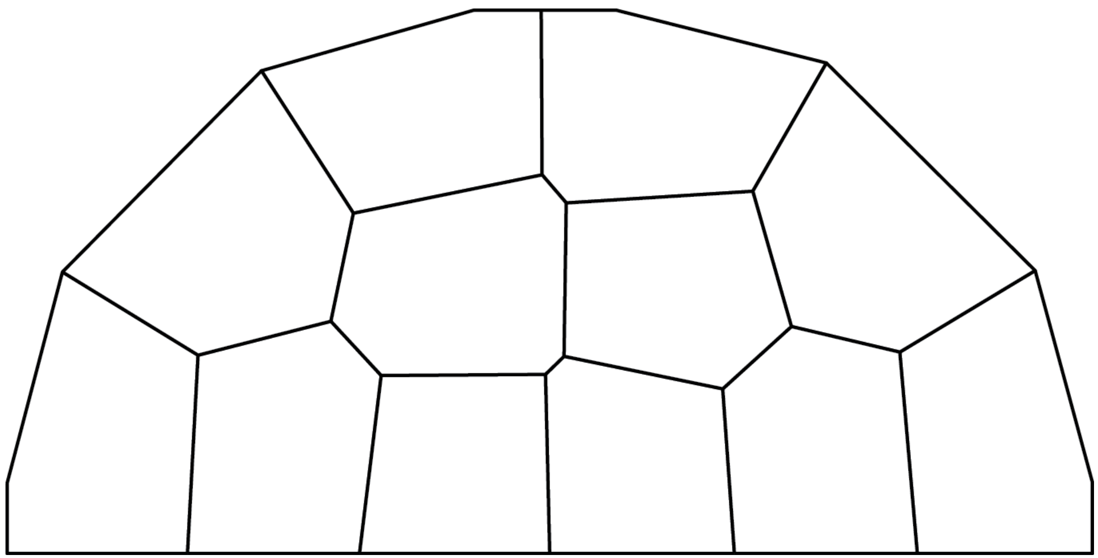
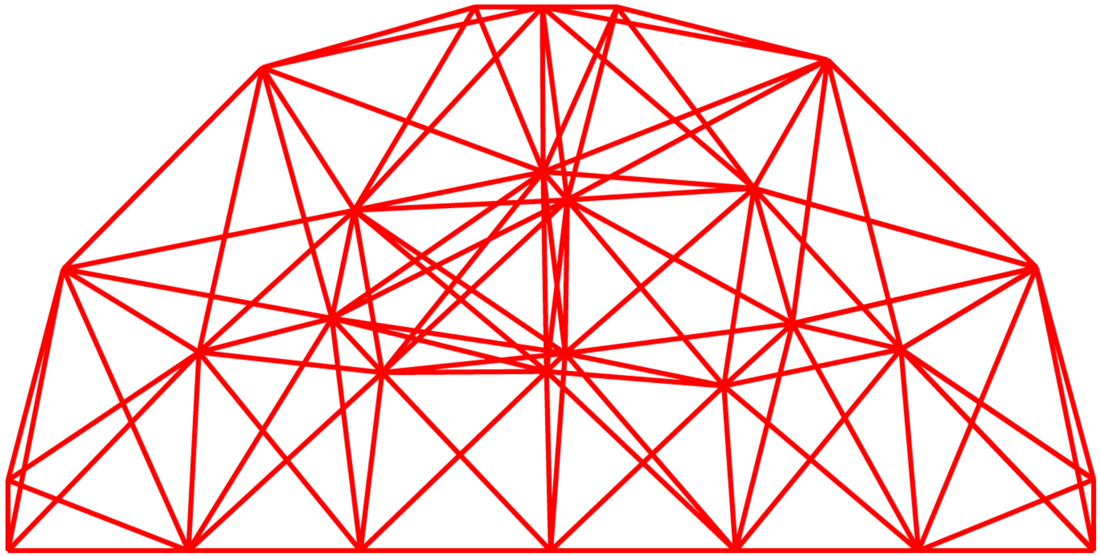


2) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 1

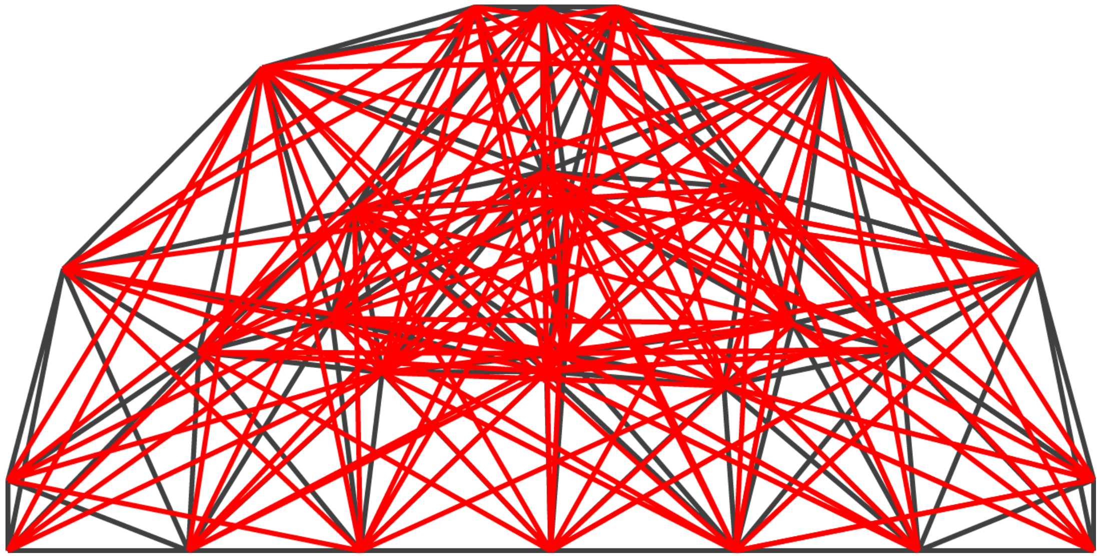
$$\mathbf{L}_1 = \mathbf{L}$$



2) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 2

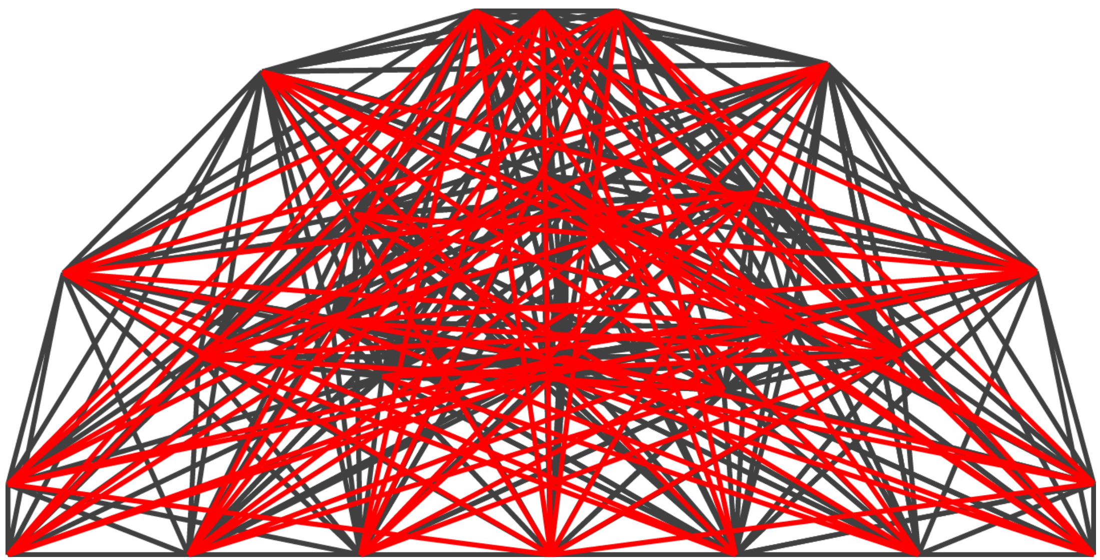
$$\mathbf{L_2 = LL}$$



2) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 3

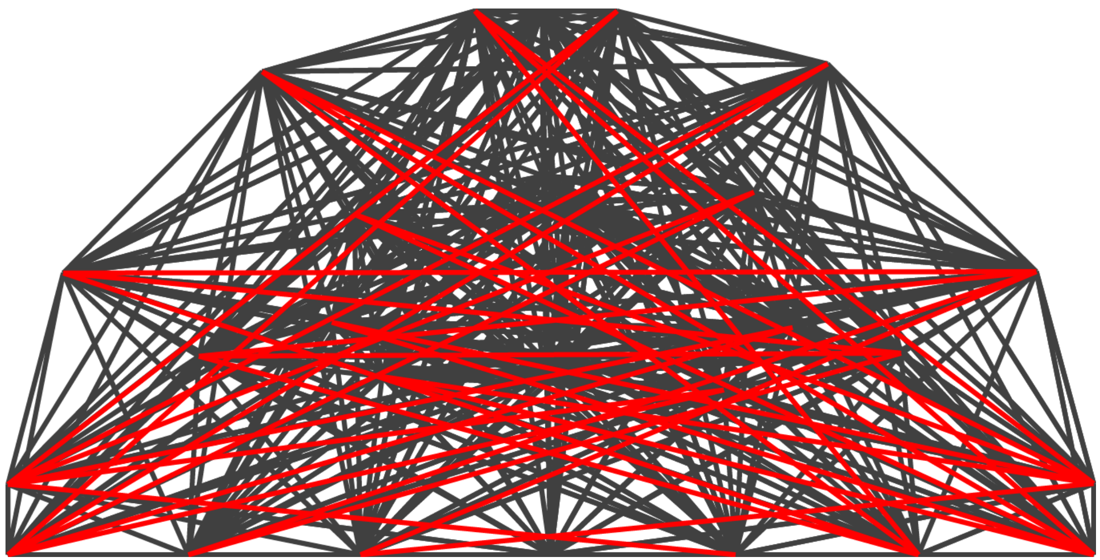
$$\mathbf{L}_3 = \mathbf{LLL}$$



2) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 4

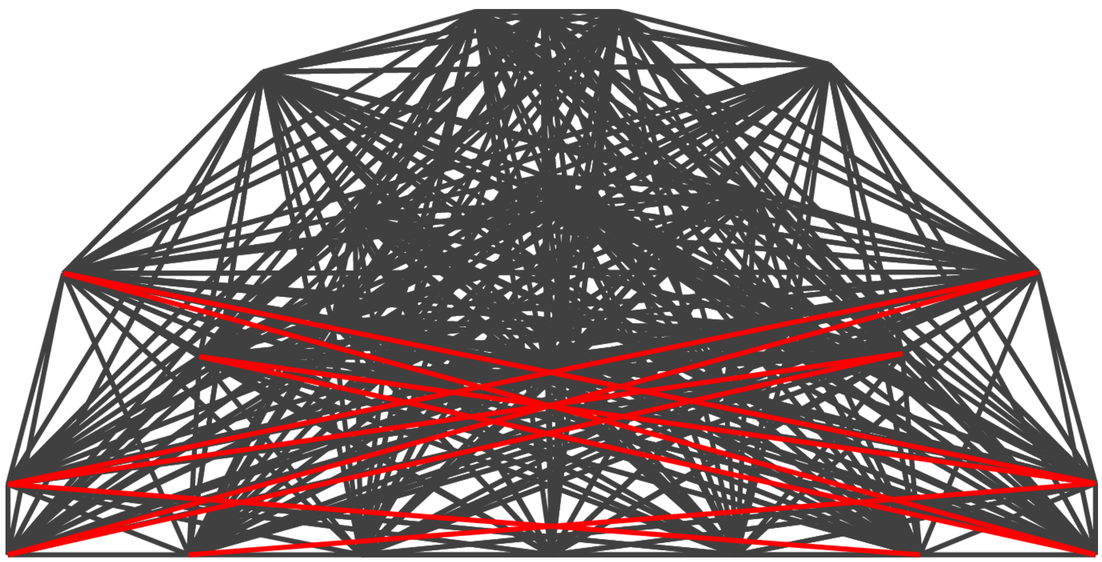
$$\mathbf{L}_4 = \mathbf{LLLL}$$



2) GROUND STRUCTURES IN 2D

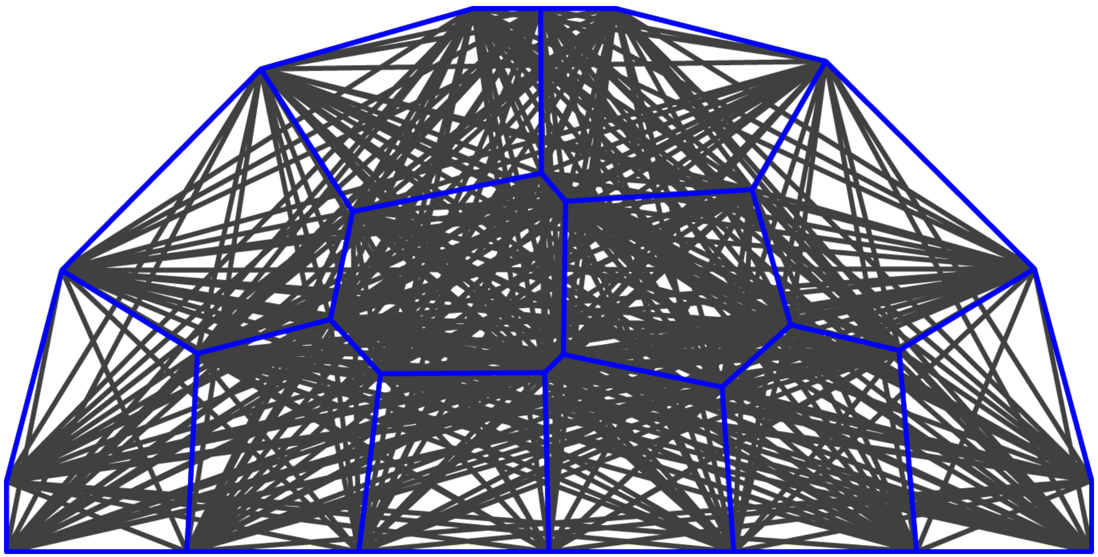
- EXAMPLE
  - CONNECTIVITY: LEVEL 5

$\mathbf{L_5 = LLLLL}$



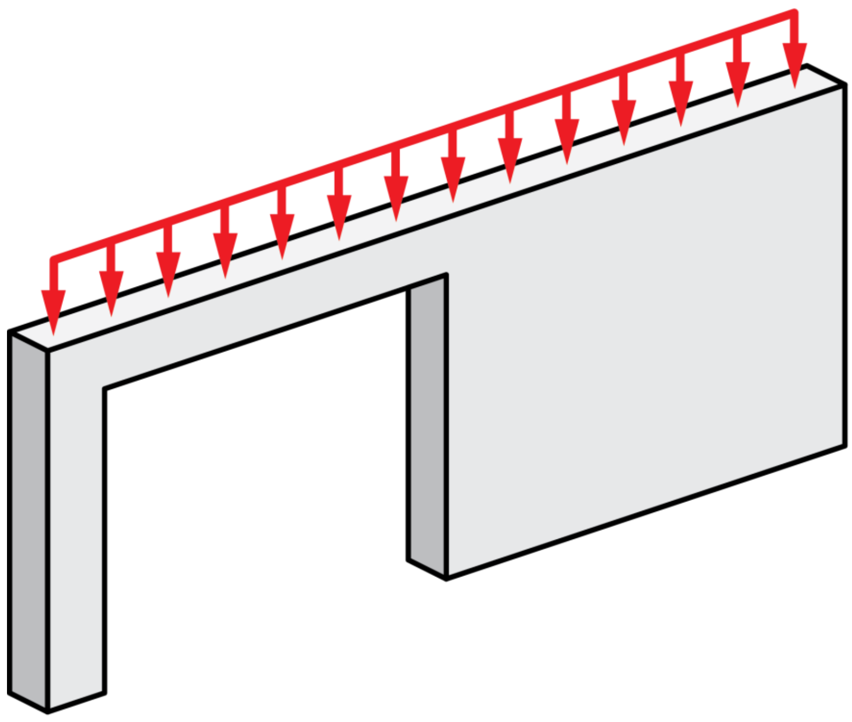
2) GROUND STRUCTURES IN 2D

- EXAMPLE
  - CONNECTIVITY: LEVEL 5



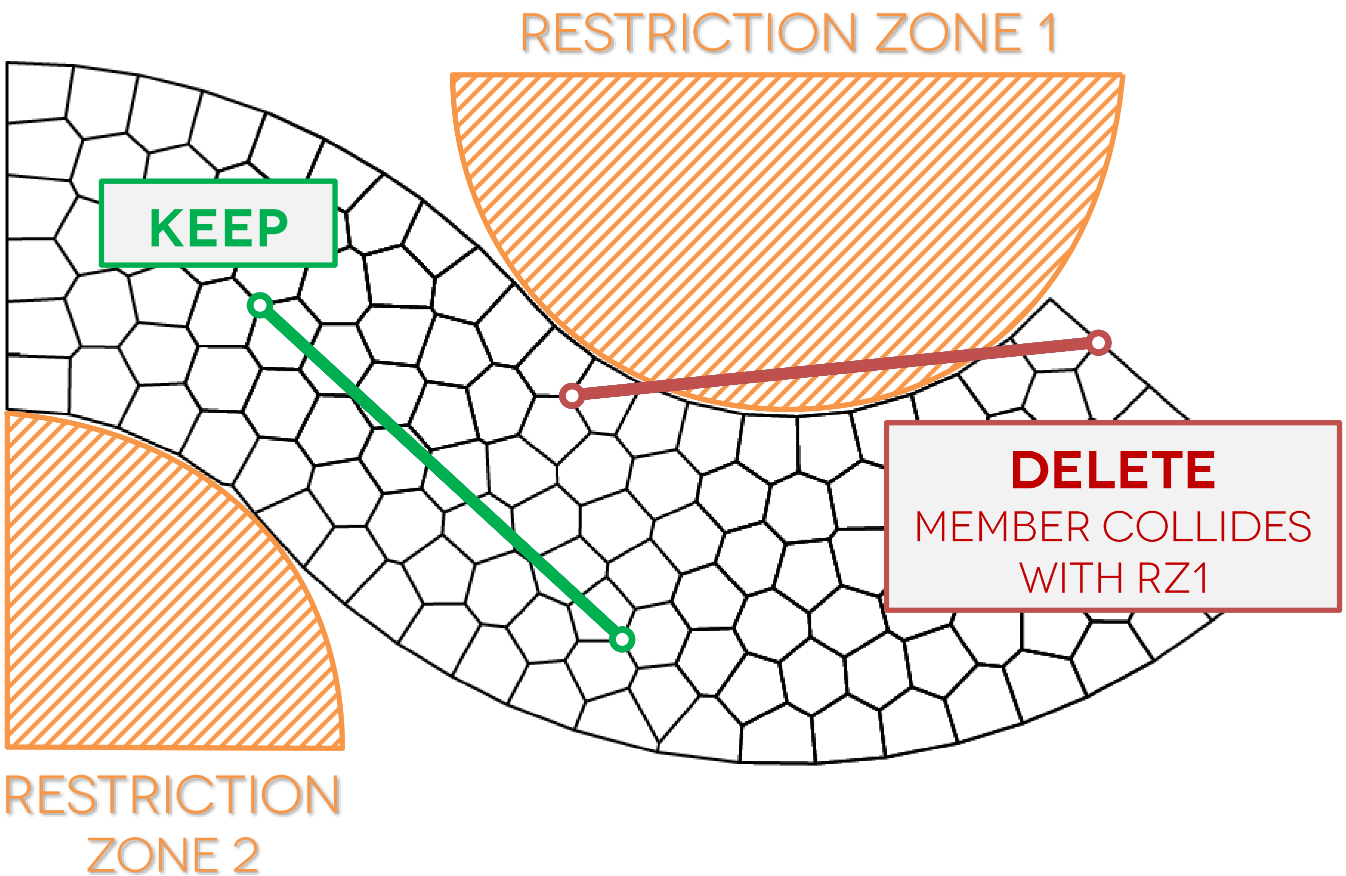
2) GROUND STRUCTURES IN 2D

- THERE CANNOT BE BARS **EVERYWHERE**
  - DEFINE ZONES WHERE NO BARS CAN BE



2) GROUND STRUCTURES IN 2D

- INTERSECTION TESTS FROM VIDEO-GAME AND COMPUTER GRAPHICS INDUSTRY

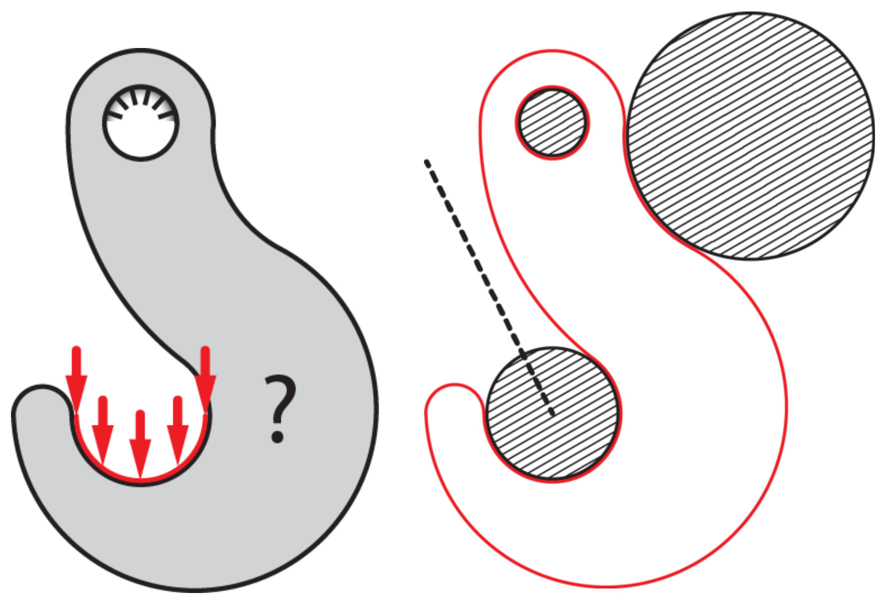


2) GROUND STRUCTURES IN 2D

- RESTRICTION ZONE PRIMITIVES
  - CIRCLE
  - SEGMENT (LINE)
  - RECTANGLE
  - POLYGON

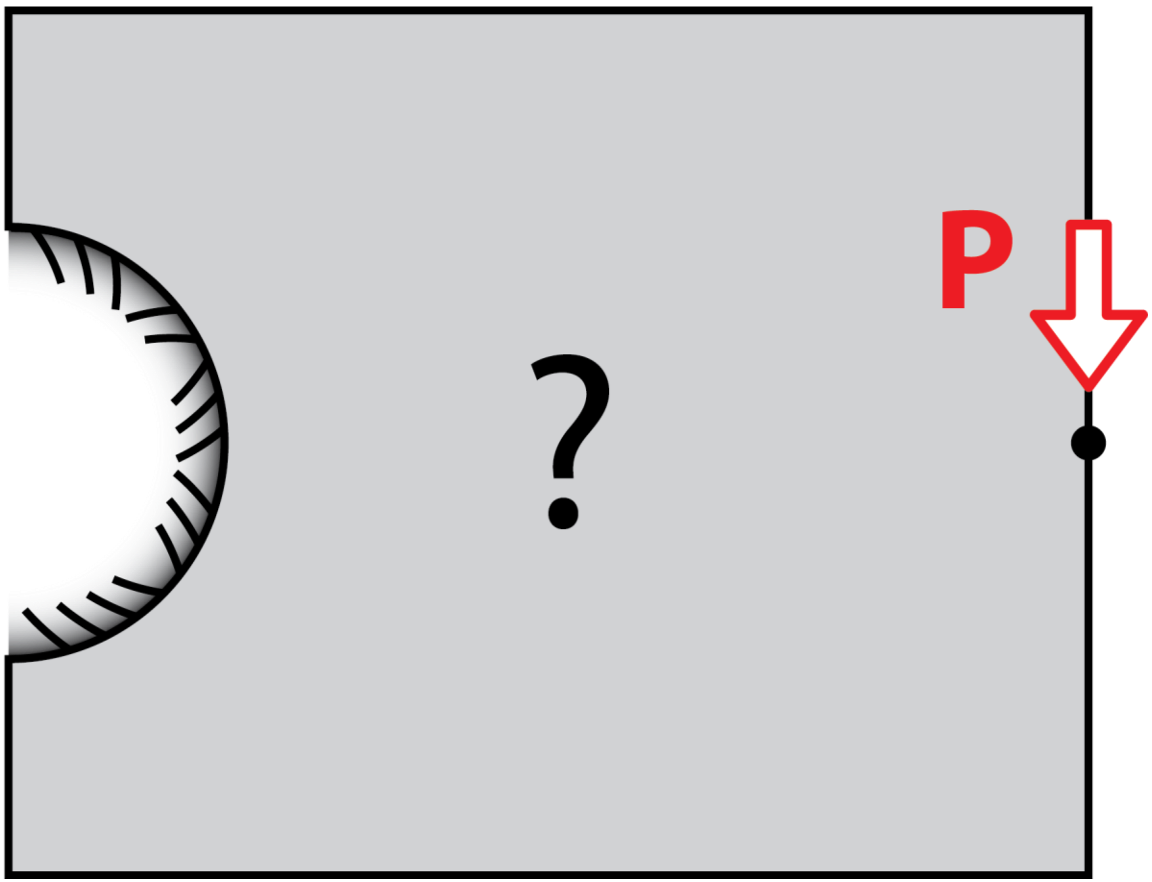
3 CIRCLES
+
1 SEGMENT

- CAN BE COMBINED...



2) GROUND STRUCTURES IN 2D

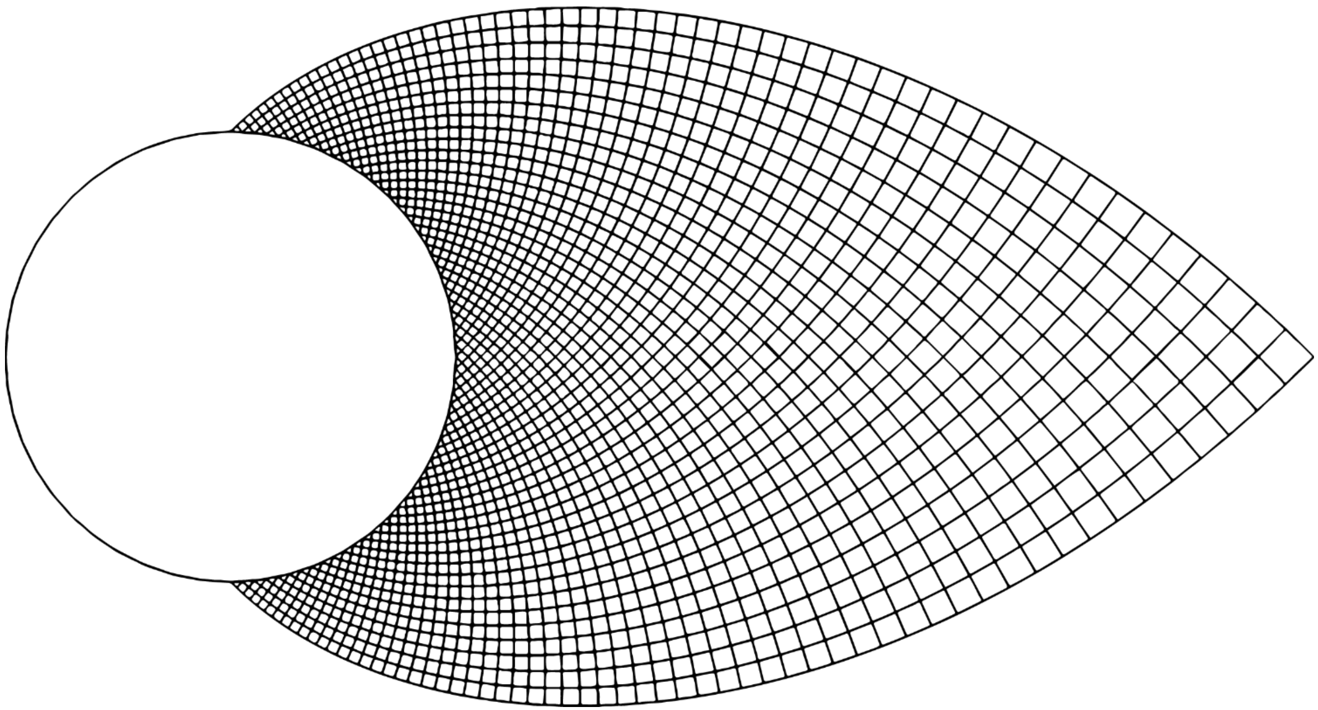
- MICHELL CANTILEVER



MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES"
PHILOS. MAGAZINE SERIES 6, 8(47), 589-597

2) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER



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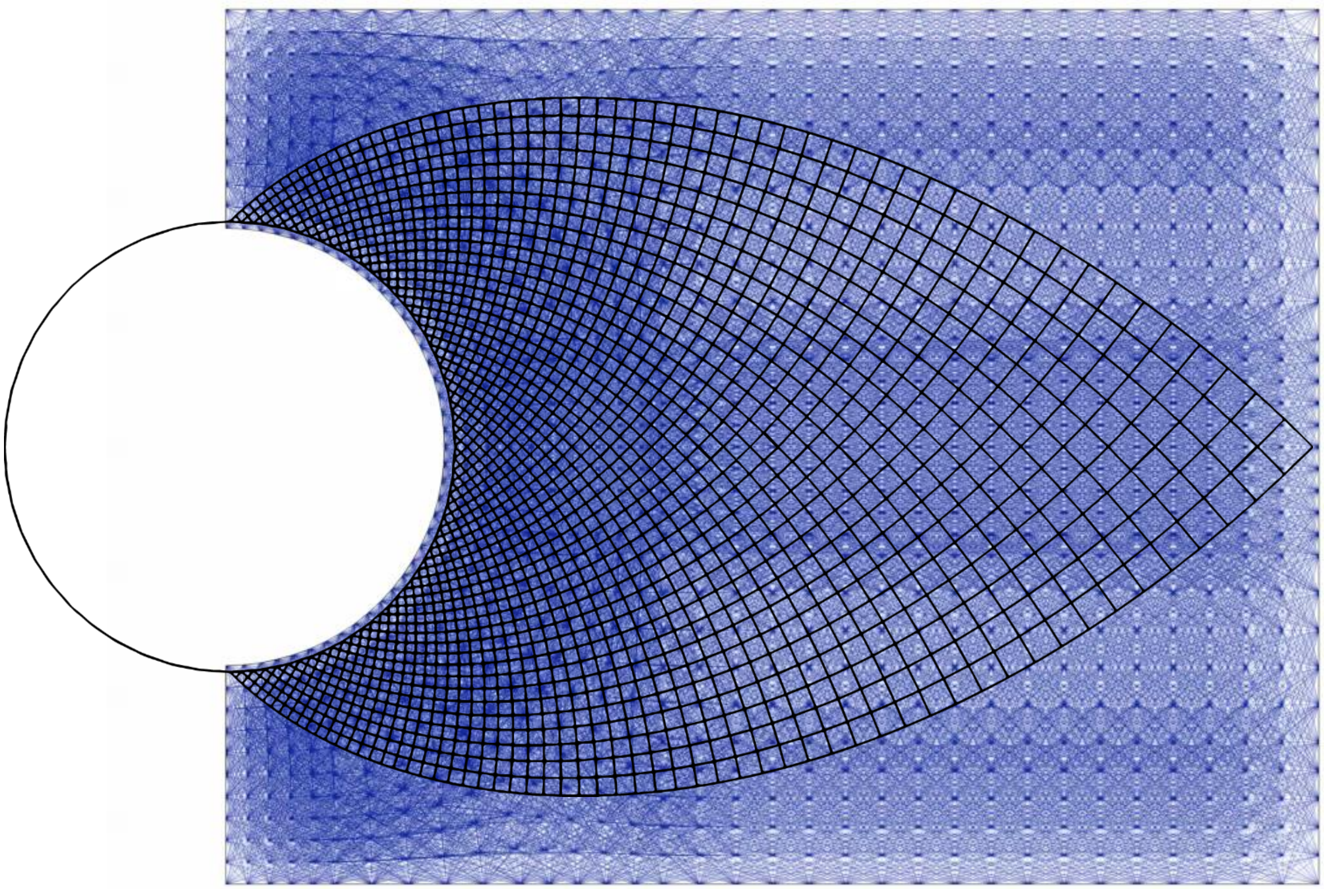
2) GROUND STRUCTURES IN 2D

- MICHELL CANTILEVER 28,256 BARS



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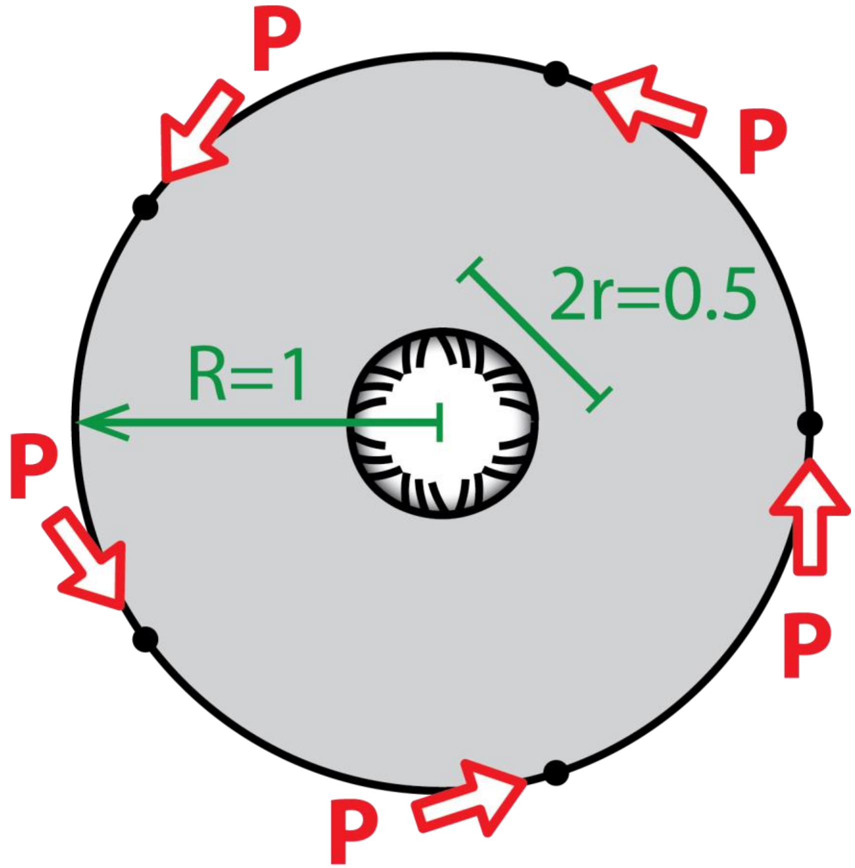
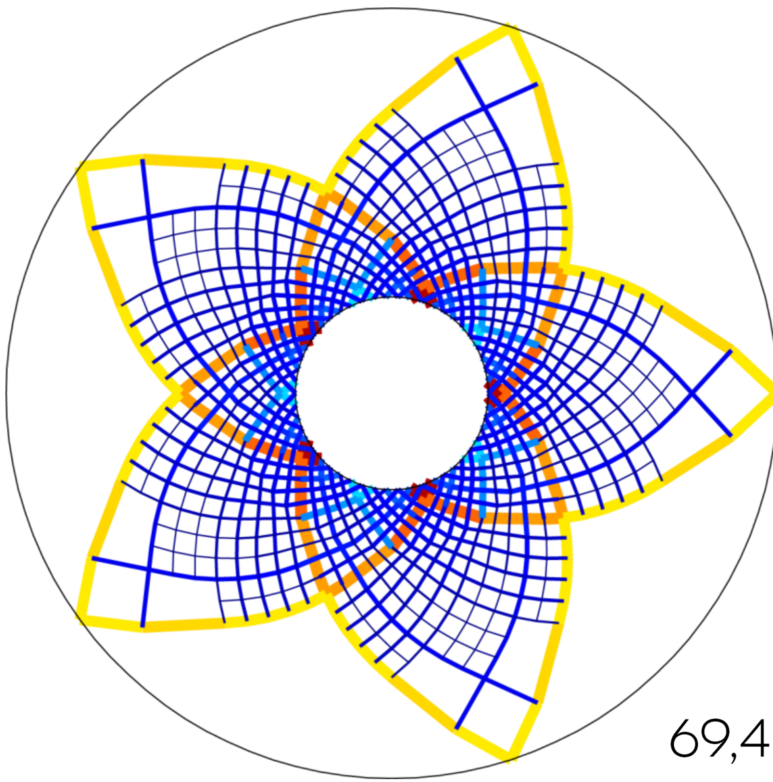
2) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM



2) GROUND STRUCTURES IN 2D

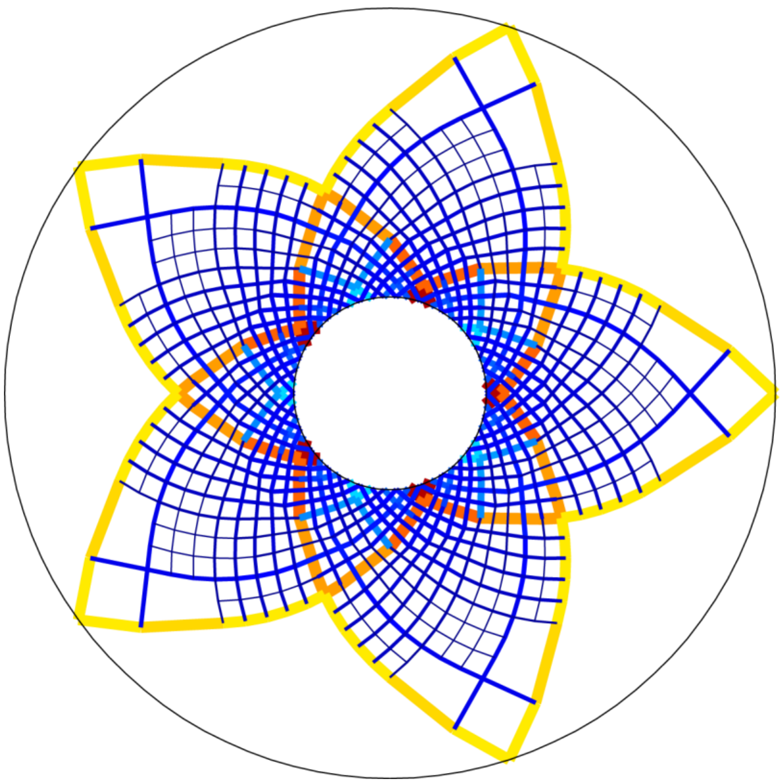
- FLOWER PROBLEM



69,400 BARS

2) GROUND STRUCTURES IN 2D

- FLOWER PROBLEM



2) GROUND STRUCTURES IN 2D

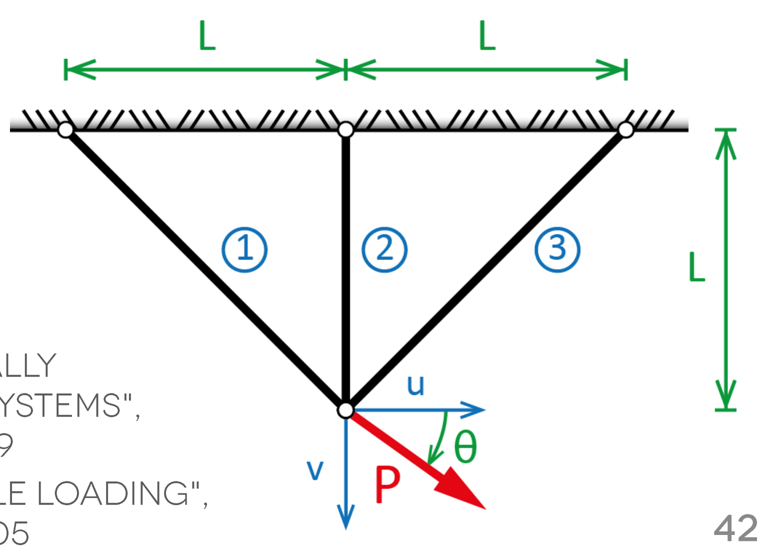
- STRESS DISCONTINUITY PROBLEM
  - GIVEN AN (ADMISSIBLE) DISPLACEMENT FIELD $\mathbf{u}$

$$\boldsymbol{\delta} = \mathbf{B}^{\mathrm{T}}\mathbf{u}$$
$$\boldsymbol{\delta} = \boldsymbol{\varepsilon} : \mathbf{l}$$
$$\boldsymbol{\sigma} = \mathbf{E}\boldsymbol{\varepsilon}$$

  - STRESSES $\boldsymbol{\sigma}$ ARE INDEPENDENT OF AREAS…

$\sigma \neq 0$ EVEN FOR $a_i = 0$

- 3 BAR PROBLEM



SCHMIDT LC (1960), "MINIMUM WEIGHT LAYOUTS OF ELASTIC, STATICALLY DETERMINATE, TRIANGULATED FRAMES UNDER ALTERNATIVE LOAD SYSTEMS", JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS 10(2), 139–149

SVED G, GINOS Z (1968), "STRUCTURAL OPTIMIZATION UNDER MULTIPLE LOADING", INTERNATIONAL JOURNAL OF MECHANICAL SCIENCES 10(10), 803–805

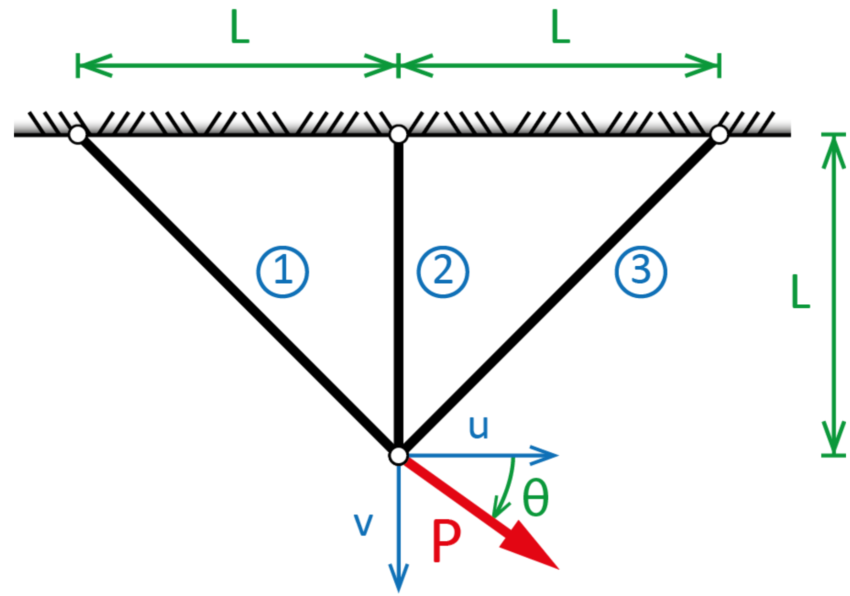
2) GROUND STRUCTURES IN 2D

- PROBLEM STATEMENT
  - THREE LOAD CASES

$$f_A = 40 \ @ \ \frac{\pi}{4}$$

$$f_B = 30 \ @ \ \frac{\pi}{2}$$

$$f_C = 20 \ @ \frac{3\pi}{4}$$

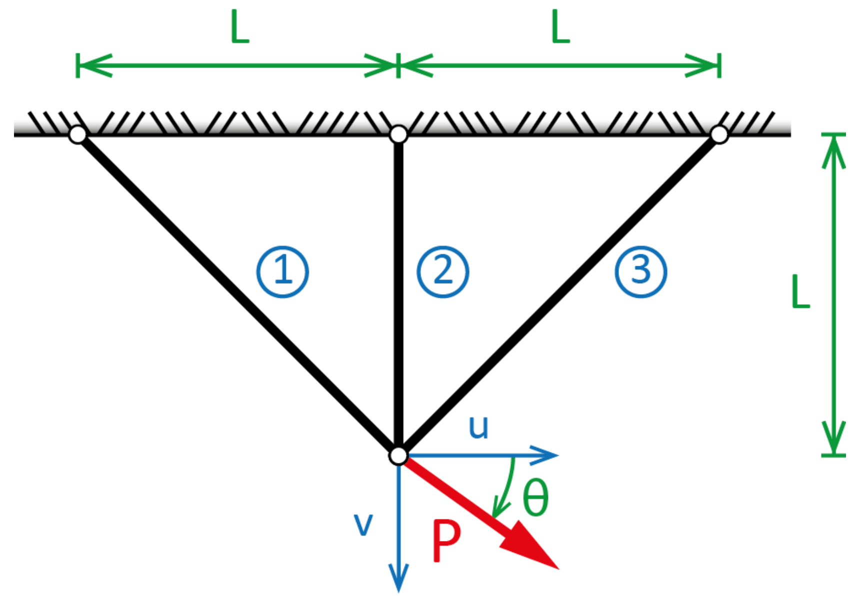


  - STRESS LIMITS $\sigma_T = \sigma_C = \bar{\sigma}$

$$\bar{\sigma}_1 = 5$$
$$\bar{\sigma}_2 = 20$$
$$\bar{\sigma}_3 = 5$$

$$\begin{aligned} \min_{\mathbf{a}} \quad & V = \mathbf{a}^T \mathbf{l} \\ \text{s.t.} \quad & \mathbf{Ku} = \mathbf{f} \\ & -\sigma_C \leq \boldsymbol{\sigma} \leq \sigma_T \quad \text{if} \quad a_i > 0 \\ & \mathbf{a} \geq 0 \end{aligned}$$

2) GROUND STRUCTURES IN 2D



- SOLUTION $f(\mathbf{a}^{\star}) = 15.97$

  $\mathbf{a}^{\star} = \{7.02 \quad 2.14 \quad 2.76\}$

	Bar 1: $\sigma = 5$	Bar 2: $\sigma = 20$	Bar 3: $\sigma = 5$
$f_A = 40 @ \frac{\pi}{4}$	**5.0**	1.7	−0.9
$f_B = 30 @ \frac{\pi}{2}$	3.2	6.1	4.1
$f_C = 20 @ \frac{3\pi}{4}$	−1.8	4.4	**5.0**

$= \mathbf{a}^T\mathbf{l}$

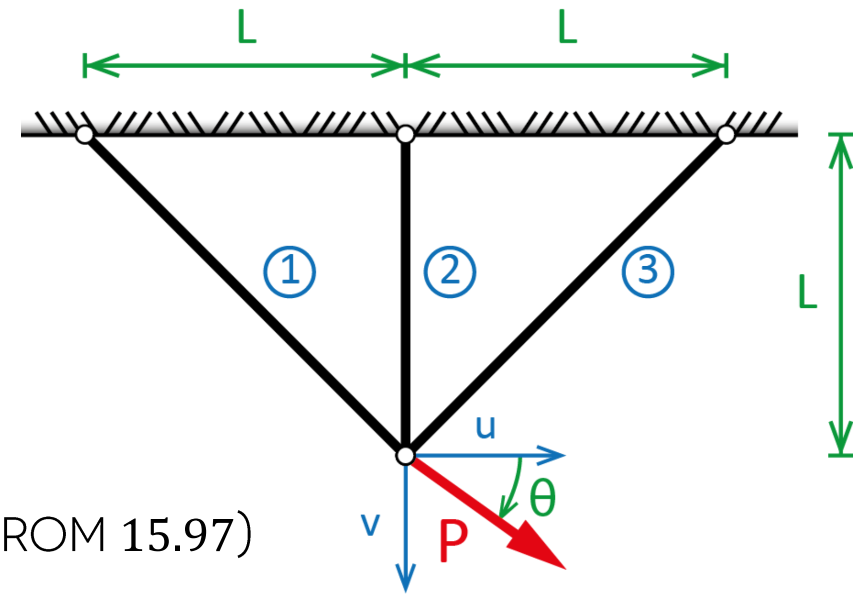
$\mathbf{u} = \mathbf{f}$

$\leq \boldsymbol{\sigma} \leq \sigma_T \quad \text{if} \quad a_i > 0$

≥ 0

2) GROUND STRUCTURES IN 2D

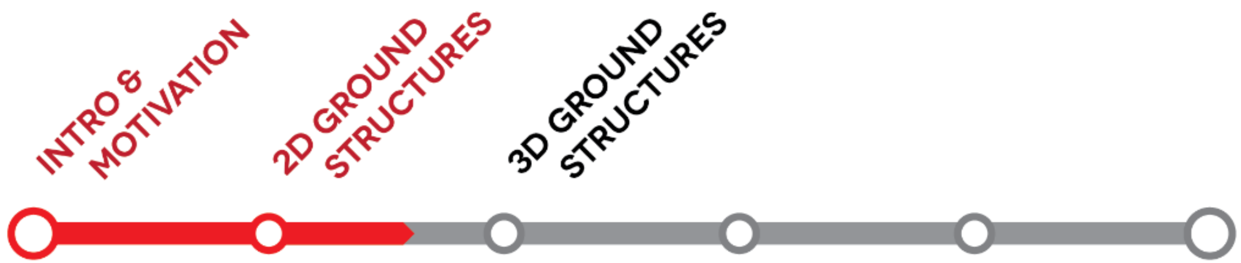
$$\begin{aligned} \min_{\mathbf{a}} \quad & V = \mathbf{a}^T \mathbf{l} \\ \text{s.t.} \quad & \mathbf{Ku} = \mathbf{f} \\ & -\sigma_C \leq \boldsymbol{\sigma} \leq \sigma_T \quad \text{if} \quad a_i > 0 \\ & \mathbf{a} \geq 0 \end{aligned}$$



- SOLUTION $f(\mathbf{a}^\star) = 12.81$ (FROM 15.97)
  $\mathbf{a}^\star = \{8.00 \quad 1.50 \quad 0.00\}$

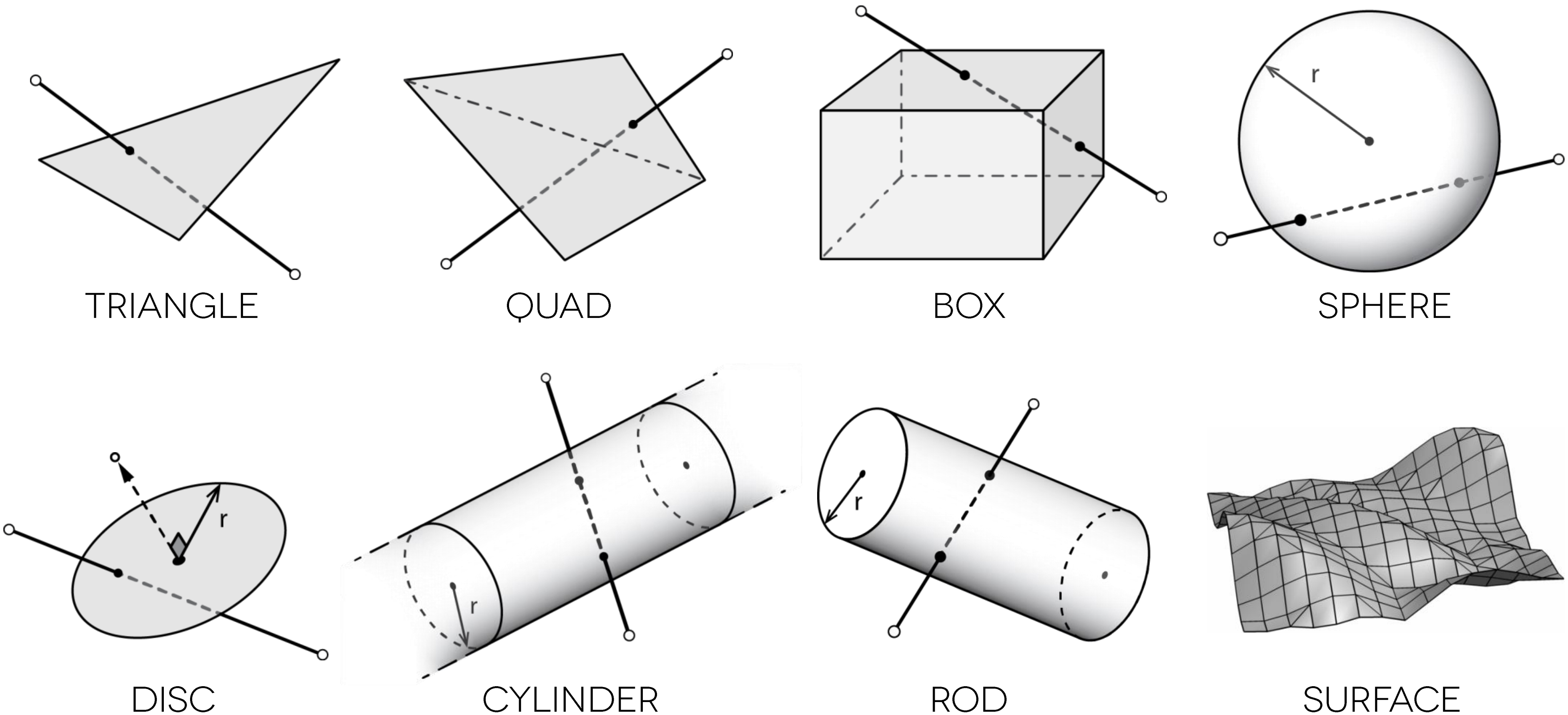
	Bar 1: $\sigma = 5$	Bar 2: $\sigma = 20$	Bar 3: $\sigma = 5$
$f_A = 40 @ \frac{\pi}{4}$	$\mathbf{5.0}$	0.0	-2.5
$f_B = 30 @ \frac{\pi}{2}$	0.0	$\mathbf{20.0}$	$\mathbf{18.9}$
$f_C = 20 @ \frac{3\pi}{4}$	$\mathbf{-5.0}$	$\mathbf{20.0}$	$\mathbf{21.4}$

ROADMAP


INTRO & MOTIVATION
2D GROUND STRUCTURES
3D GROUND STRUCTURES

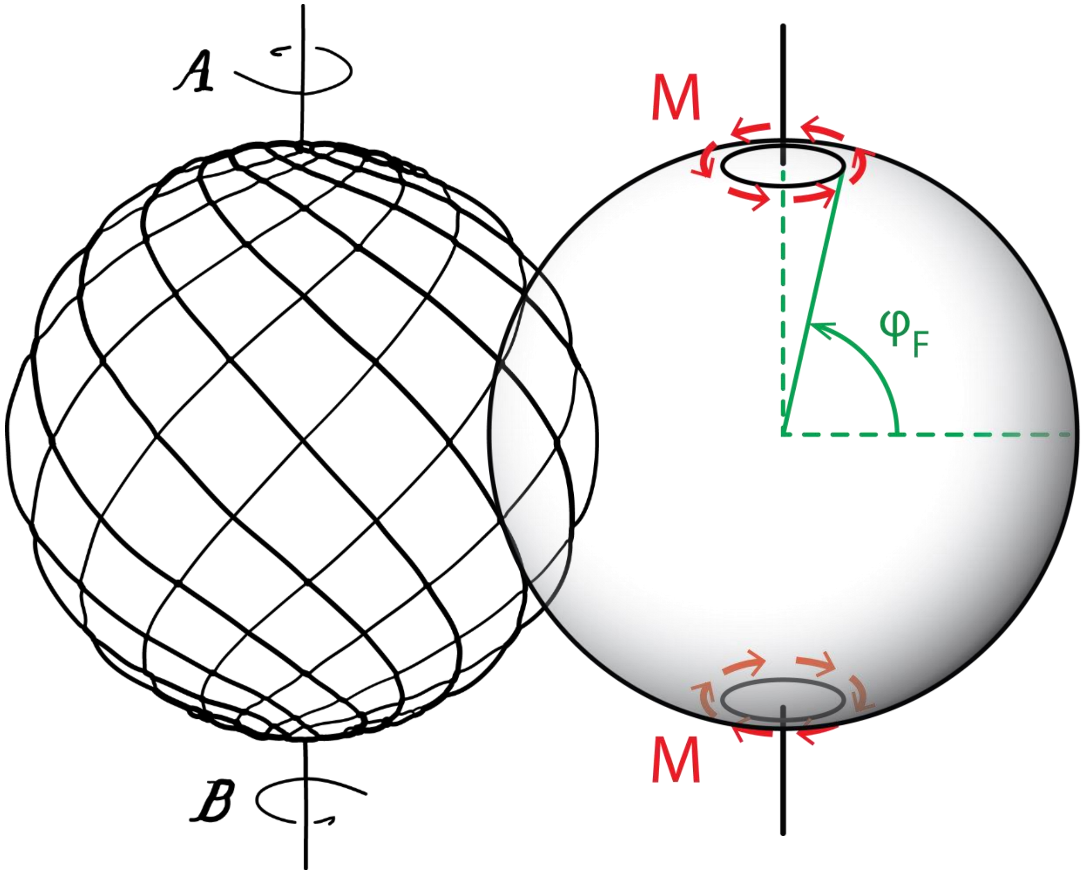
3) GROUND STRUCTURES IN 3D

- RESTRICTION PRIMITIVES:



3) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

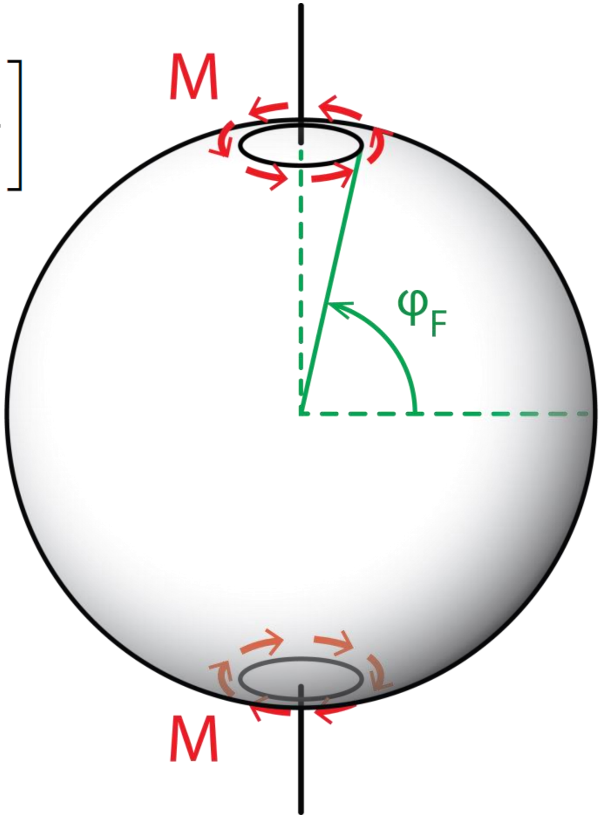


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES“
PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

3) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM

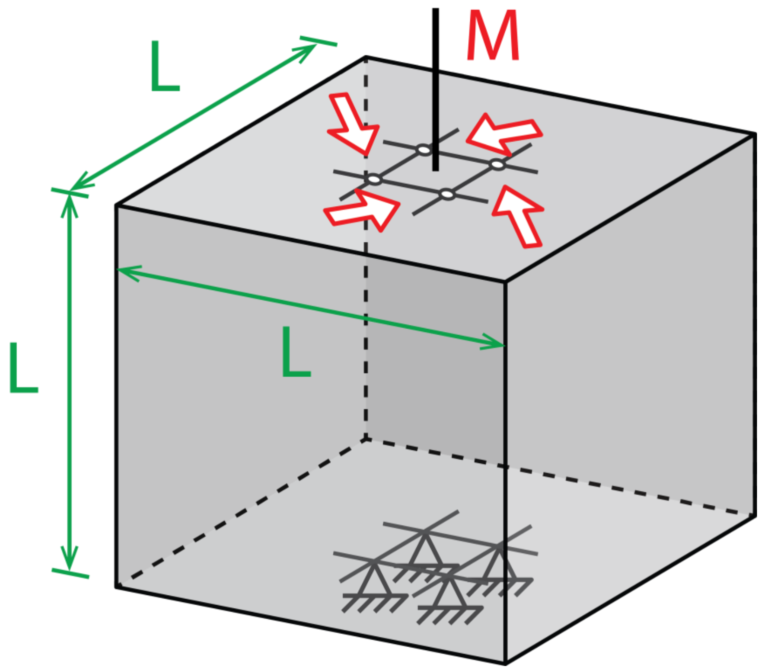
$$V_{opt} = 2M \log\left(\tan\left\{\frac{\pi}{4} + \frac{\phi_F}{2}\right\}\right)\left[\frac{1}{\sigma_T} + \frac{1}{\sigma_C}\right]$$

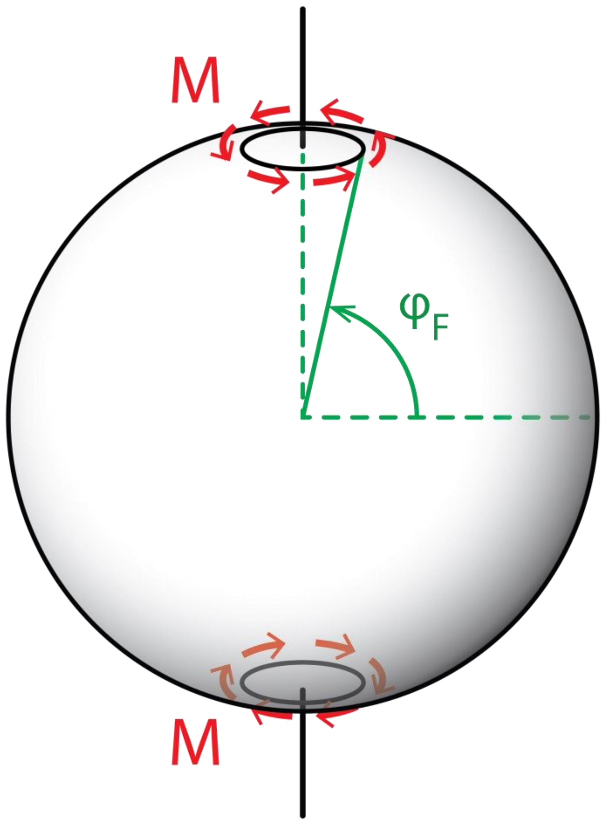


MICHELL AGM (1904), "THE LIMITS OF ECONOMY OF MATERIAL IN FRAME-STRUCTURES"
PHILOS. MAGAZINE SERIES 6, 8(47), 589–597

3) GROUND STRUCTURES IN 3D

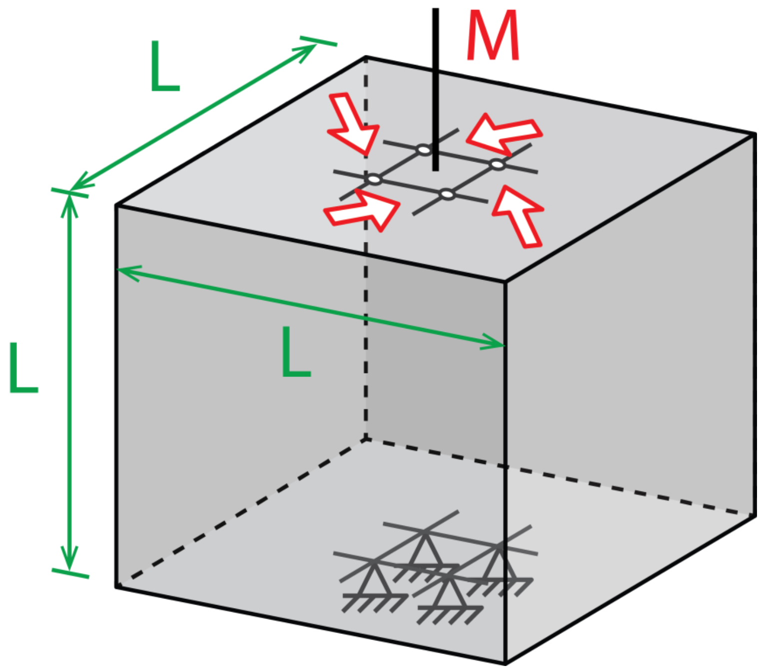
- TORSION BALL PROBLEM

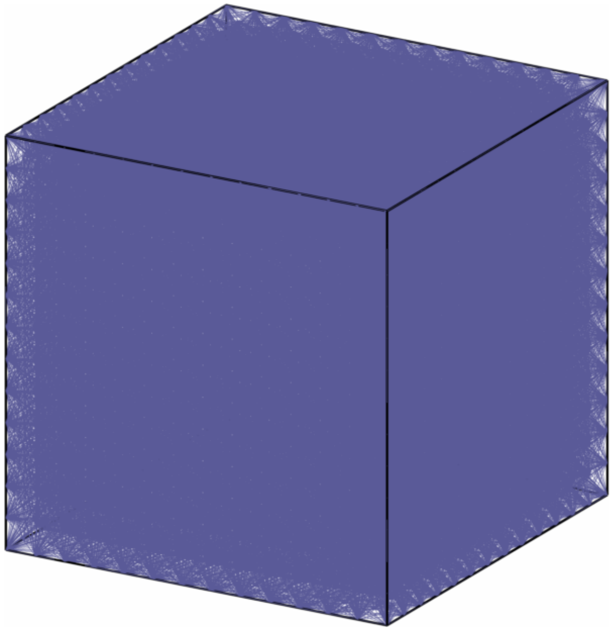




3) GROUND STRUCTURES IN 3D

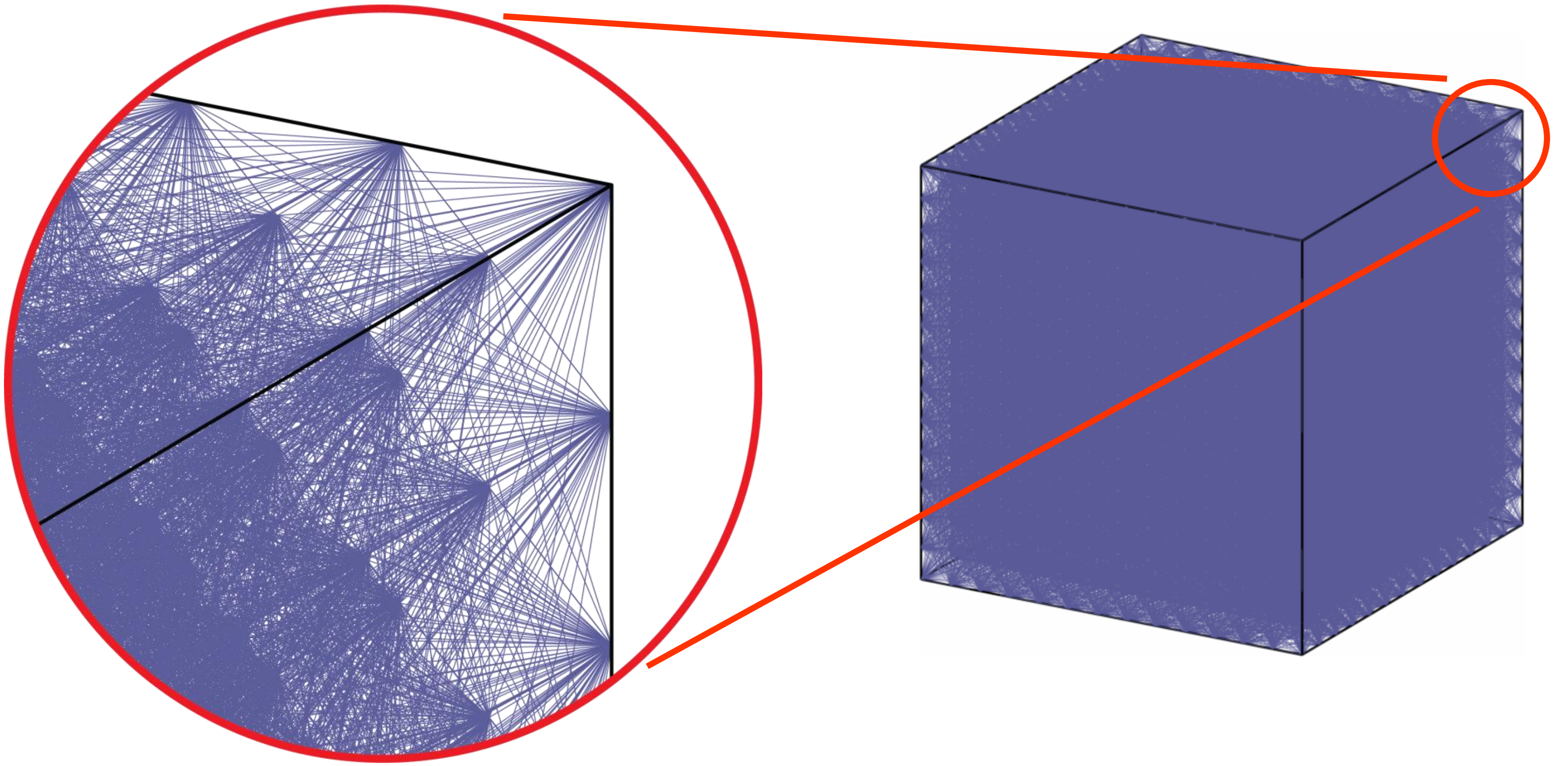
- TORSION BALL PROBLEM





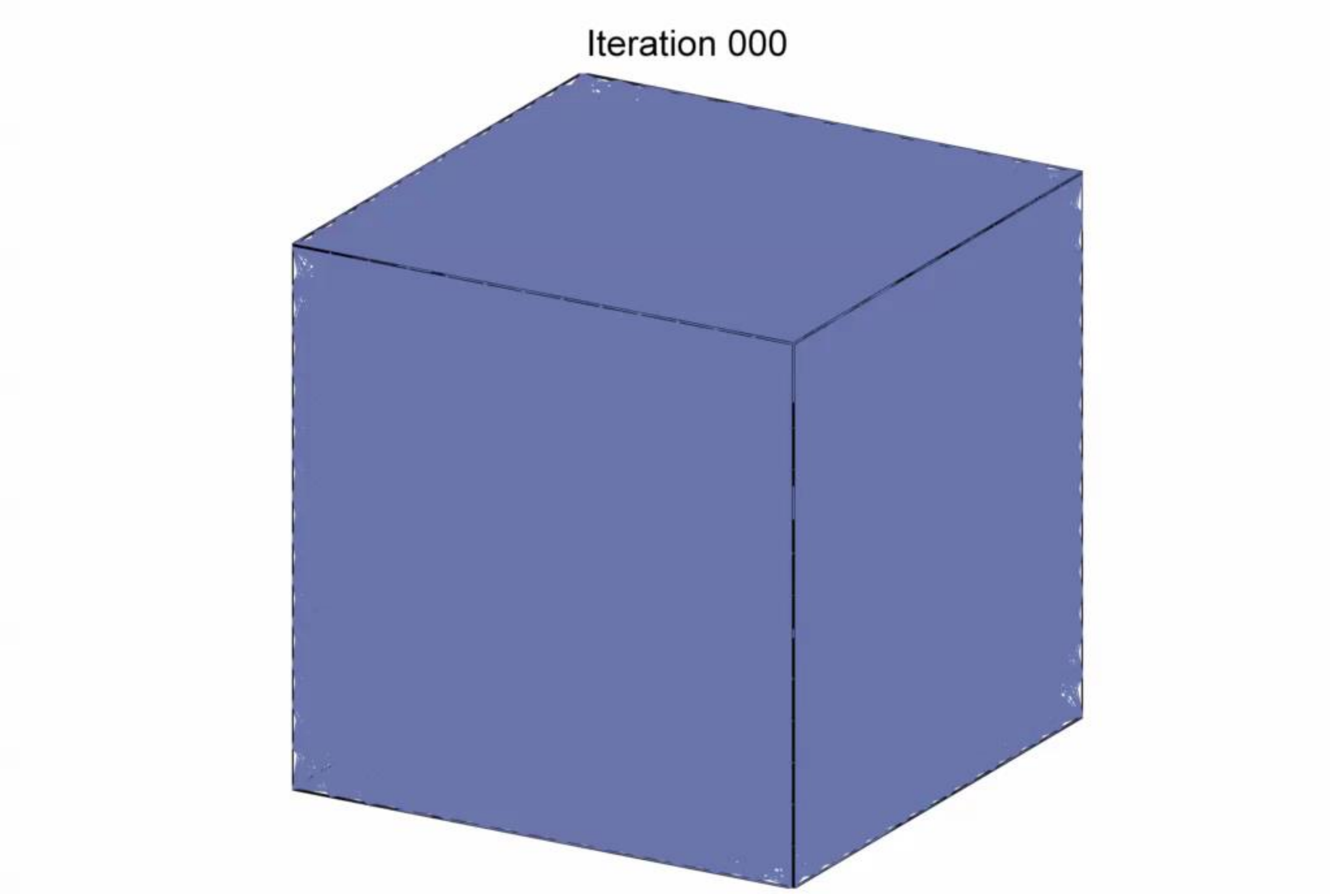
3) GROUND STRUCTURES IN 3D

- TORSION BALL PROBLEM



3) GROUND STRUCTURES IN 3D

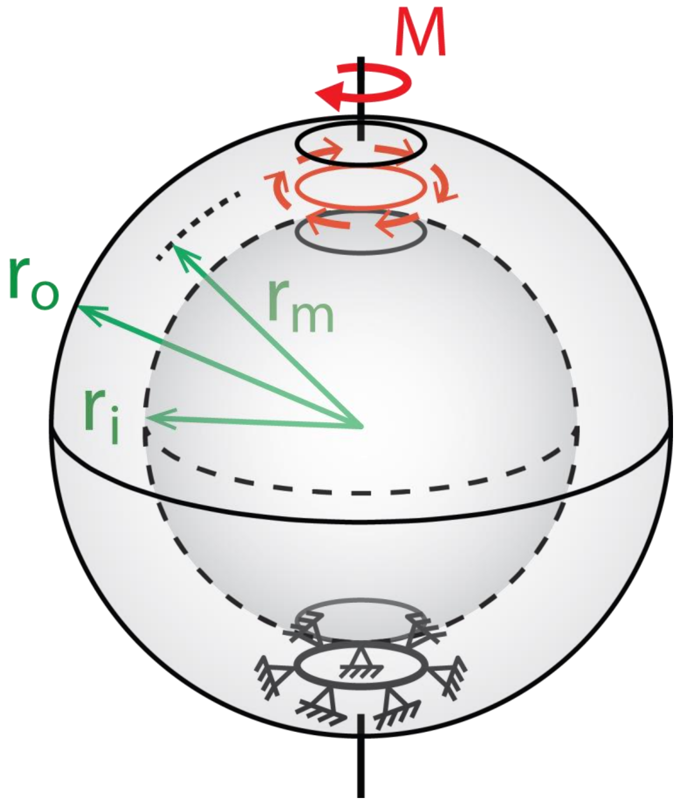
- TORSION BALL PROBLEM

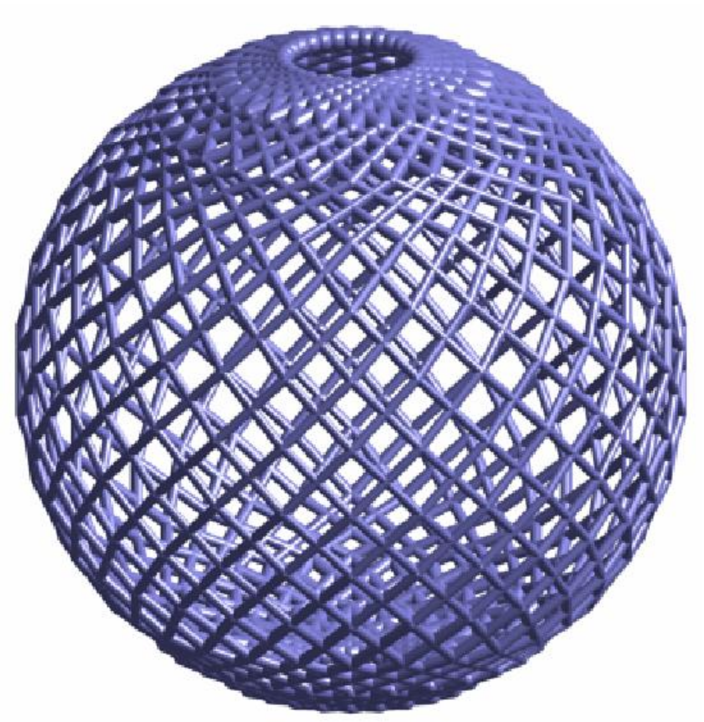


268,636 BARS

3) GROUND STRUCTURES IN 3D

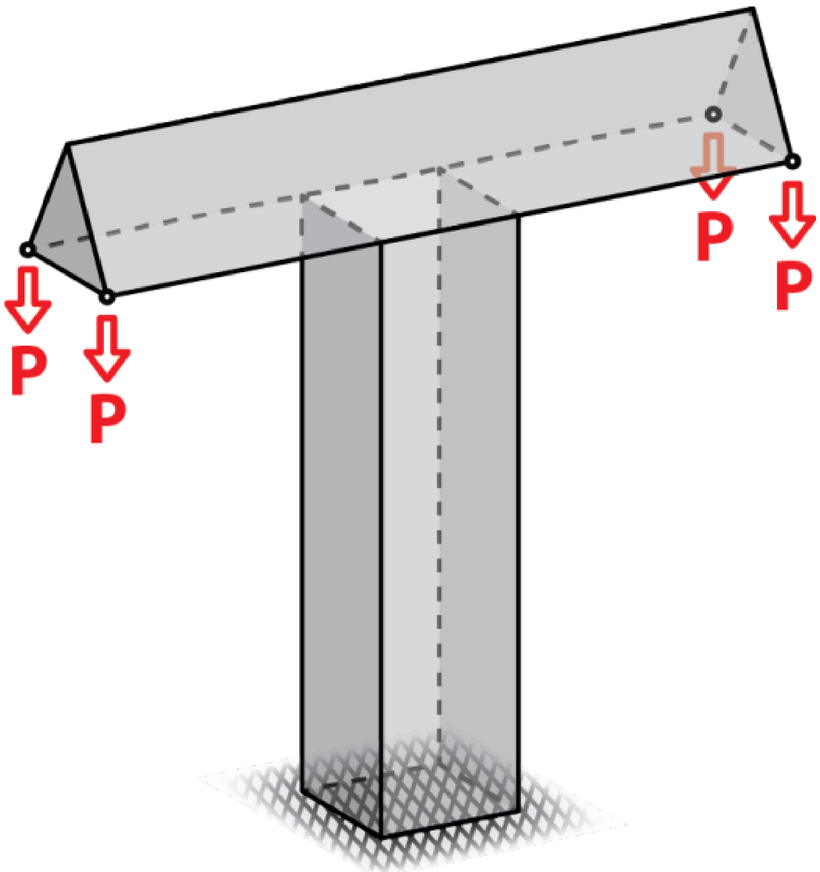
- TORSION BALL PROBLEM
  IMPROVING THE BASE MESH: SPHERICAL COORDINATES

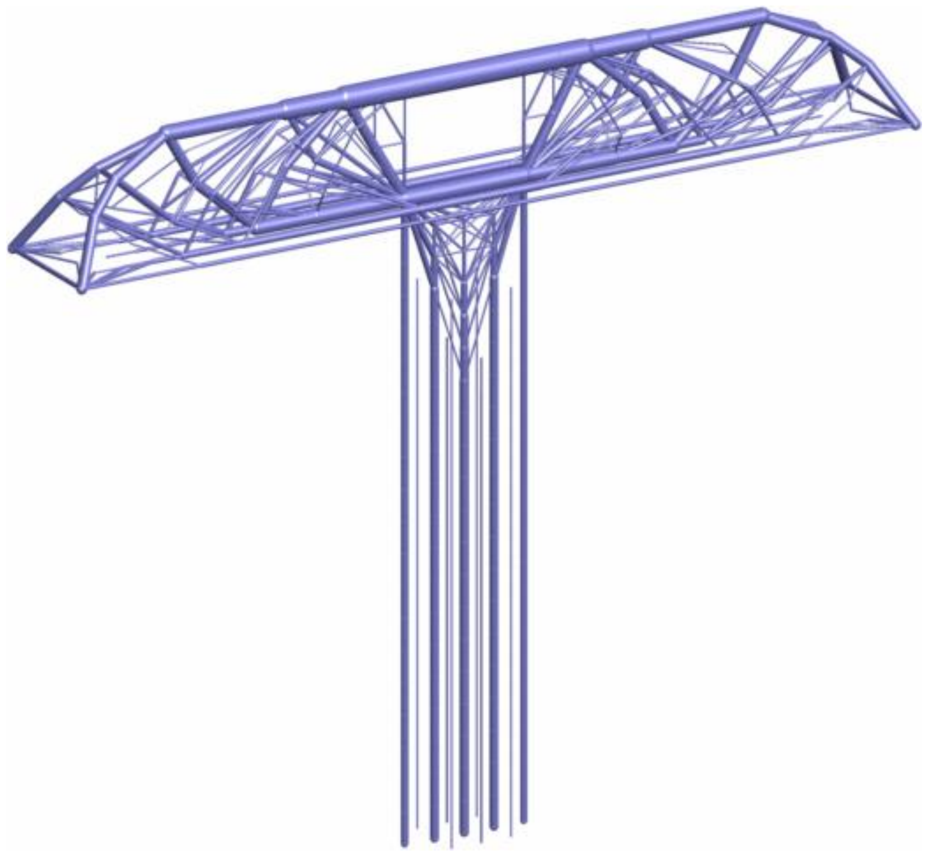




3) GROUND STRUCTURES IN 3D

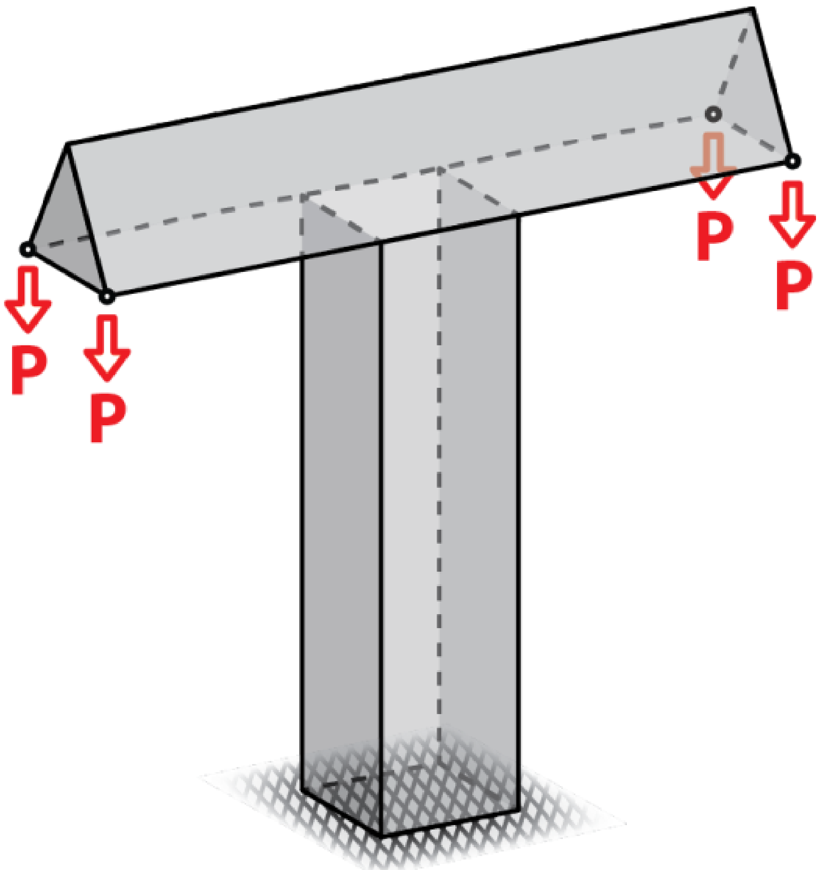
- MORE APPLIED PROBLEMS?

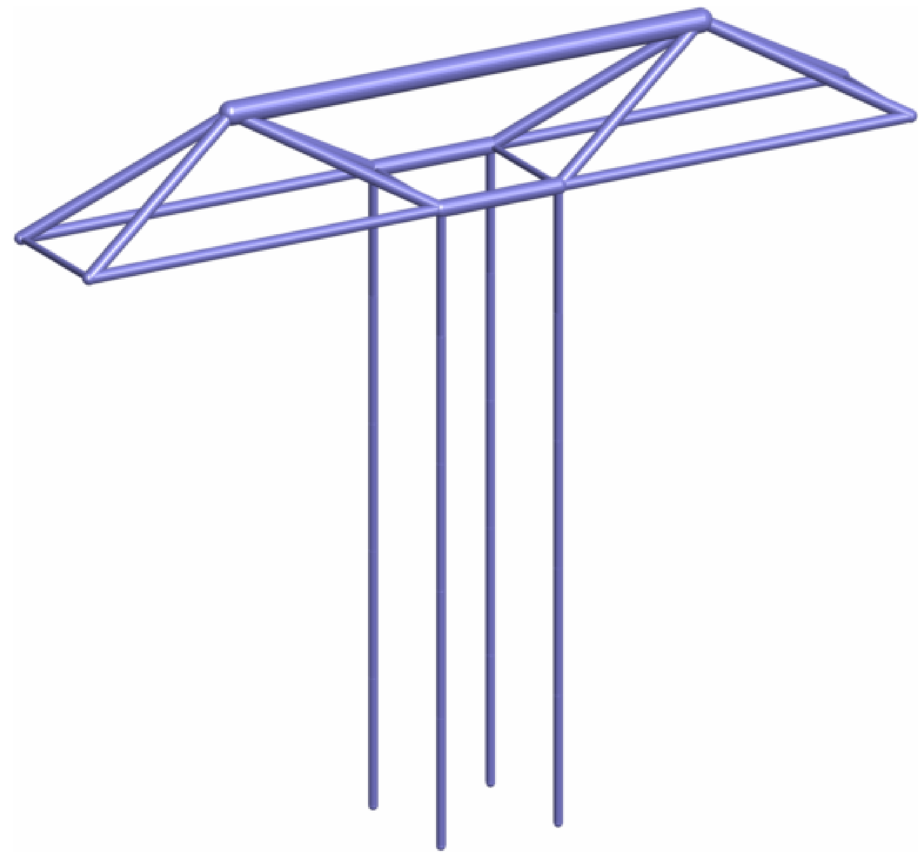




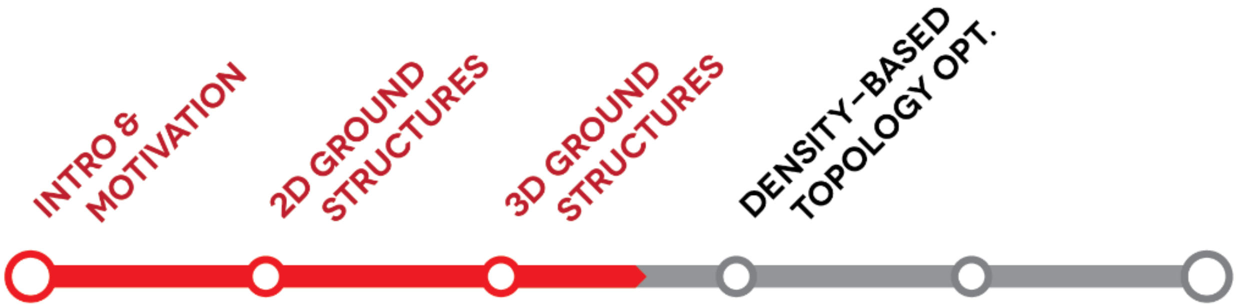
3) GROUND STRUCTURES IN 3D

- MORE APPLIED PROBLEMS?





ROADMAP



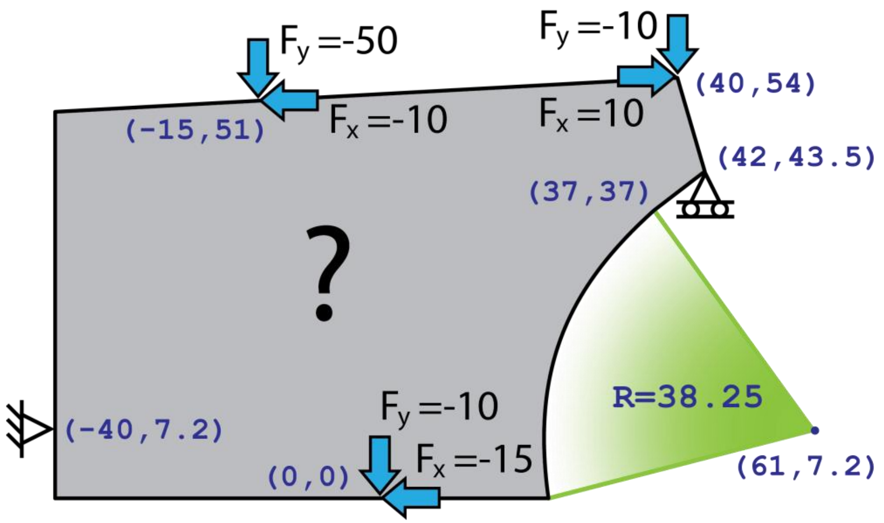
INTRO &
MOTIVATION
2D GROUND
STRUCTURES
3D GROUND
STRUCTURES
DENSITY-BASED
TOPOLOGY OPT.

4) DENSITY-BASED TOPOLOGY OPT

- DENSITY-BASED TOPOLOGY OPTIMIZATION

HTTP://WWW.CANNONDALE.COM

CANNONDALE CAPO
(URBAN COMMUTER BIKE)

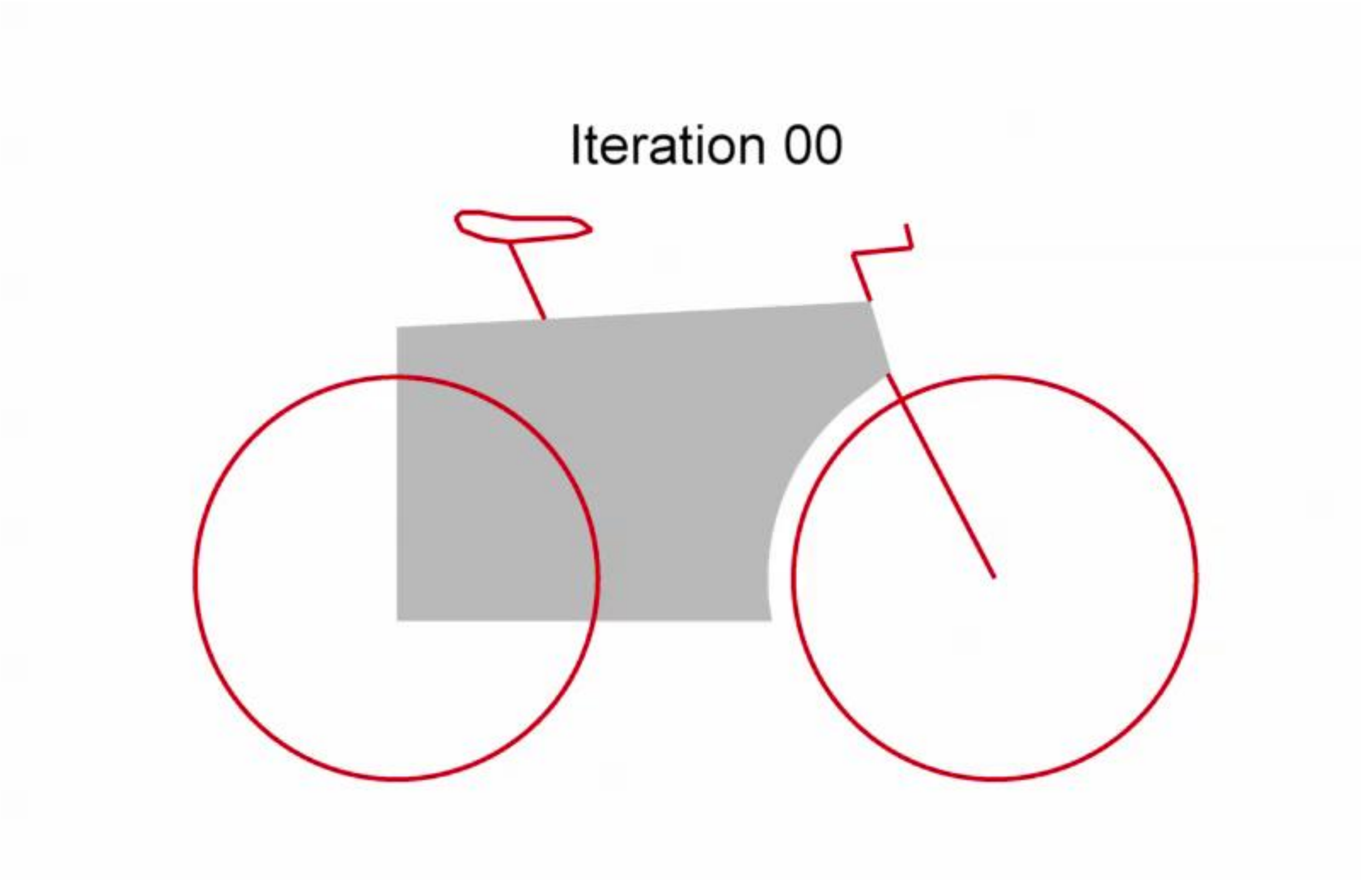


BIKE DOMAIN AND LOADS

ZEGARD T, PAULINO GH (2013) "TOWARD GPU ACCELERATED TOPOLOGY OPTIMIZATION ON UNSTRUCTURED MESHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 48(3):473-485

4) DENSITY-BASED TOPOLOGY OPT

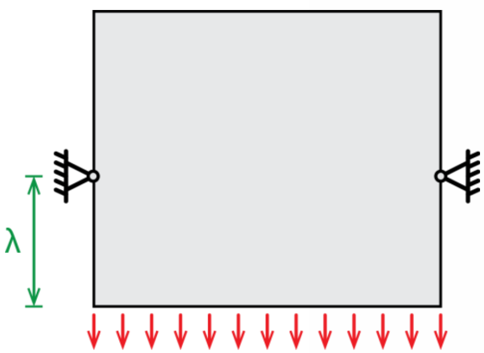
- DENSITY-BASED TOPOLOGY OPTIMIZATION

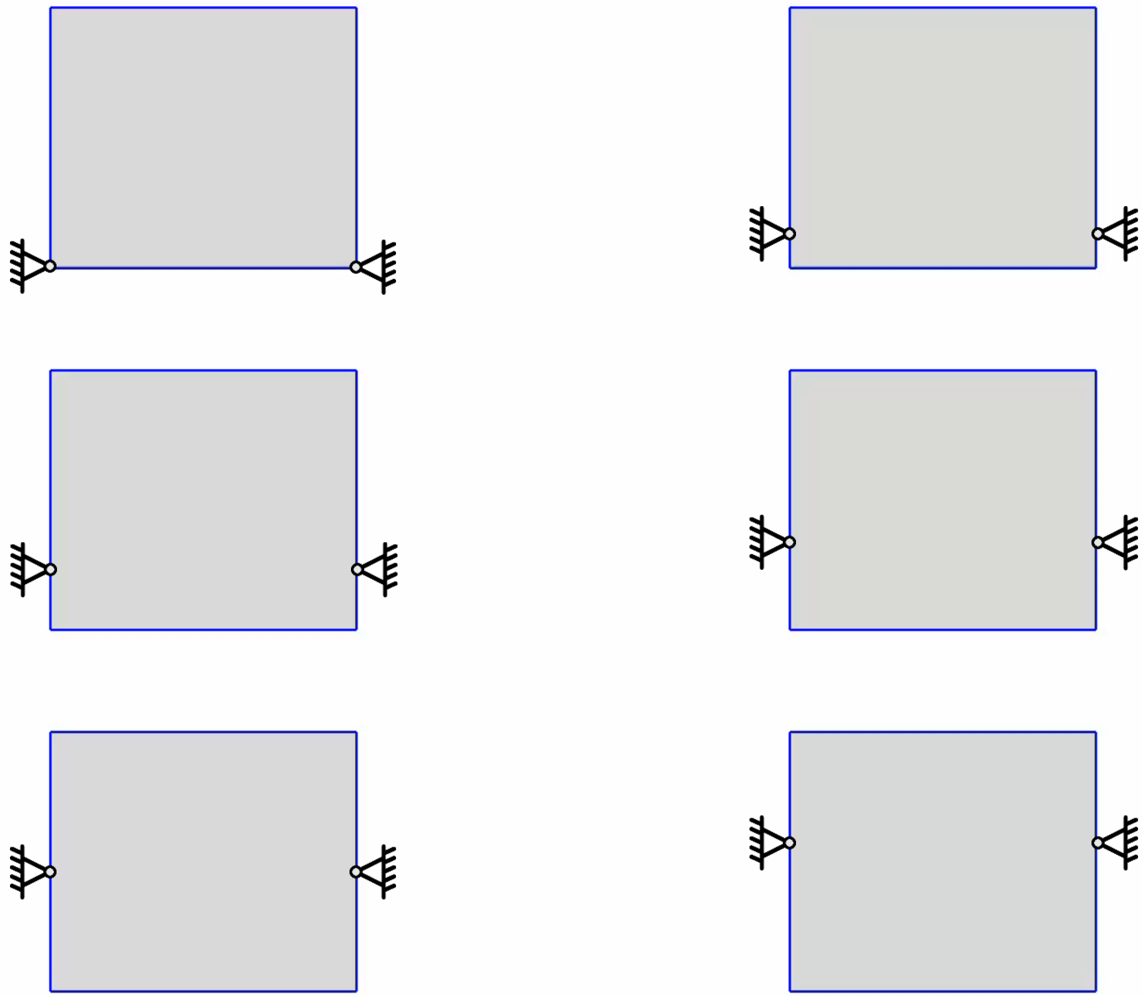


ZEGARD T, PAULINO GH (2013) "TOWARD GPU ACCELERATED TOPOLOGY OPTIMIZATION ON UNSTRUCTURED MESHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 48(3):473-485

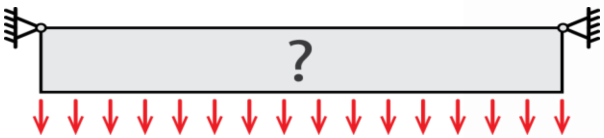
4) DENSITY-BASED TOPOLOGY OPT

- ARCH OR SUSPENDED?



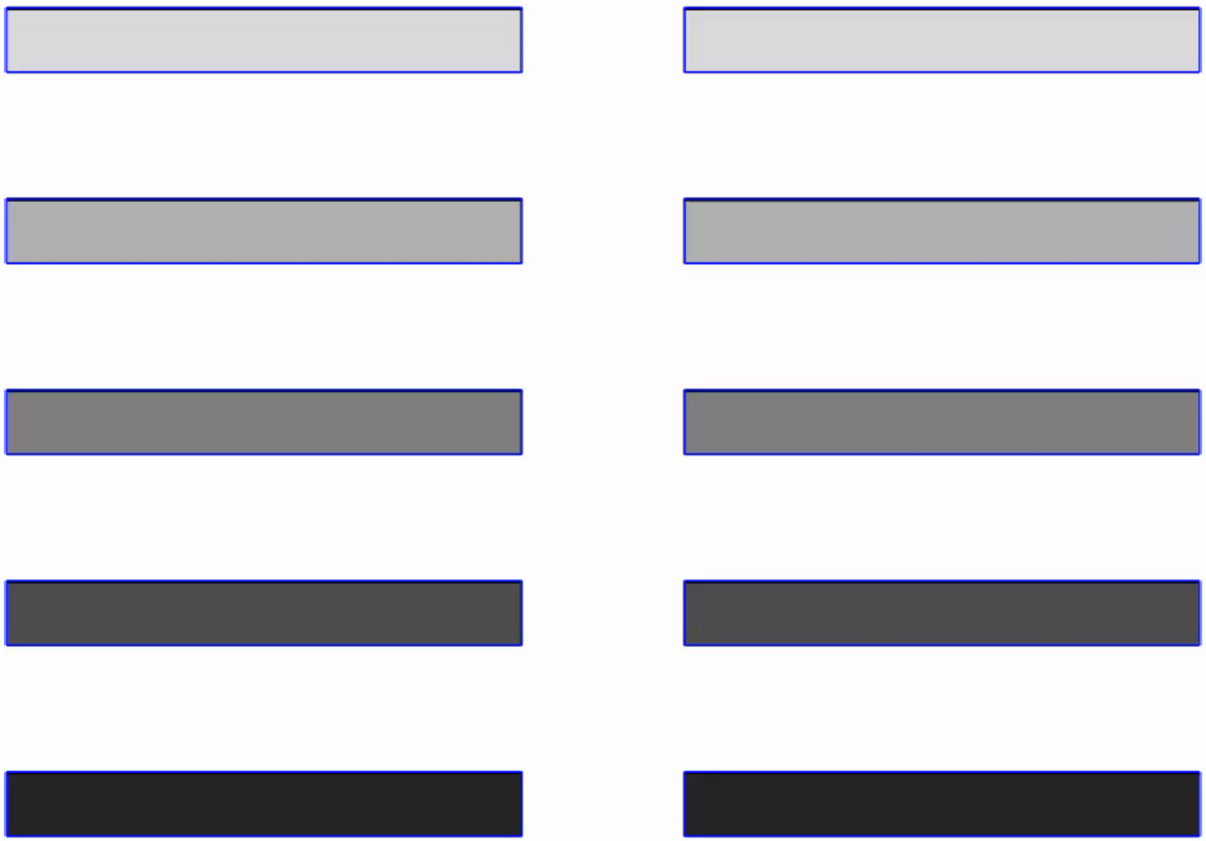


4) DENSITY-BASED TOPOLOGY OPT



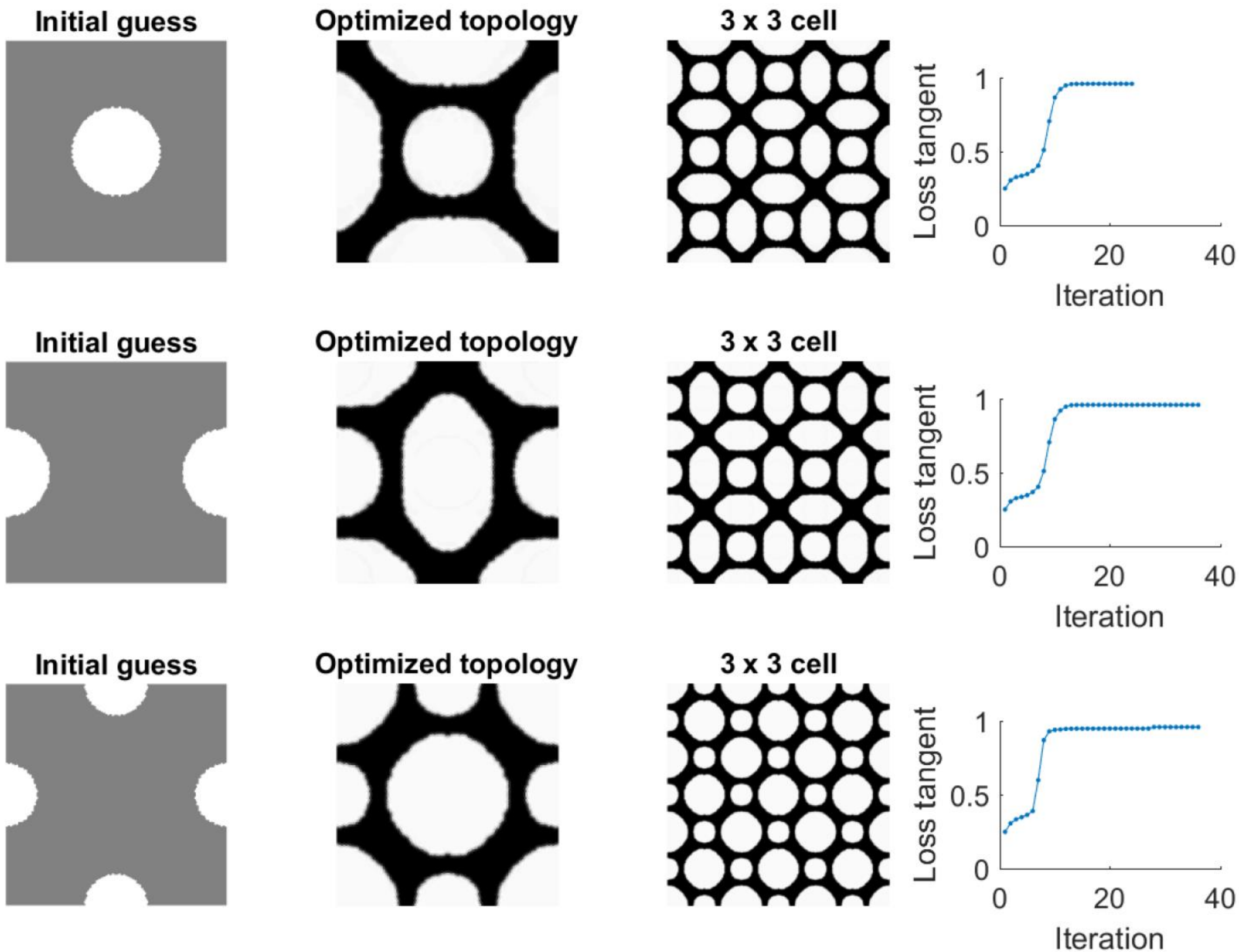
LOW FILTER

HIGH FILTER



4) Density-based topology opt

- Maximization of loss tangent



4) DENSITY-BASED TOPOLOGY OPT

- DENSITY-BASED (NESTED) FORMULATION:
  - USING A DENSITY FILTER[1]
  - MODIFIED SIMP[234]

$$
\begin{aligned}
\min_{\boldsymbol{\rho}} \quad & J\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \\
\text{s.t.} \quad & \overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho} \\
& \textstyle\sum_i^{N_e} \overline{\rho}_i v_i - (f)\left(V_0\right) \leq 0 \\
& g_i\left(\boldsymbol{\rho}, \mathbf{u}\left(\boldsymbol{\rho}\right)\right) \leq 0 \qquad i = 1 \ldots N_c \\
& 0 \leq \rho_j \leq 1 \qquad j = 1 \ldots N_e \\
& E_k\left(\overline{\rho}_k\right) = E_{min} + \overline{\rho}_k^p\left(E_0 - E_{min}\right) \qquad k = 1 \ldots N_e \\
\text{with} \quad & \mathbf{K}\left(\overline{\rho}\right)\mathbf{u} = \mathbf{f}
\end{aligned}
$$

FILTERING ($\overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho}$)

VOLUME CONSTRAINT ($\sum_i^{N_e} \overline{\rho}_i v_i - (f)(V_0) \leq 0$)

1 = SOLID
0 = VOID ($0 \leq \rho_j \leq 1$)

MOD-SIMP ($E_k(\overline{\rho}_k) = E_{min} + \overline{\rho}_k^p(E_0 - E_{min})$)

1) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143-2158

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193-202

3) ZHOU M, ROZVANY G (1991) "THE COC ALGORITHM, PART II: TOPOLOGICAL, GEOMETRICAL AND GENERALIZED SHAPE OPTIMIZATION. "COMP METH APPL MECH ENGRG 89:309-336

4) SIGMUND O (2007) "MORPHOLOGY-BASED BLACK AND WHITE FILTERS FOR TOPOLOGY OPTIMIZATION." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION, 33(4-5):401-424.

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- DENSITY-BASED (NESTED) FORMULATION:
  - USING A DENSITY FILTER[1]
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\text{with} \quad & \mathbf{K}\left(\overline{\rho}\right)\mathbf{u} = \mathbf{f}
\end{aligned}
$$

P=1
VARIABLE THICKNESS SHEET PROBLEM (CONVEX)

1) BOURDIN B (2001) "FILTERS IN TOPOLOGY OPTIMIZATION." INTERNATIONAL JOURNAL FOR NUMERICAL METHODS IN ENGINEERING, 50(9):2143-2158

2) BENDSOE MP (1989) "OPTIMAL SHAPE DESIGN AS A MATERIAL DISTRIBUTION PROBLEM." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 1(4):193-202

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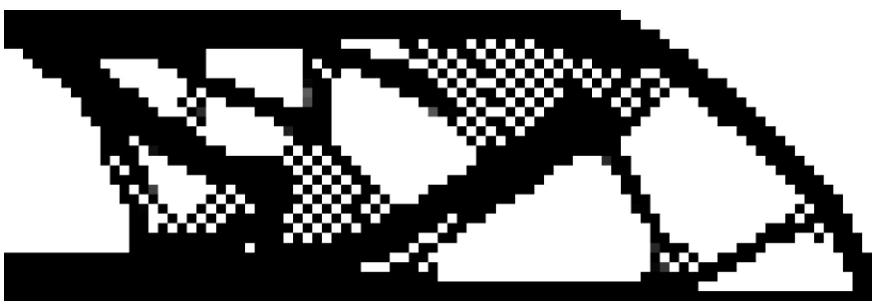
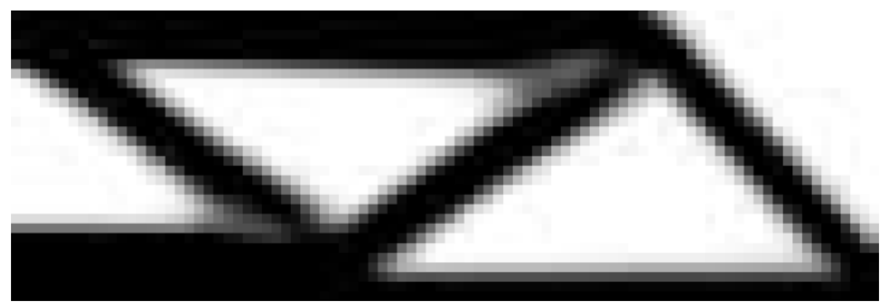
4) DENSITY-BASED TOPOLOGY OPT

- FILTERS IN DENSITY-BASED FORMULATION:
  - SENSITIVITY FILTER (1-FIELD)
  - DENSITY FILTER (2-FIELDS)
  - PROJECTION FILTER (3-FIELDS)

USED IN THIS WORK



UNFILTERED
(CHECKERBOARD)

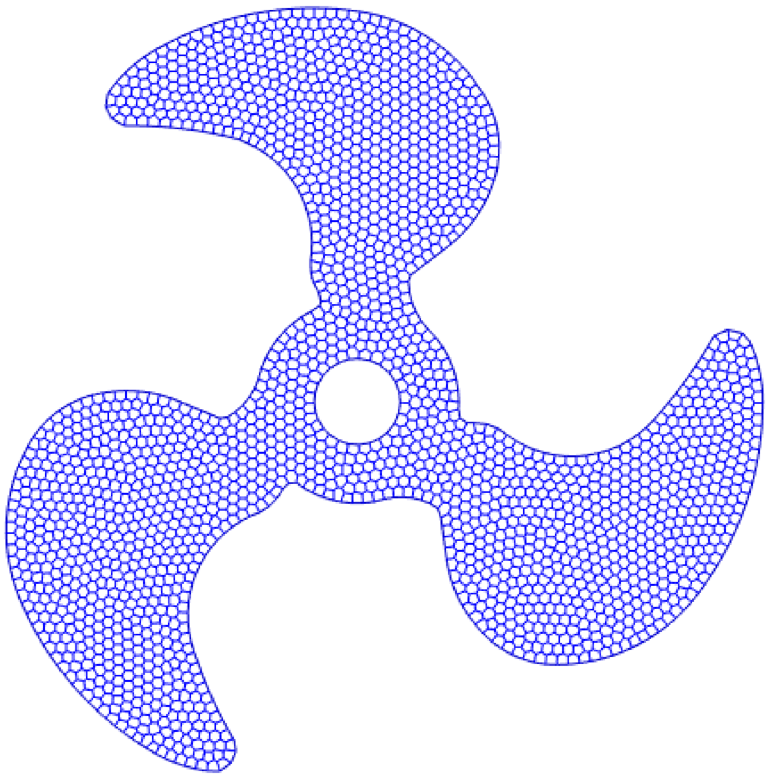


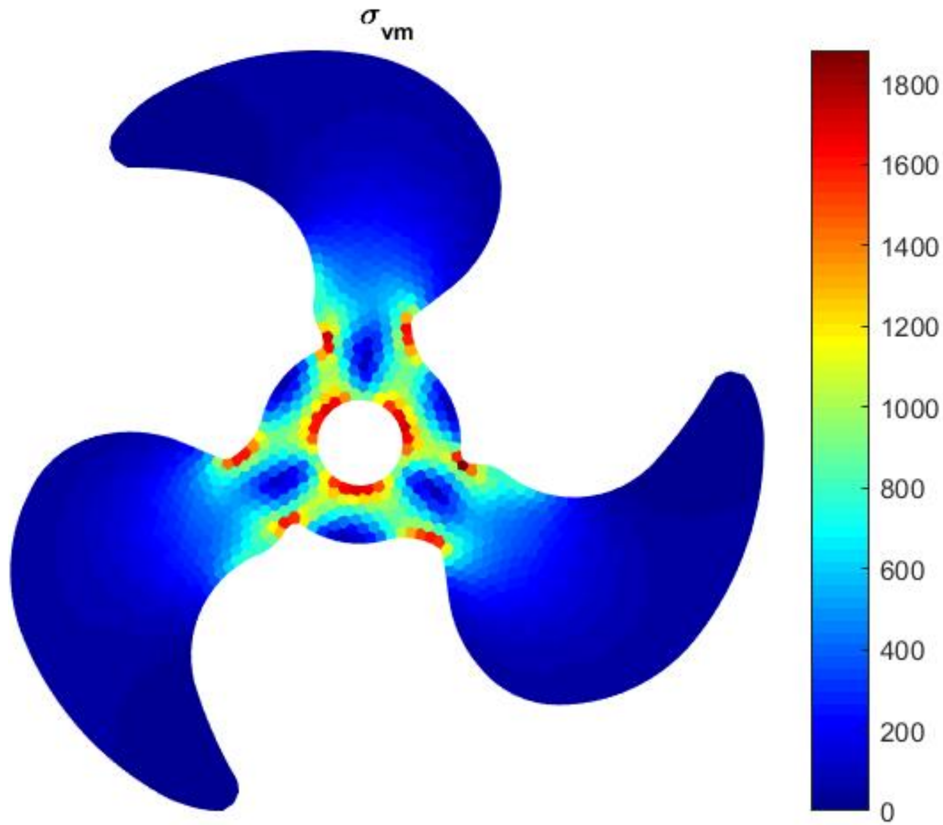
FILTERED

REVIEW ON FILTERING:
SIGMUND O, MAUTE K (2013) "TOPOLOGY OPTIMIZATION APPROACHES." STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 48(6):1031-1055

4) DENSITY-BASED TOPOLOGY OPT

- POLYMESHER: VORONOI POLYGONAL MESHER (CVT)





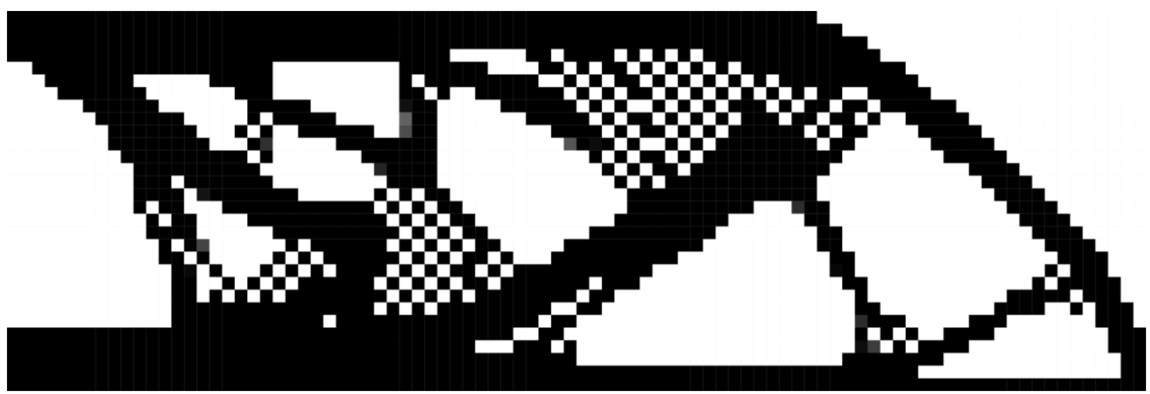
TALISCHI C, PAULINO GH, PEREIRA A, MENEZES IFM (2012), "POLYMESHER: A GENERAL-PURPOSE MESH GENERATOR FOR POLYGONAL ELEMENTS WRITTEN IN MATLAB." JOURNAL OF STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 45(3), 329–357

4) DENSITY-BASED TOPOLOGY OPT

- POLYTOP: POLYGONAL ELEMENT TOPOLOGY OPT

UNFILTERED POLYTOP (2700 ELEMENTS)

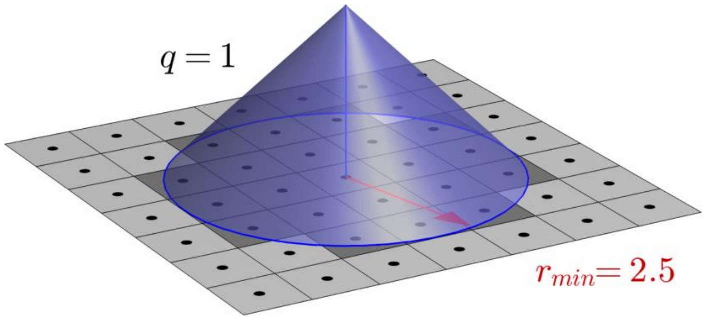
UNFILTERED Q4 (2700 ELEMENTS)



TALISCHI C, PAULINO GH, PEREIRA A, MENEZES IFM (2012), "POLYTOP: A MATLAB IMPLEMENTATION OF A GENERAL TOPOLOGY OPTIMIZATION FRAMEWORK USING UNSTRUCTURED POLYGONAL FINITE ELEMENT MESHES." JOURNAL OF STRUCTURAL AND MULTIDISCIPLINARY OPTIMIZATION 45(3), 329–357

4) DENSITY-BASED TOPOLOGY OPT

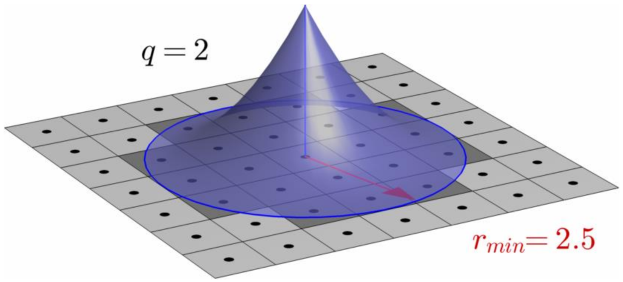
- CONVOLUTION (BLURRING) OF THE DENSITY FIELD

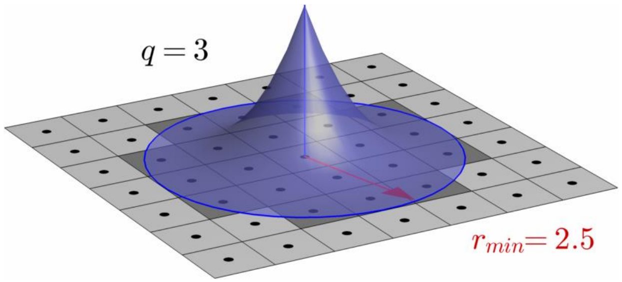


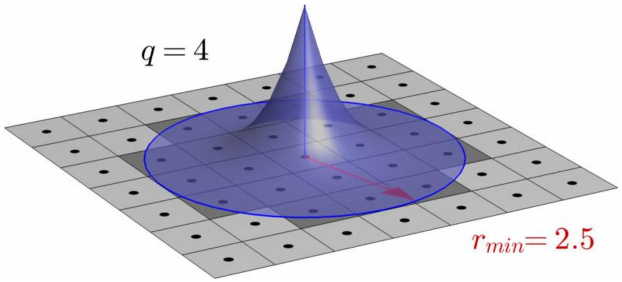
$$\overline{\boldsymbol{\rho}} = \mathbf{H}\boldsymbol{\rho}$$

$$\text{with} \quad \mathbf{H}_{ij} = \frac{h(i,j)\, v_j}{\sum_k^{N_e} h(i,k)\, v_k}$$

$$h(i,j) = \begin{cases} [r_{min} - \text{dist}(i,j)]^q & \text{for} \quad r_{min} - \text{dist}(i,j) > 0 \\ 0 & \text{otherwise} \end{cases}$$

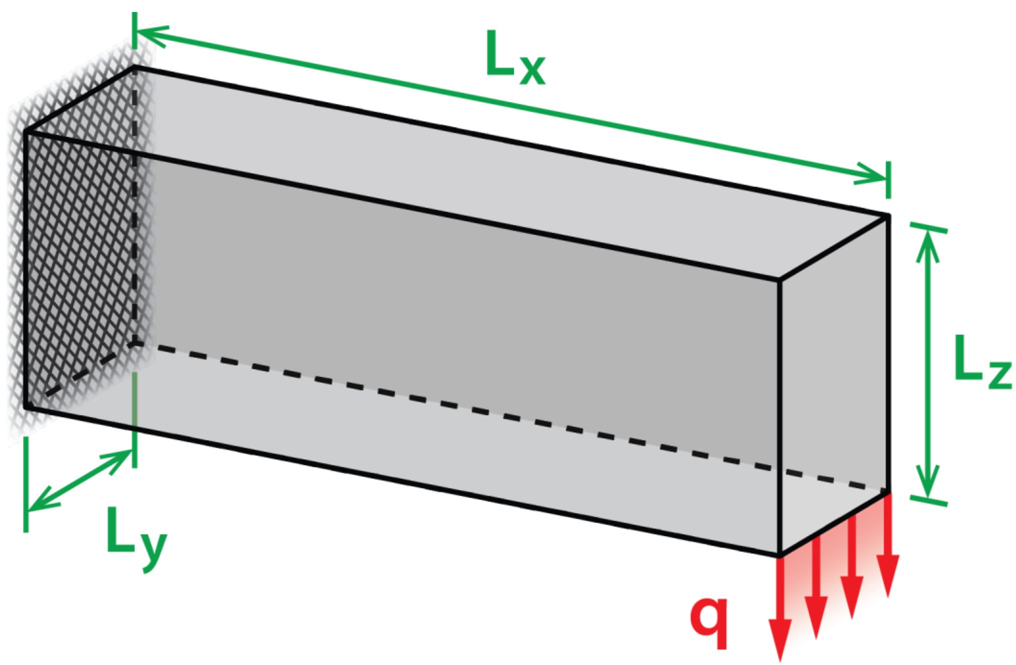






4) Density-Based Topology Opt

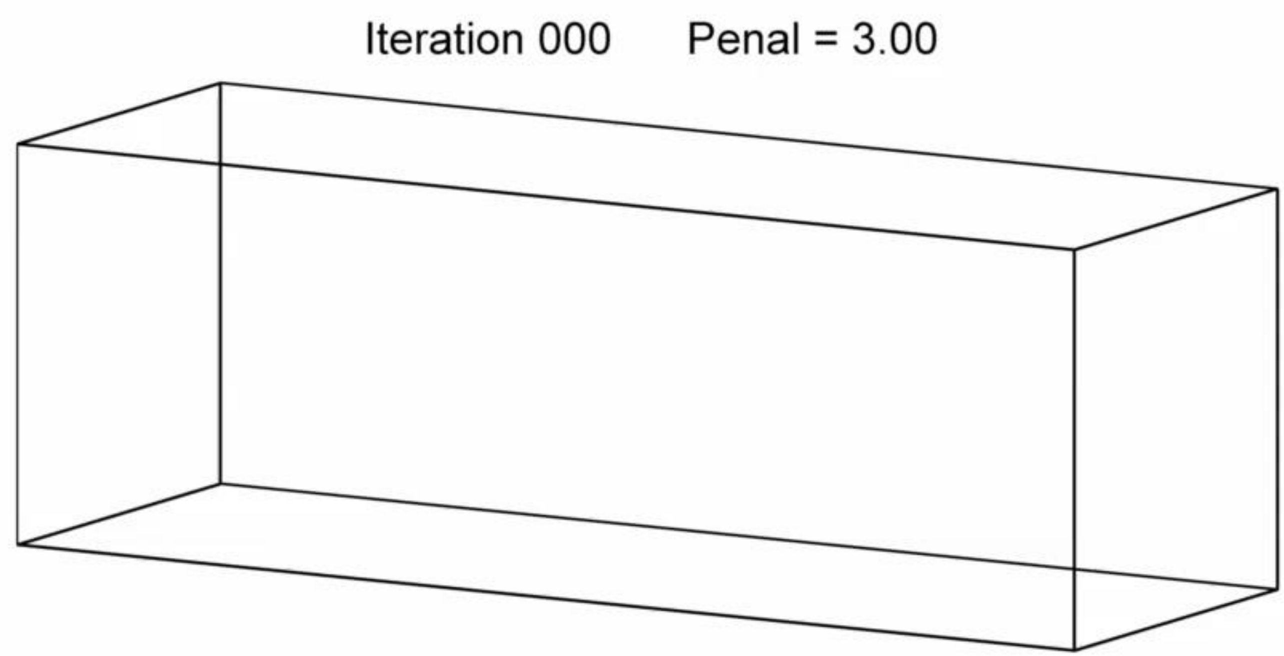
- Edge-loaded cantilever beam
  Lx=3, Ly=Lz=1
  Volfrac=10%, R=6, Q=1 and P=3



559,872 DVs for ½
(1,119,744 total)

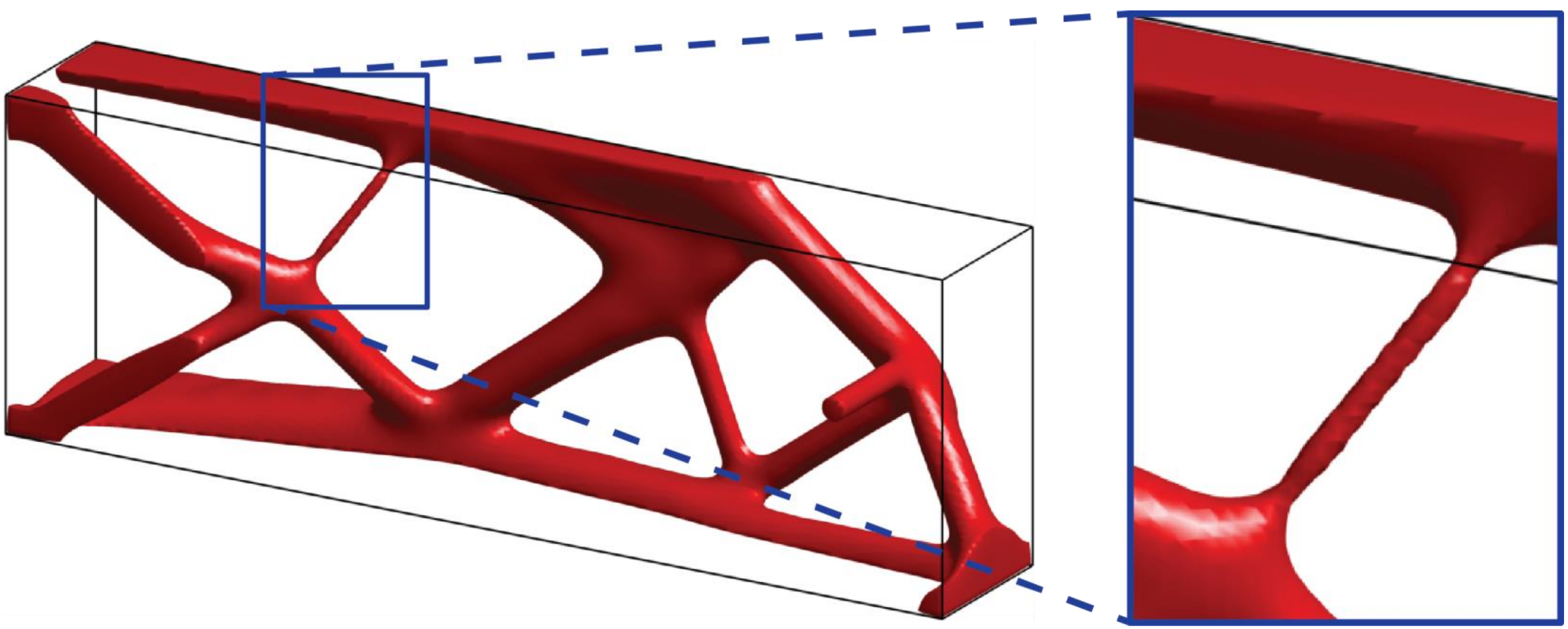
4) Density-based topology opt

Lx=3, Ly=Lz=1, Volfrac=10%, R=6, Q=1 and P=3



4) DENSITY-BASED TOPOLOGY OPT

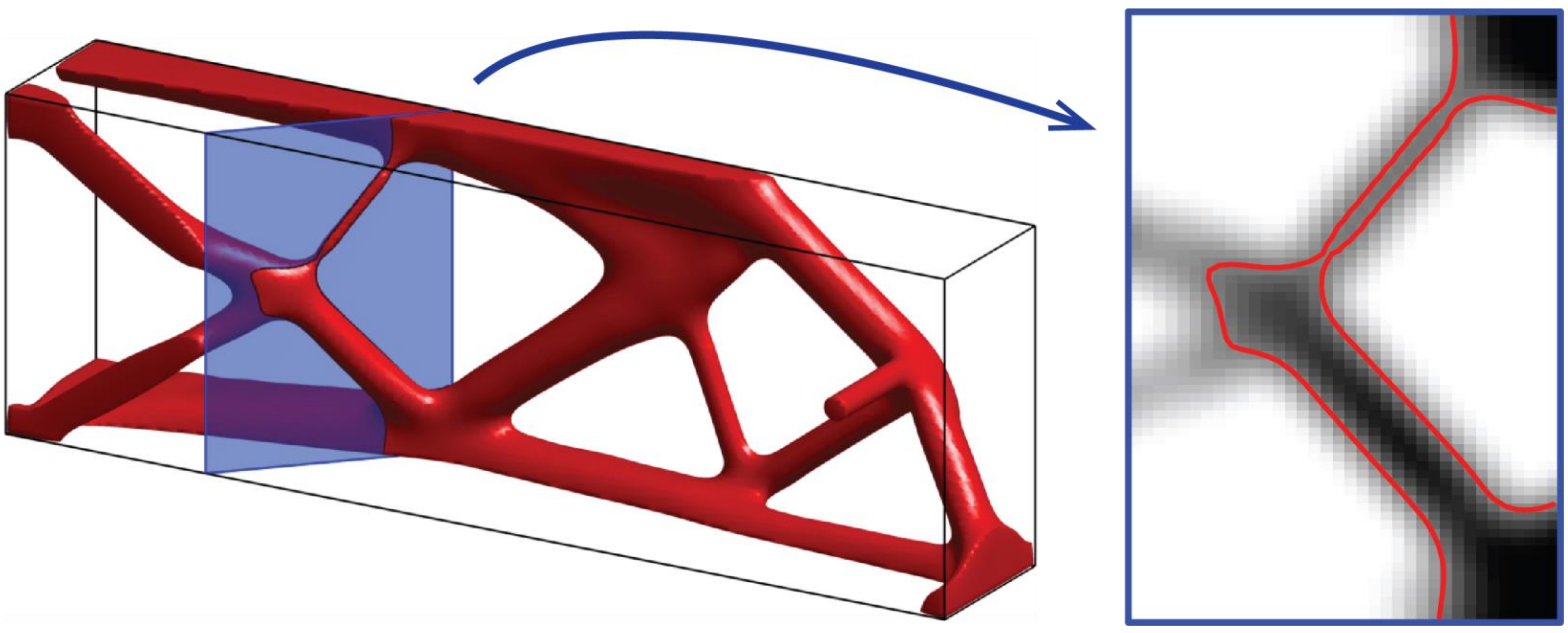
- EDGE-LOADED CANTILEVER
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=1 AND P=3



559,872 DVS FOR ½
(1,119,744 TOTAL)

4) Density-Based Topology Opt

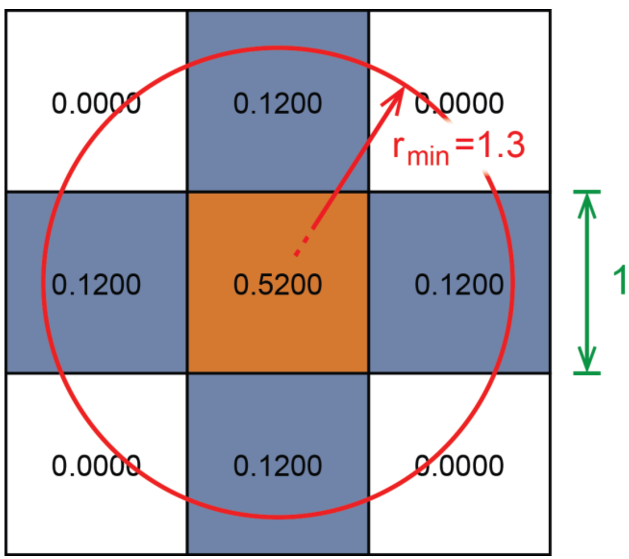
- Edge-Loaded Cantilever
  $L_x=3$, $L_y=L_z=1$
  Volfrac=10%, R=6, Q=1 and P=3



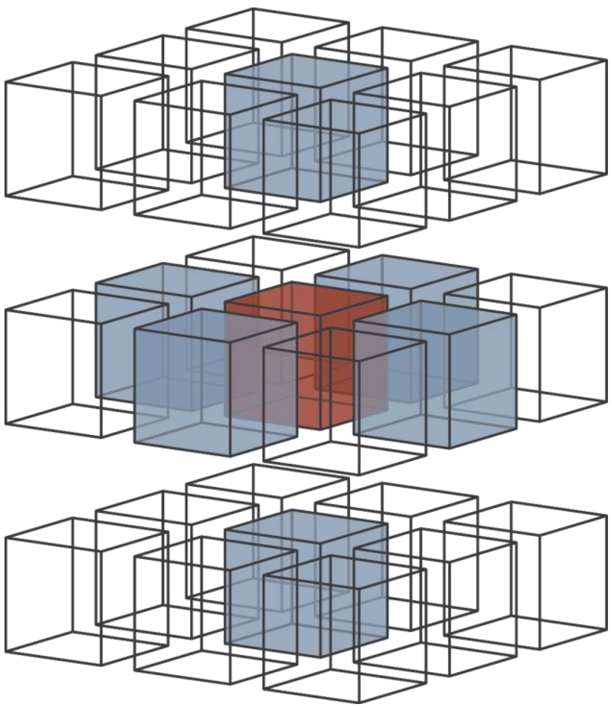
559,872 DVs for ½
(1,119,744 total)

4) DENSITY-BASED TOPOLOGY OPT

- FILTER'S WEIGHTS FOR A REGULAR MESH
  R_{MIN}=1.3, Q=1 AND ELEM SIZE IS **L=1**



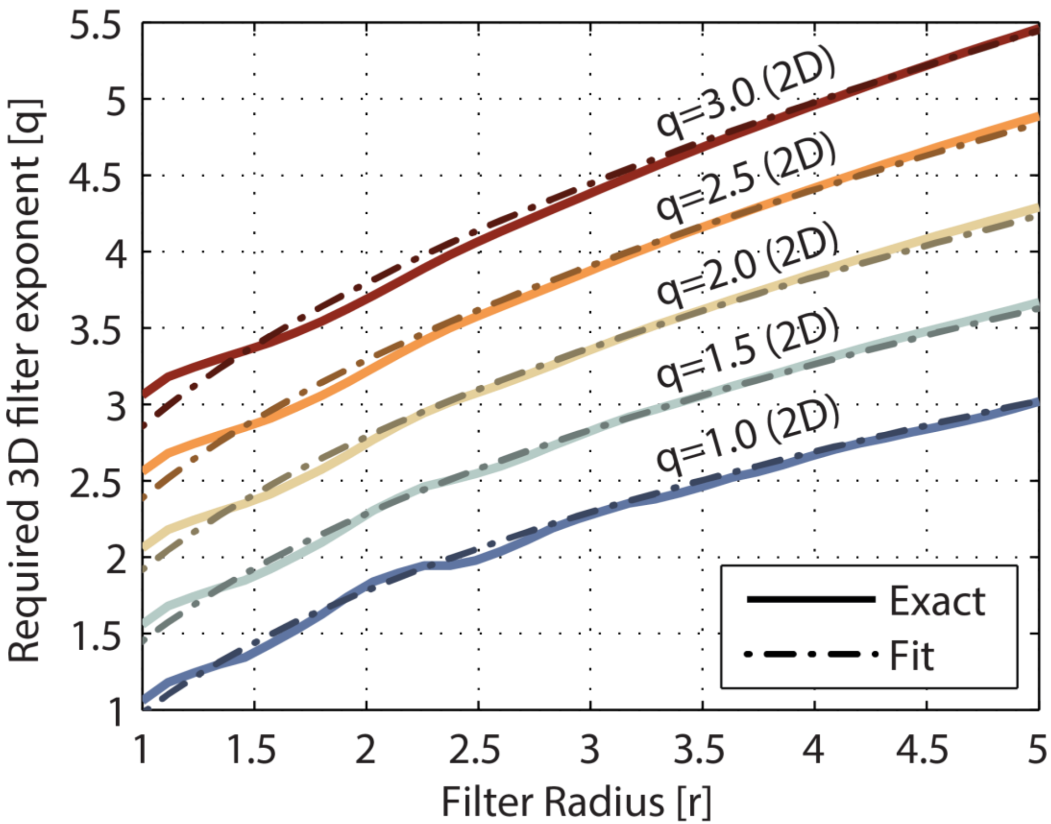
TWO-DIMENSIONS



THREE-DIMENSIONS
(H_{ii} = 0.4194)

4) DENSITY-BASED TOPOLOGY OPT

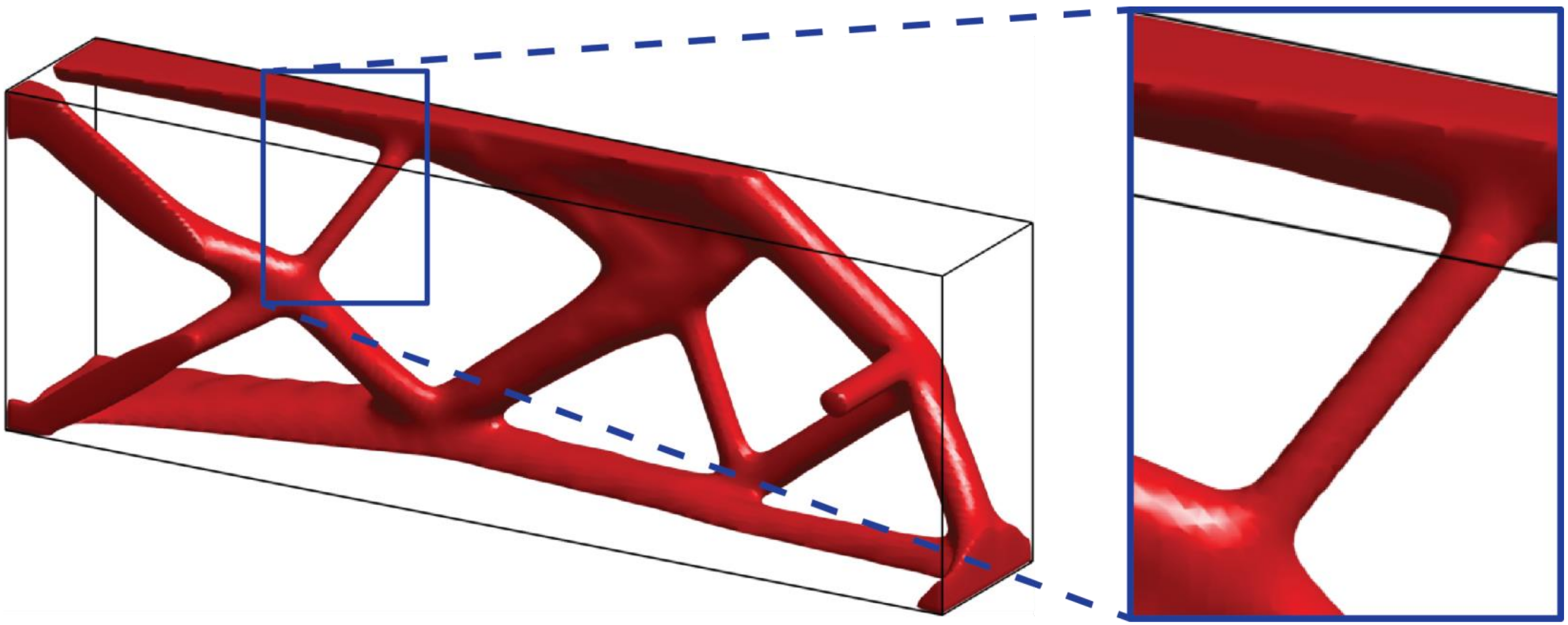
IDEA: WHAT EXPONENT Q MAKES $H_{ii}^{(2D)} = H_{ii}^{(3D)}$?



$$q^{(3D)} = \log(r_{min}) + \frac{17}{20}q^{(2D)} + \frac{4}{57}q^{(2D)}r_{min} + \frac{4}{87}r_{min}$$

4) Density-based topology opt

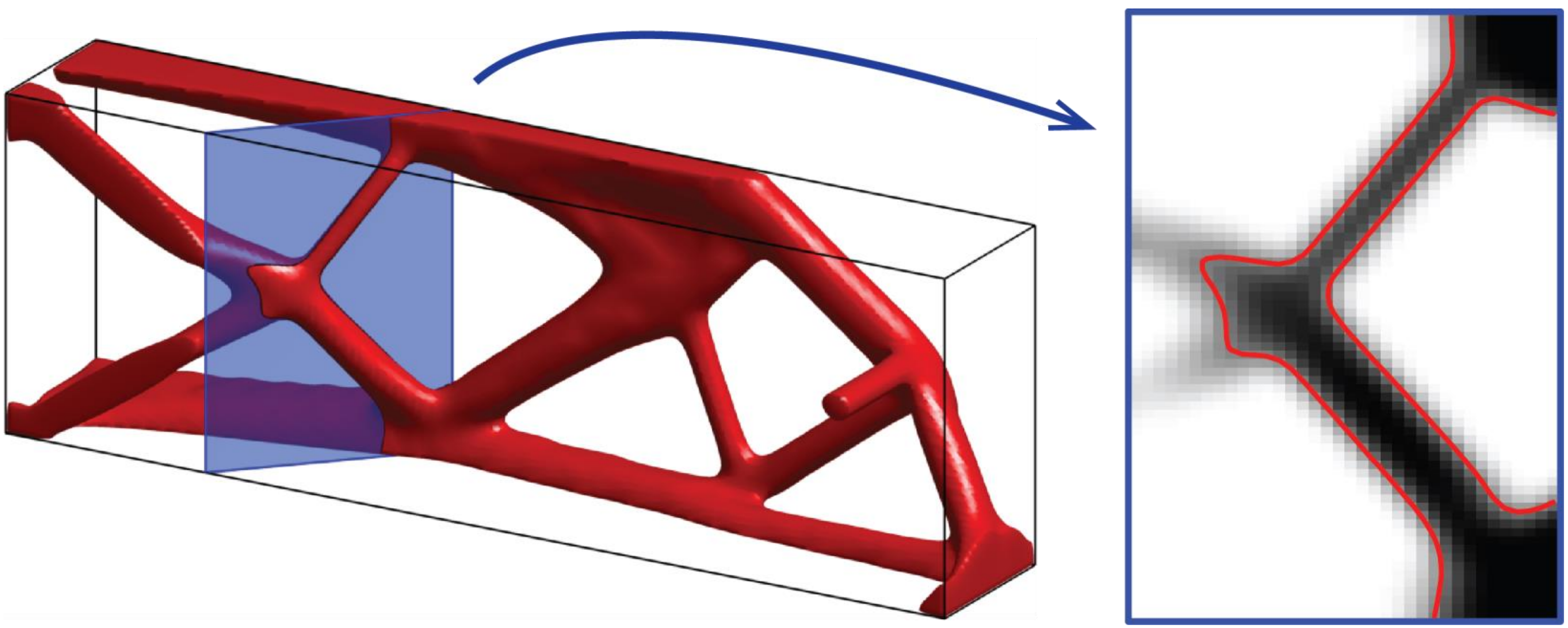
- Edge-loaded cantilever
  Lx=3, Ly=Lz=1
  Volfrac=10%, R=6, Q=3 and P=3



559,872 DVs for ½
(1,119,744 total)

4) DENSITY-BASED TOPOLOGY OPT

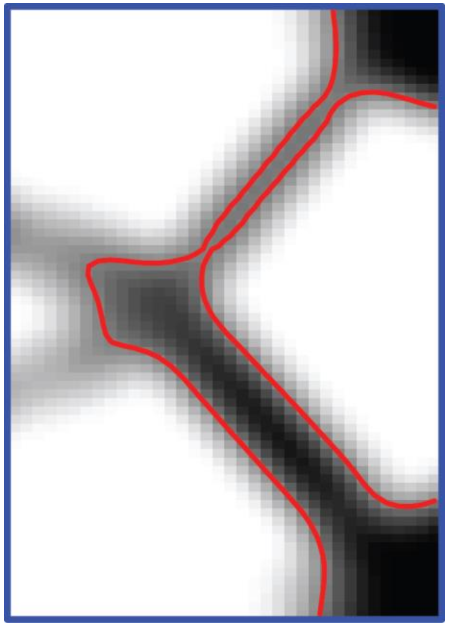
- EDGE-LOADED CANTILEVER
  LX=3, LY=LZ=1
  VOLFRAC=10%, R=6, Q=3 AND P=3



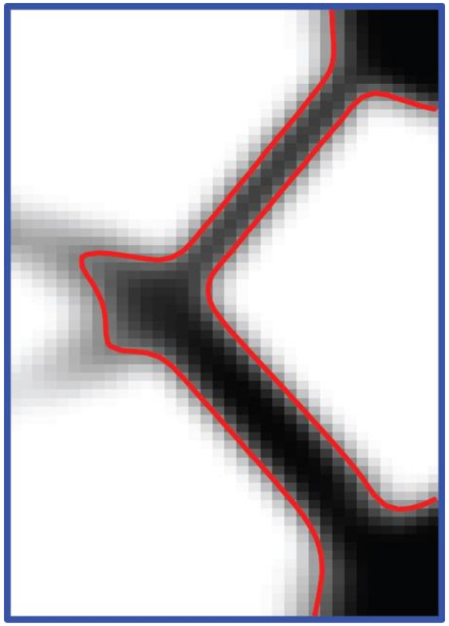
559,872 DVs FOR ½
(1,119,744 TOTAL)

4) DENSITY-BASED TOPOLOGY OPT

- EDGE-LOADED CANTILEVER
  DENSITY FILTER: R=6



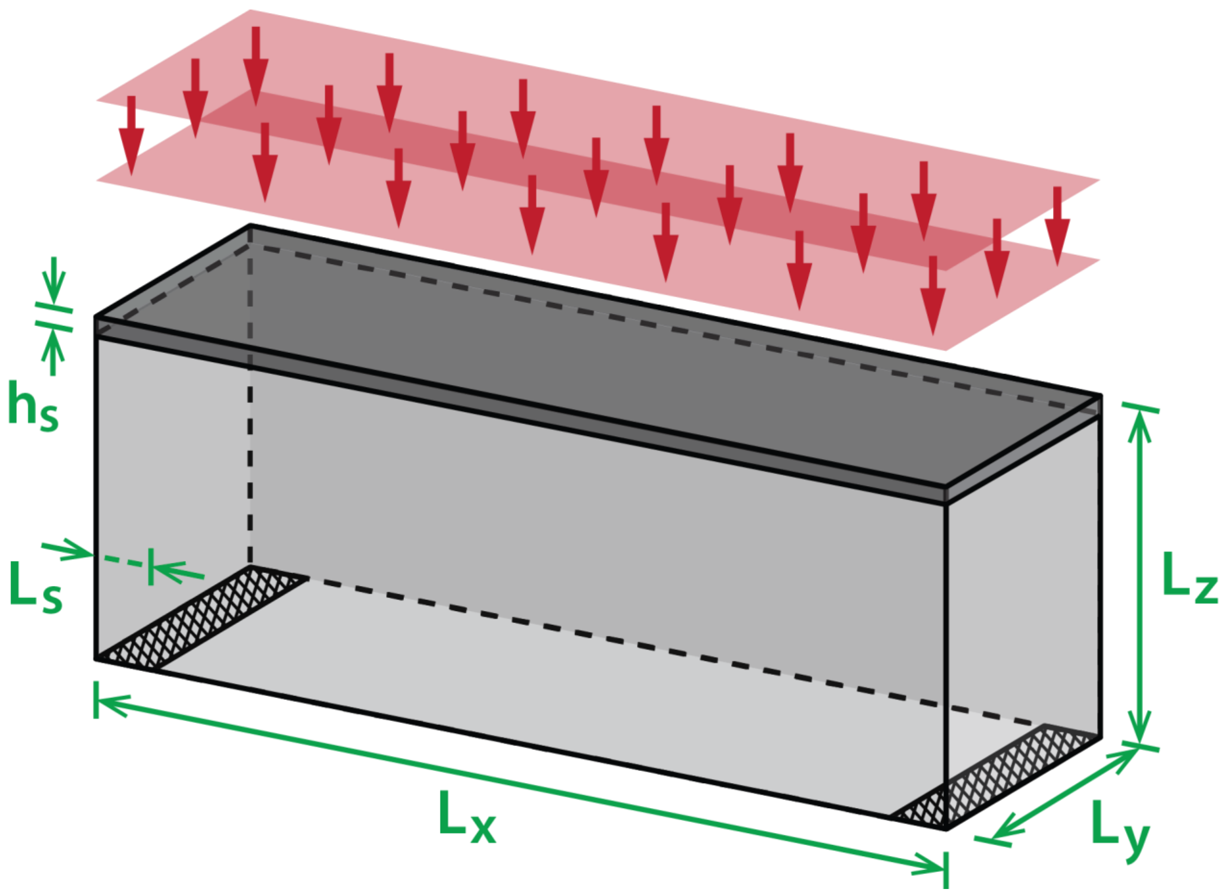
LINEAR DENSITY FILTER



CUBIC DENSITY FILTER

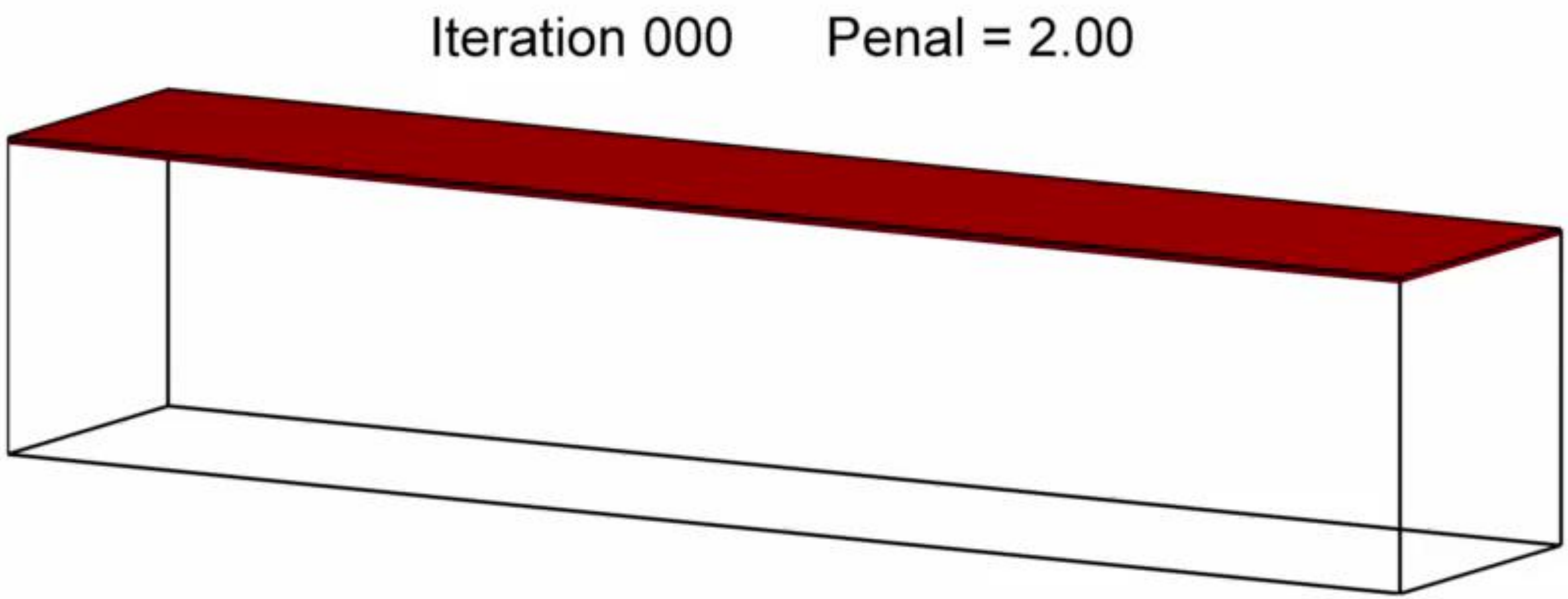
4) DENSITY-BASED TOPOLOGY OPT

- BRIDGE PROBLEM
  LX=25, LY=LZ=5, VOLFRAC=10%, R=5, Q=3 AND P=[CONT]



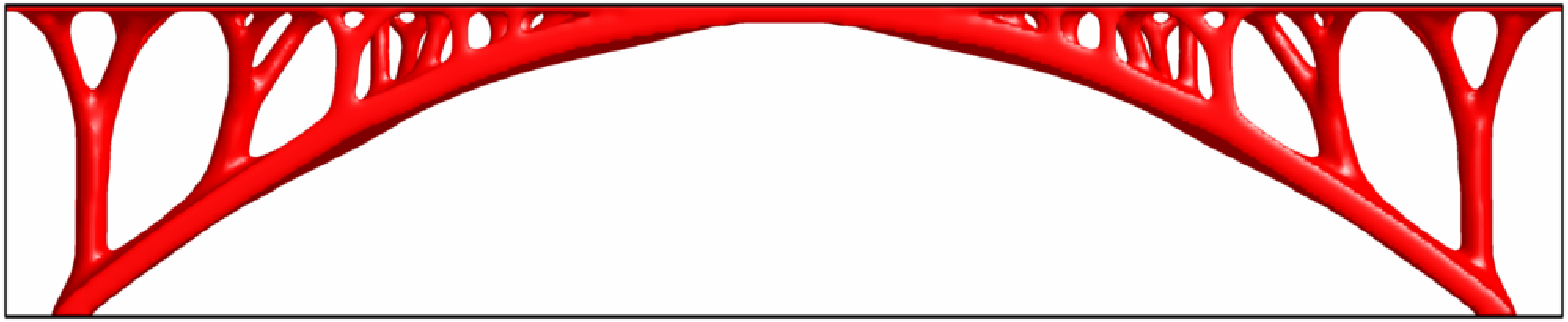
4) DENSITY-BASED TOPOLOGY OPT

- BRIDGE PROBLEM
  LX=25, LY=LZ=5, VOLFRAC=10%, R=5, Q=3 AND P=[CONT]



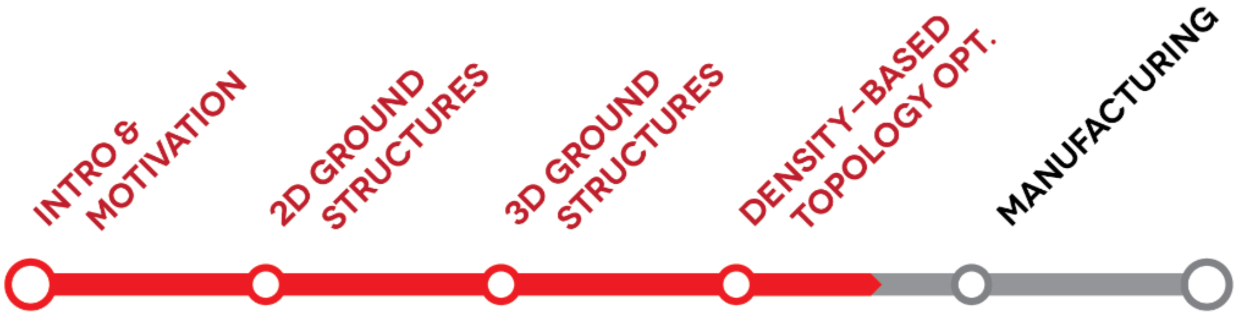
4) DENSITY-BASED TOPOLOGY OPT

- BRIDGE PROBLEM
  LX=25, LY=LZ=5, VOLFRAC=10%, R=5, Q=3 AND P=[CONT]



851,840 DVs FOR ¼
(3,407,360 TOTAL)

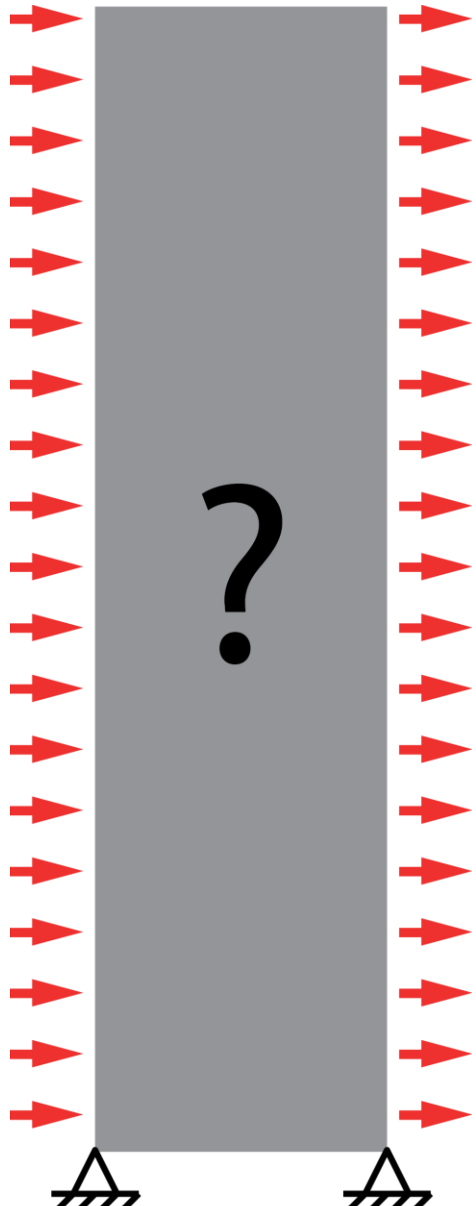
ROADMAP

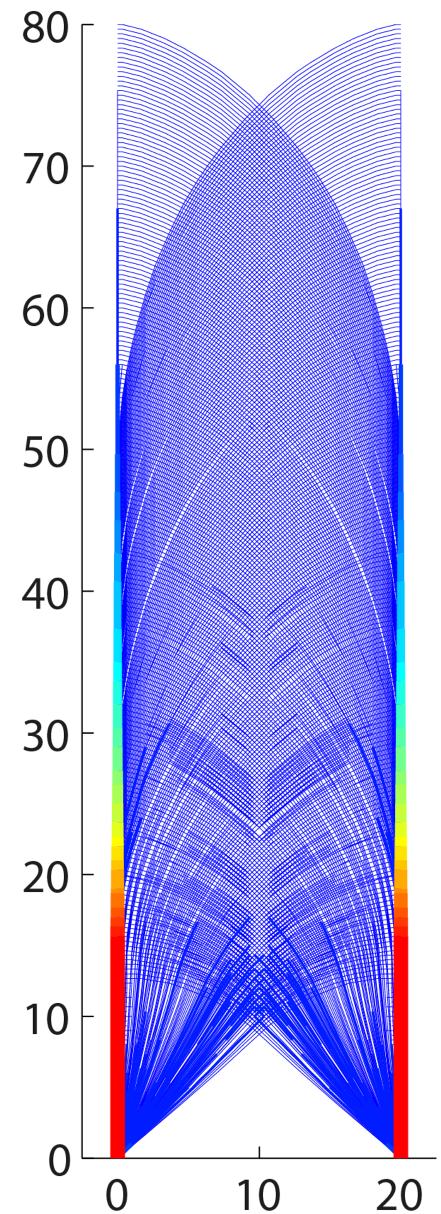


INTRO &
MOTIVATION
2D GROUND
STRUCTURES
3D GROUND
STRUCTURES
DENSITY-BASED
TOPOLOGY OPT.
MANUFACTURING

5) MANUFACTURING & DESIGNS

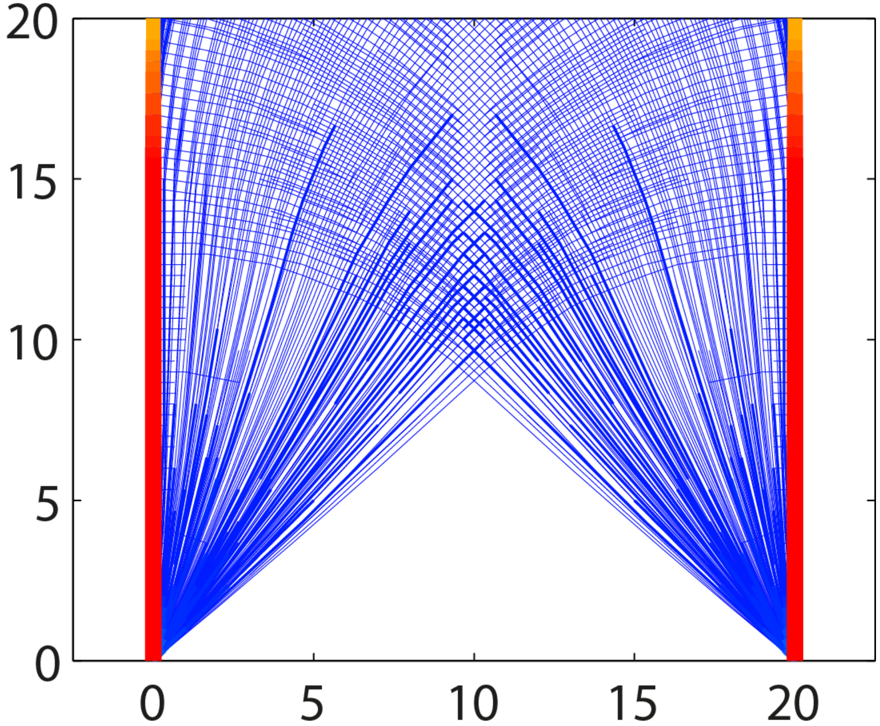
- BRACED TOWER



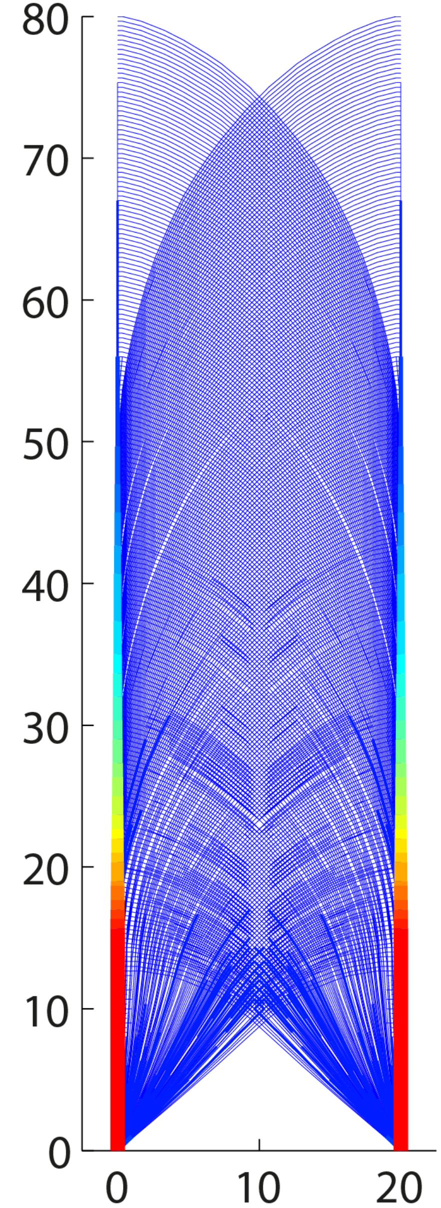


5) MANUFACTURING & DESIGNS

- BRACED TOWER

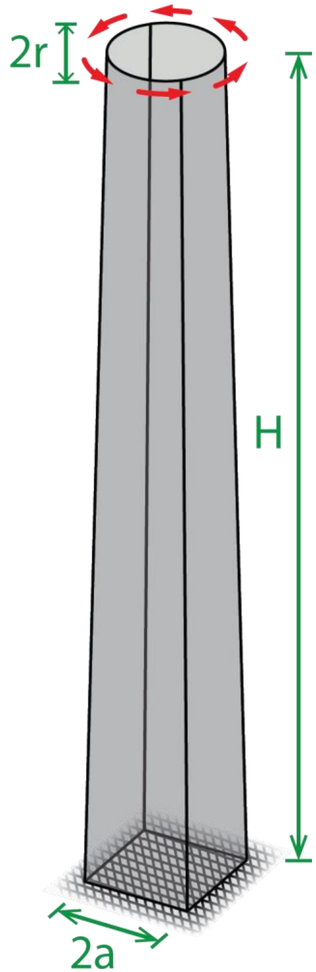


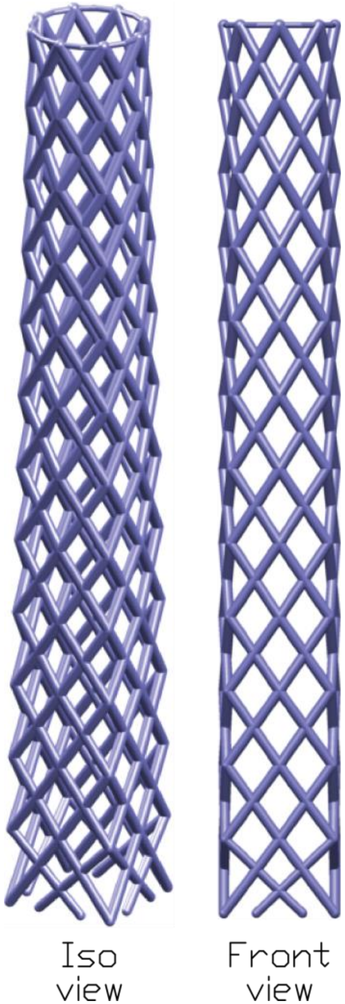
11.5 MILLION
BARS



5) MANUFACTURING & DESIGNS

- MORE APPLIED PROBLEMS?

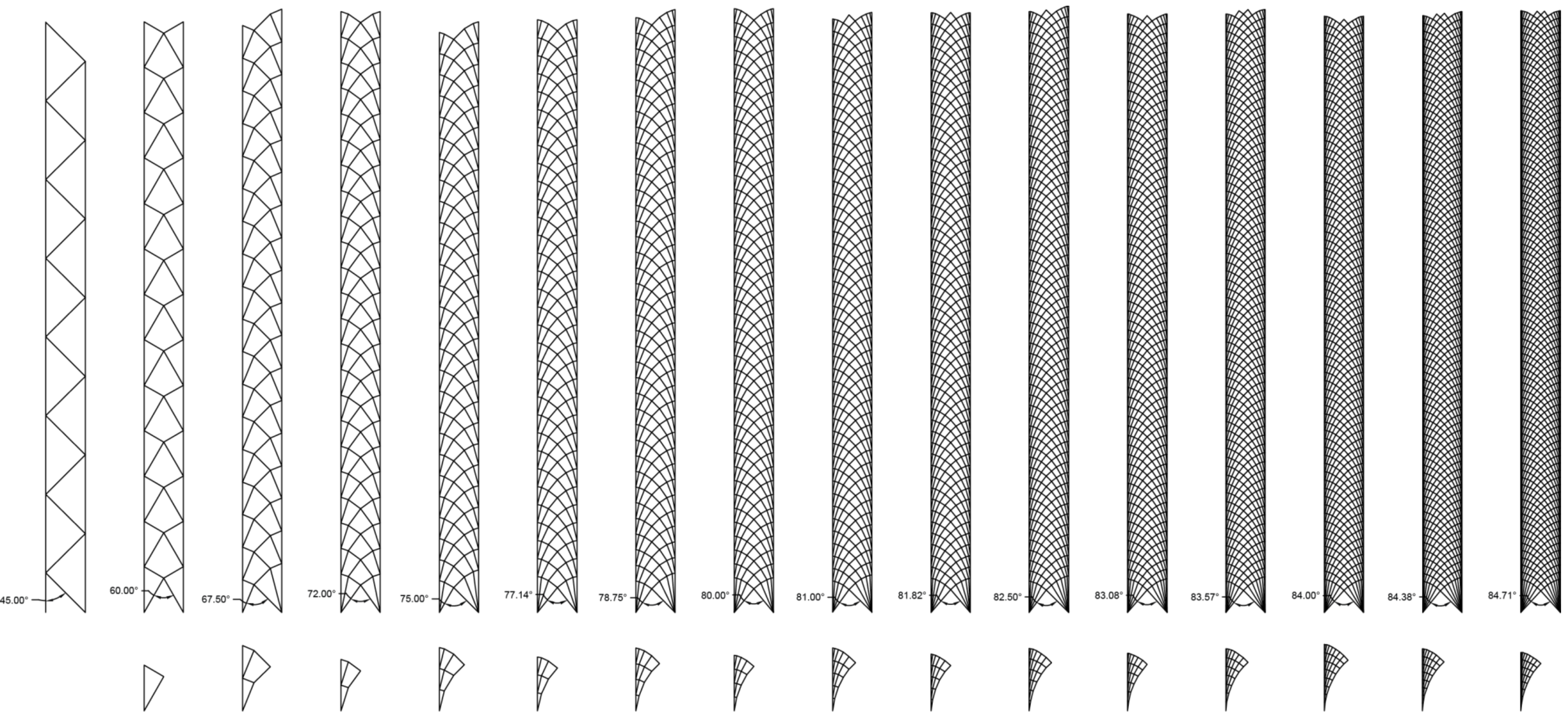




4,100 BARS

5) MANUFACTURING & DESIGNS

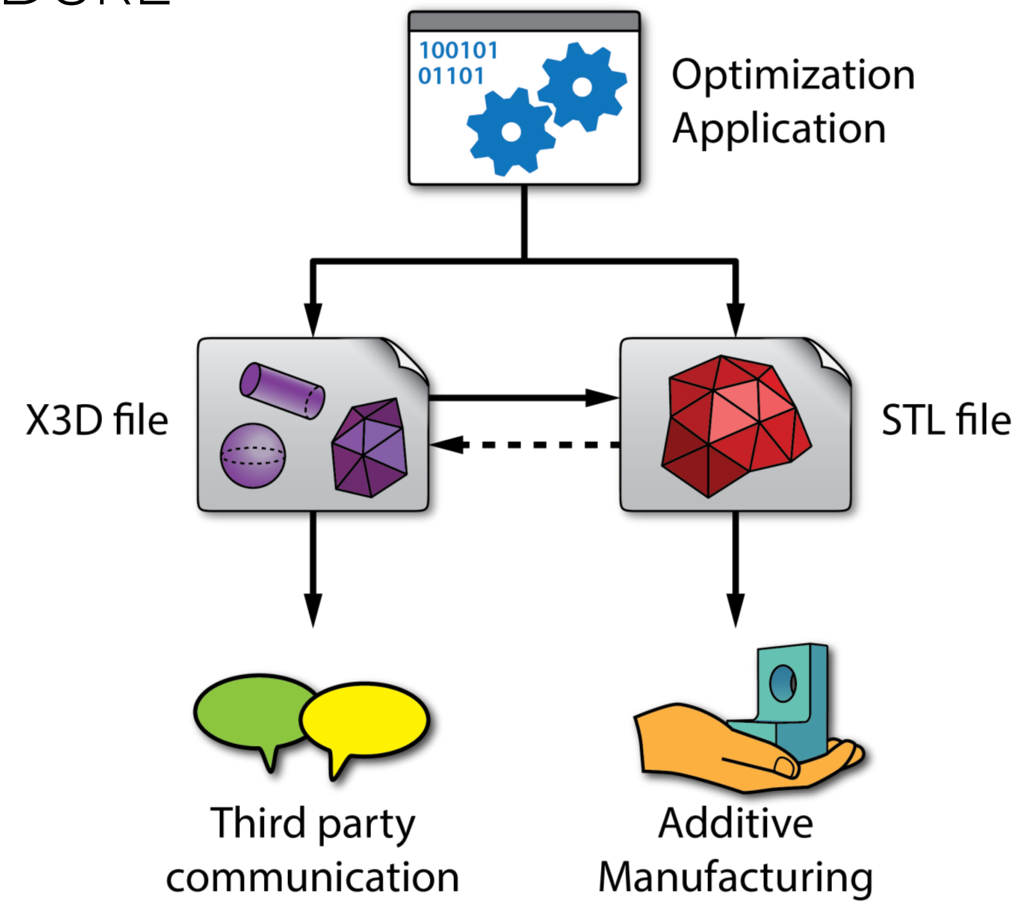
- BRACED TOWERS



5) MANUFACTURING & DESIGNS

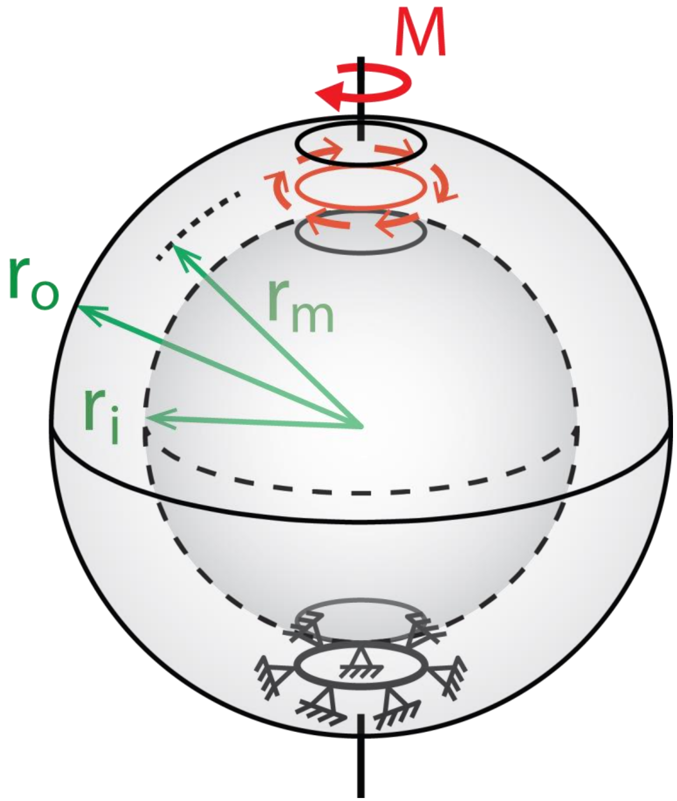
5) MANUFACTURING & DESIGNS

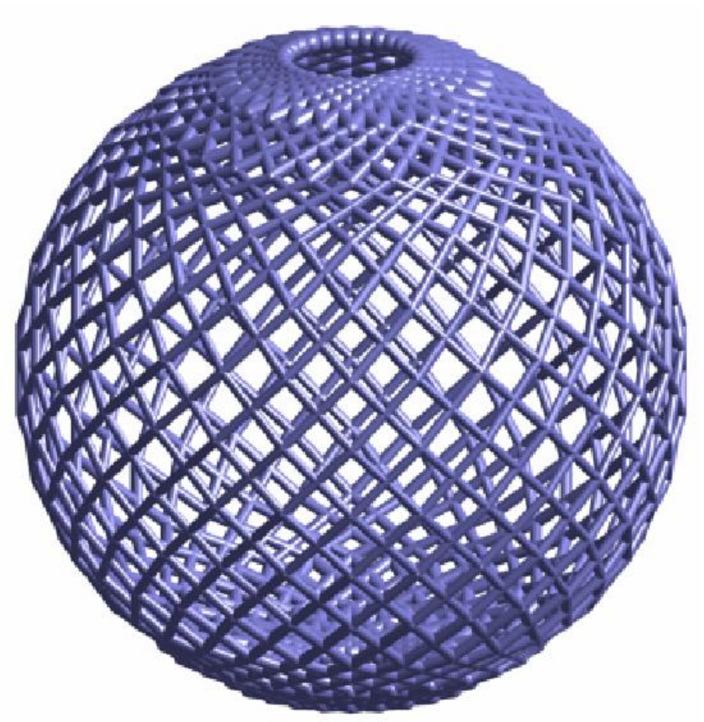
- PROCEDURE



5) MANUFACTURING & DESIGNS

- TORSION BALL



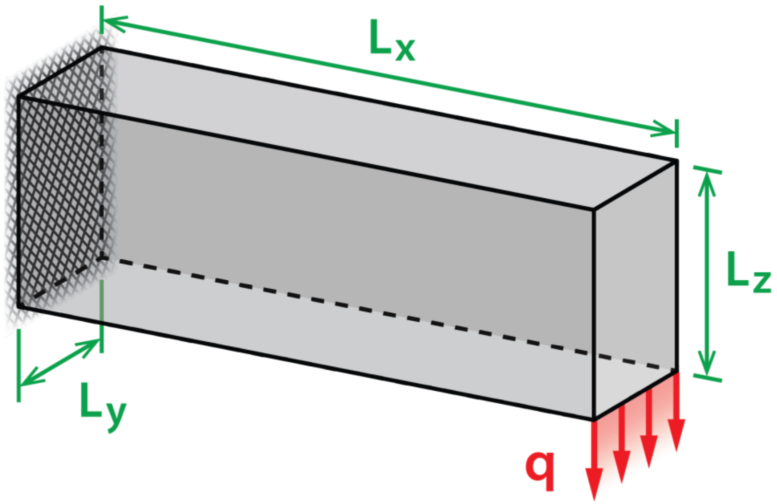


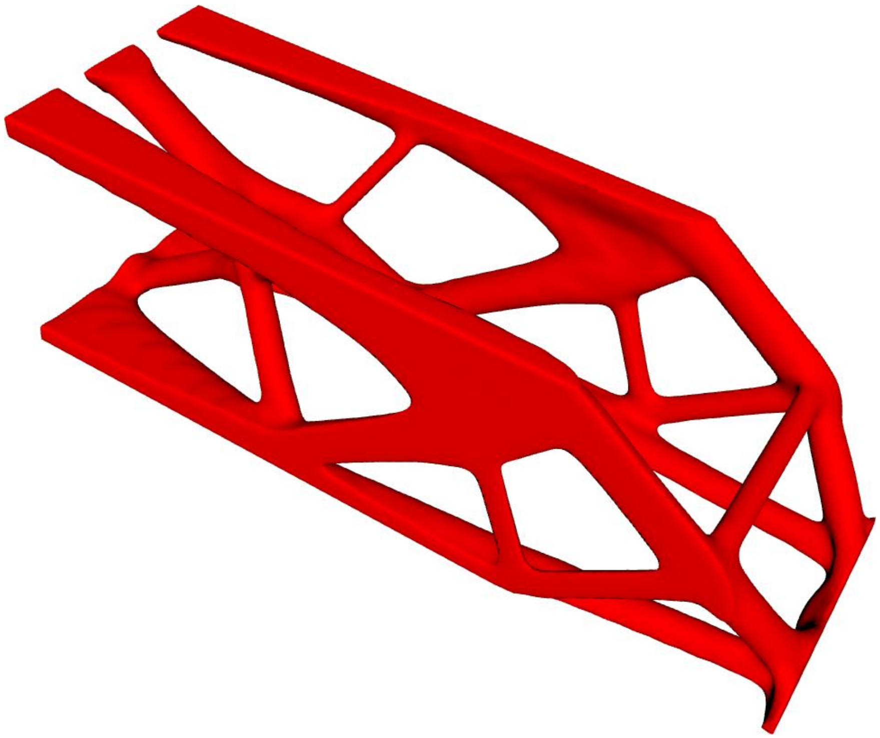
5) MANUFACTURING & DESIGNS

- TORSION BALL

5) MANUFACTURING & DESIGNS

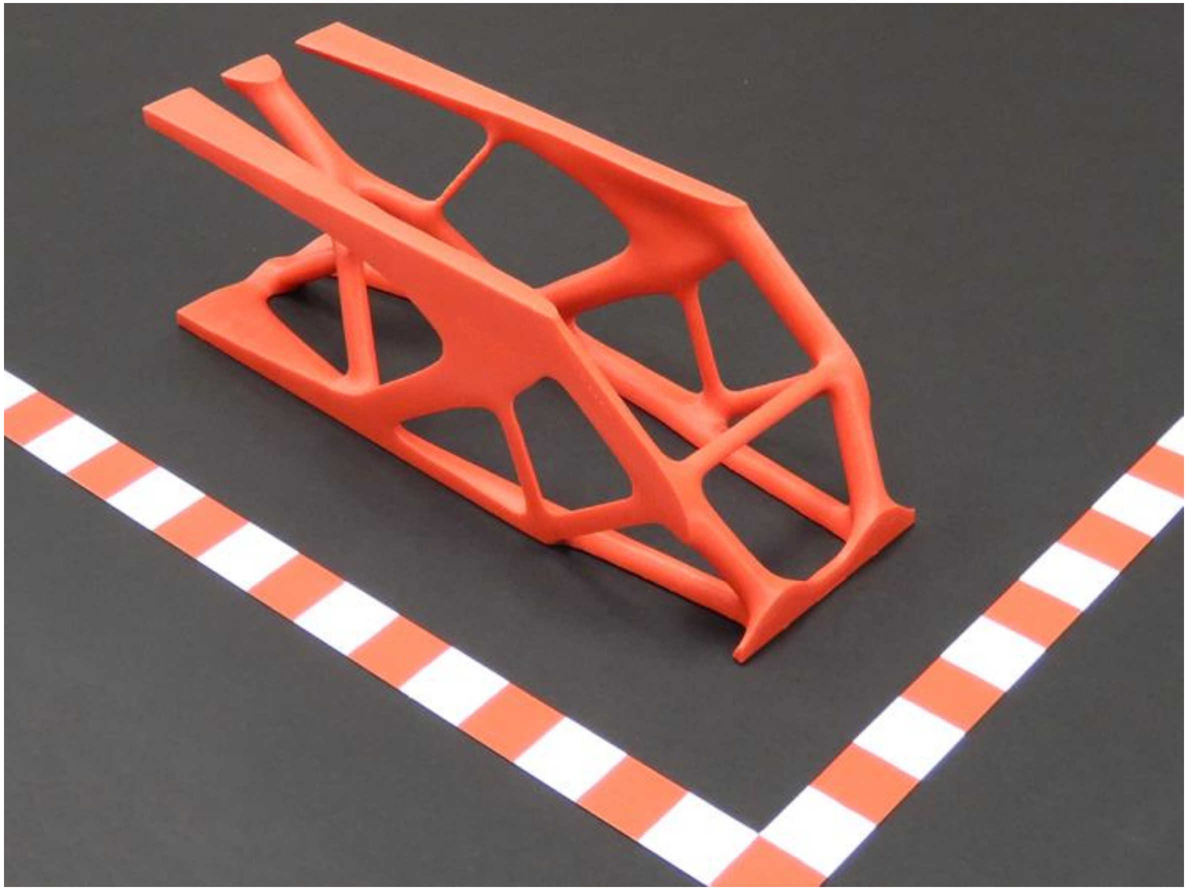
- 3D CANTILEVER





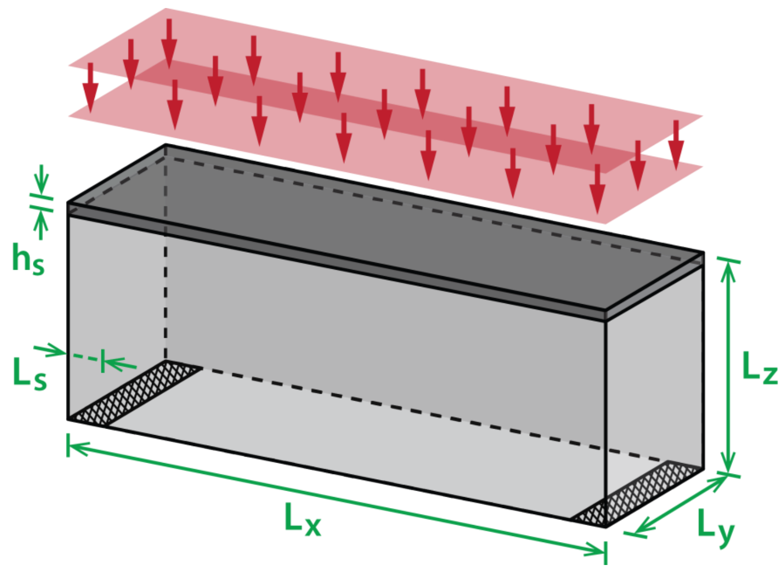
5) MANUFACTURING & DESIGNS

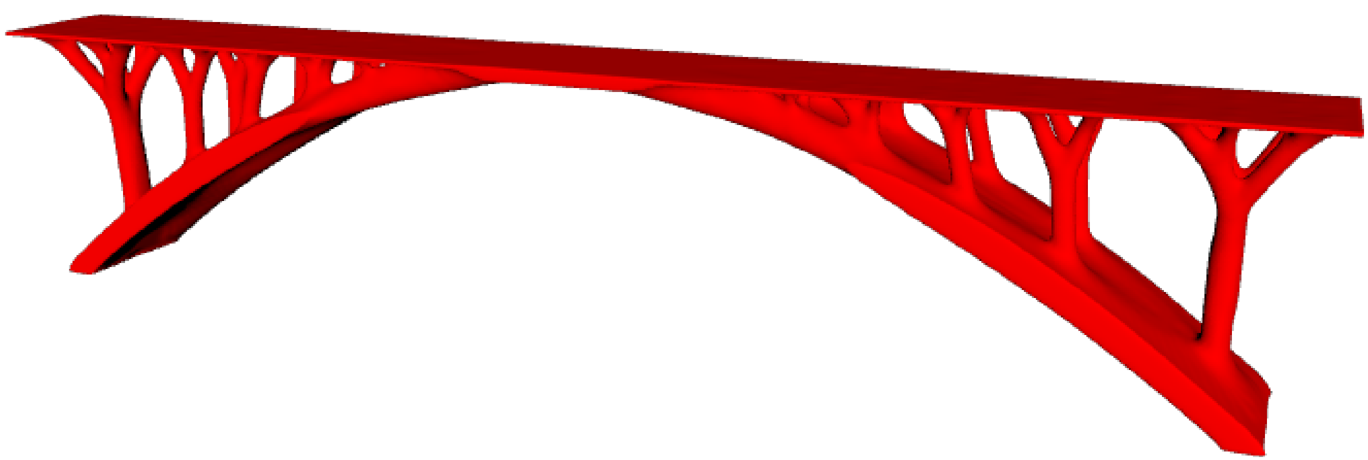
- 3D CANTILEVER



5) MANUFACTURING & DESIGNS

- BRIDGE



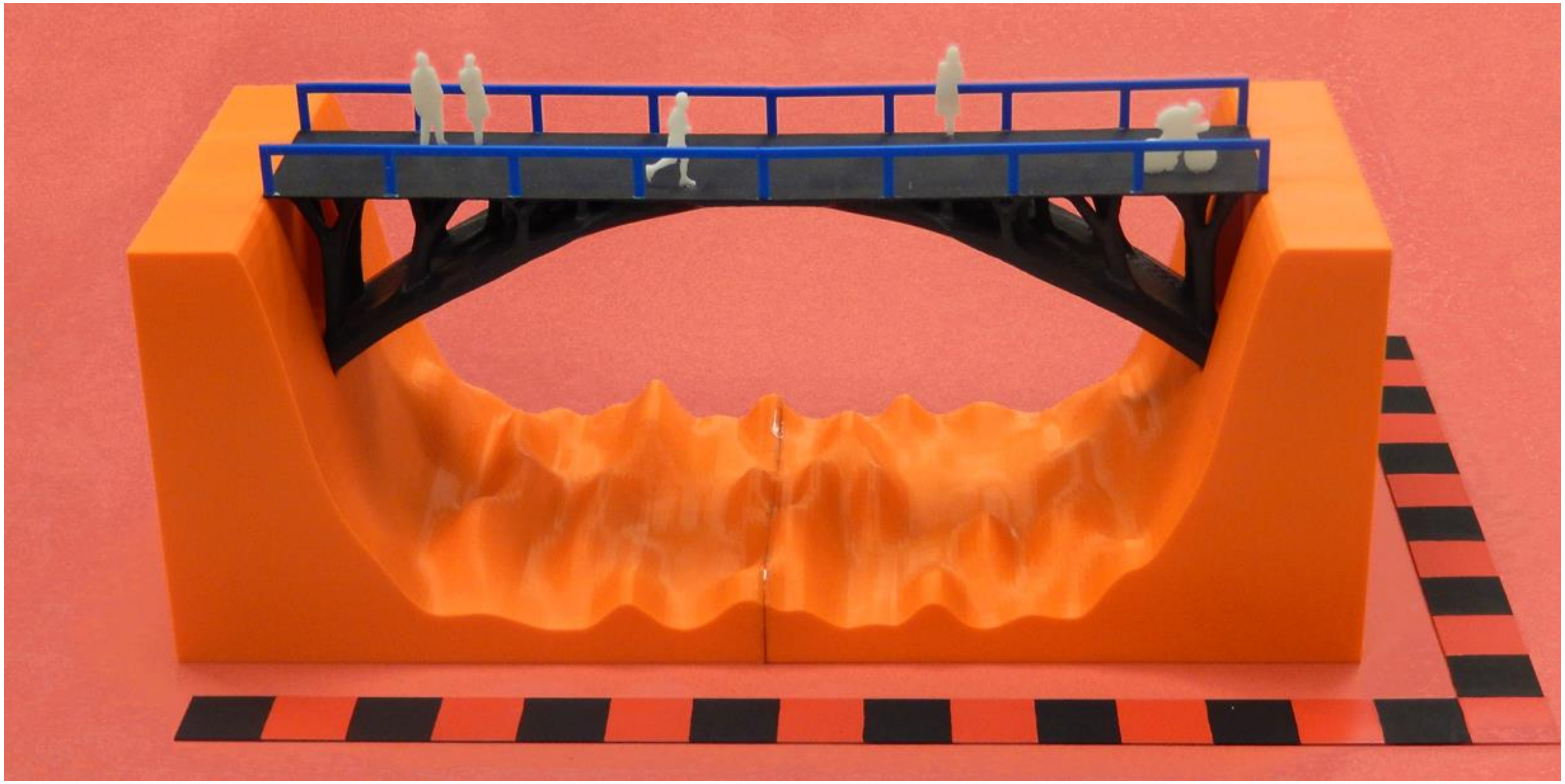


5) MANUFACTURING & DESIGNS

- BRIDGE

5) MANUFACTURING & DESIGNS

- BRIDGE

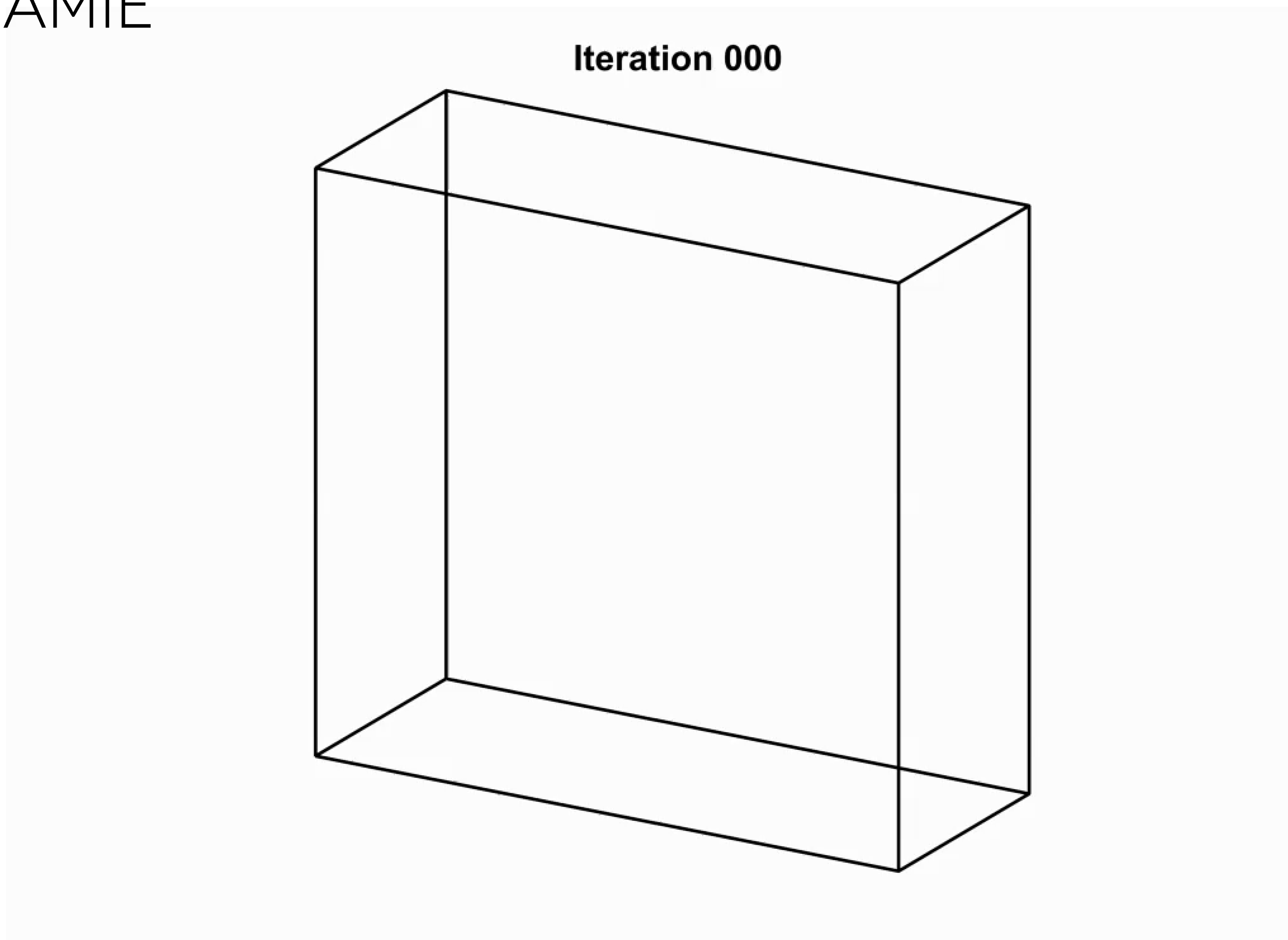


5) MANUFACTURING & DESIGNS

- BRIDGE: ACHIEVING LARGE SCALES

5) MANUFACTURING & DESIGNS

- AMIE



5) MANUFACTURING & DESIGNS

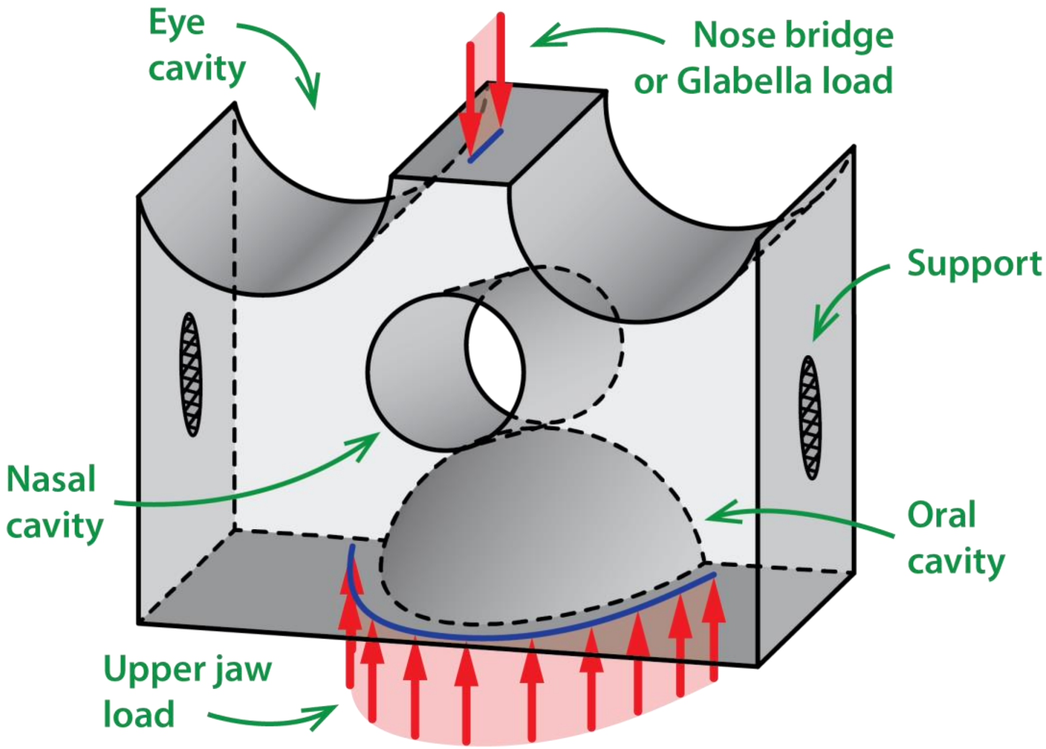
- AMIE

5) MANUFACTURING & DESIGNS

- AMIE

5) MANUFACTURING & DESIGNS

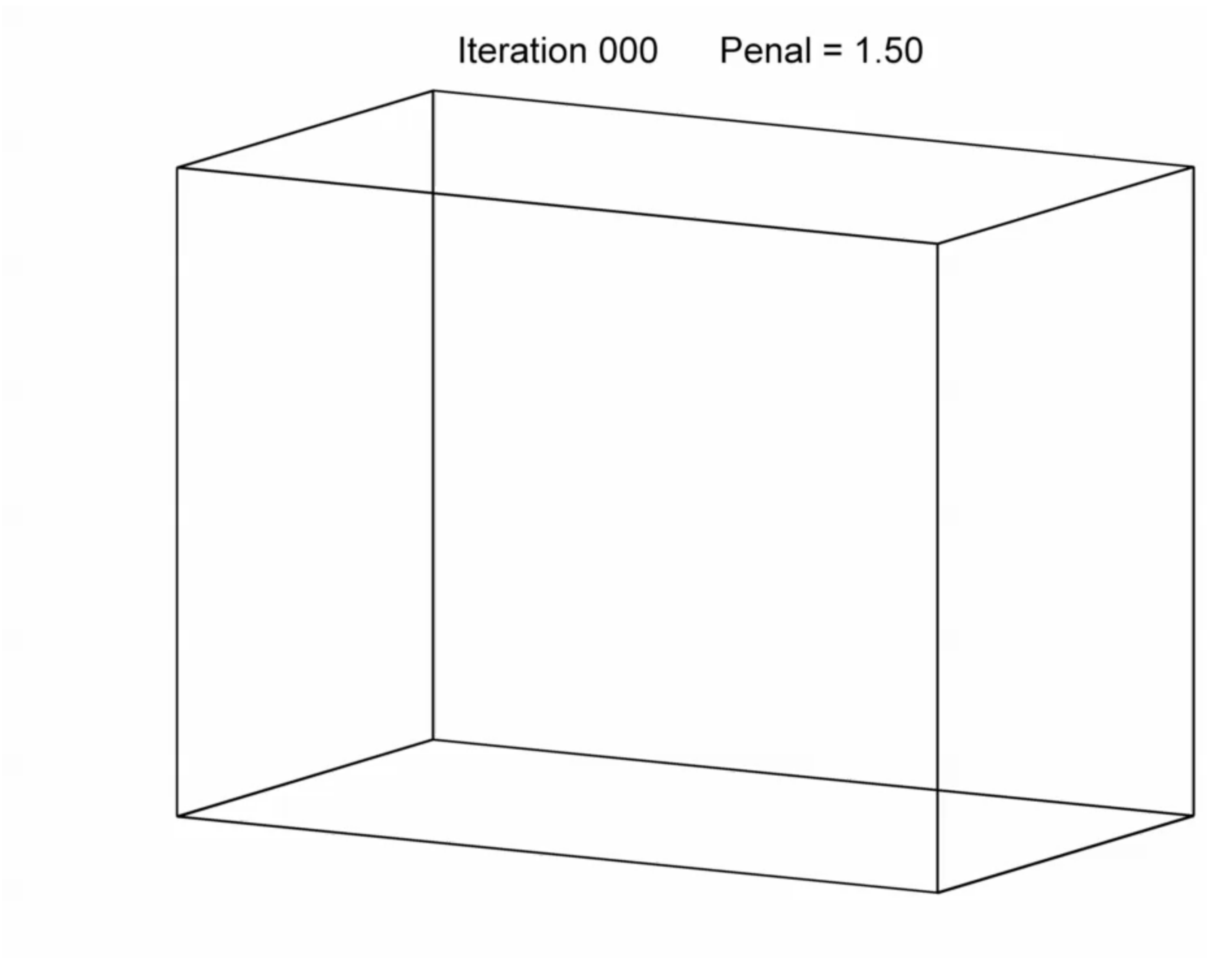
- CRANIOFACIAL RECONSTRUCTION



SUTRADHAR A, PAULINO GH, MILLER MJ, NGUYEN TH (2010) "TOPOLOGICAL OPTIMIZATION FOR DESIGNING PATIENT-SPECIFIC LARGE CRANIOFACIAL SEGMENTAL BONE REPLACEMENTS." PNAS, 107(30):13222-13227

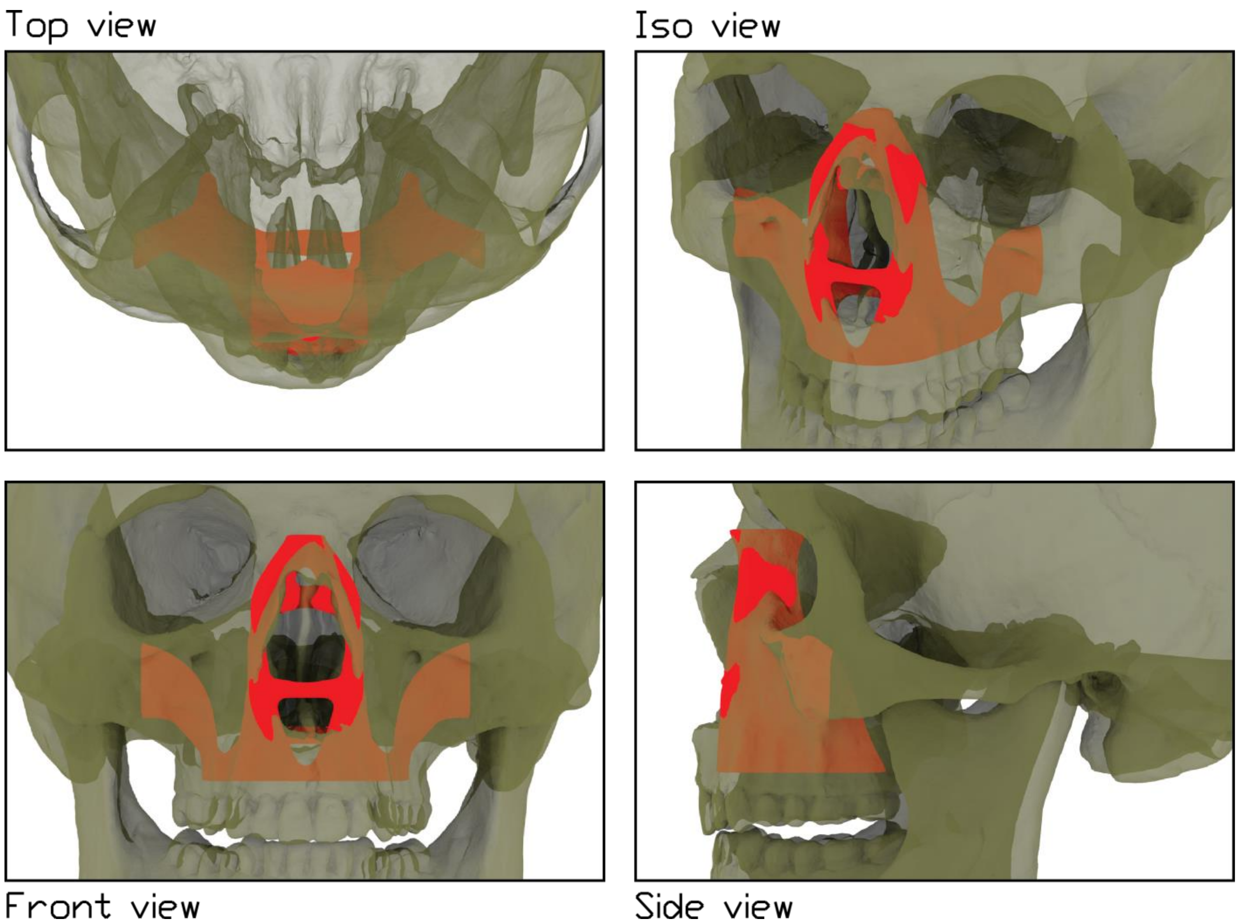
5) MANUFACTURING & DESIGNS

- CRANIOFACIAL RECONSTRUCTION



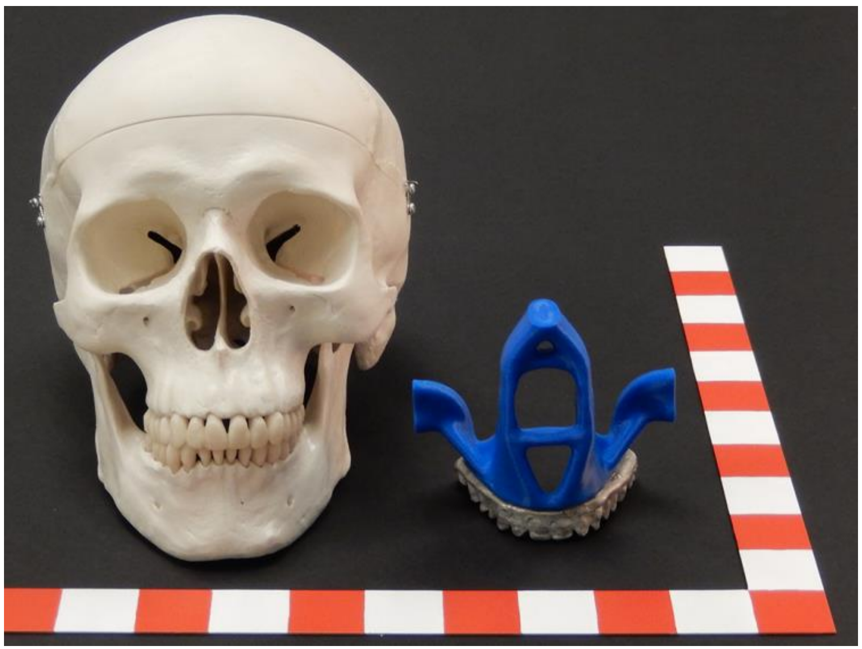
5) MANUFACTURING & DESIGNS

- CRANIOFACIAL RECONSTRUCTION

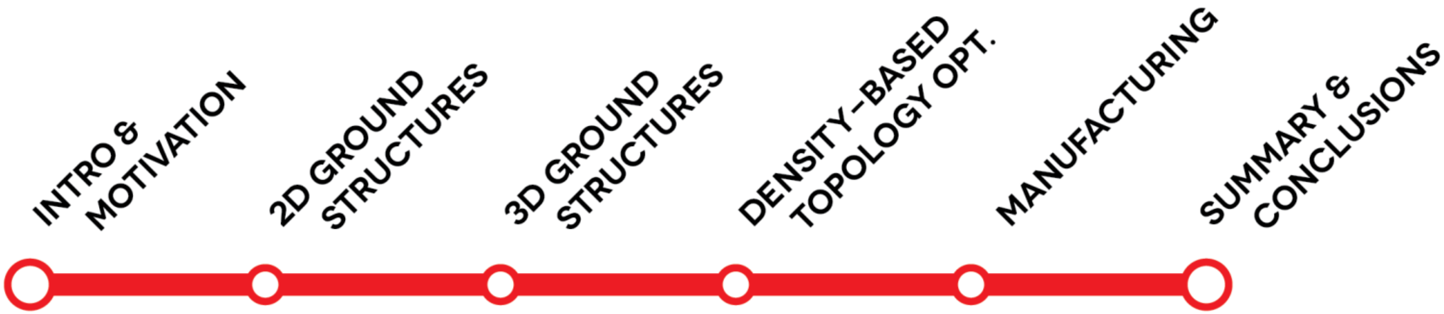


5) MANUFACTURING & DESIGNS

- CRANIOFACIAL RECONSTRUCTION

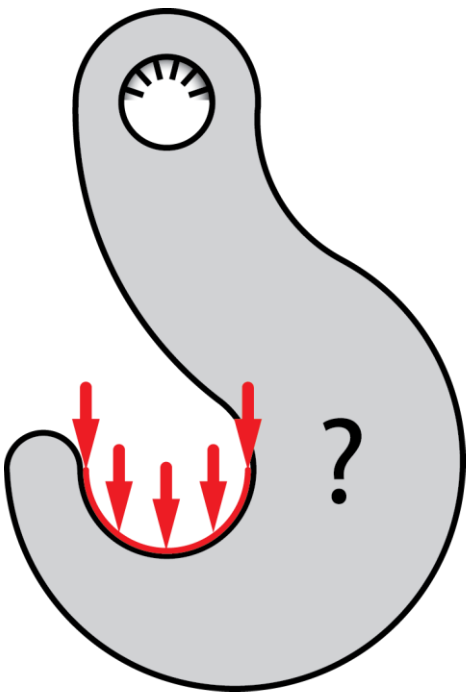


ROADMAP

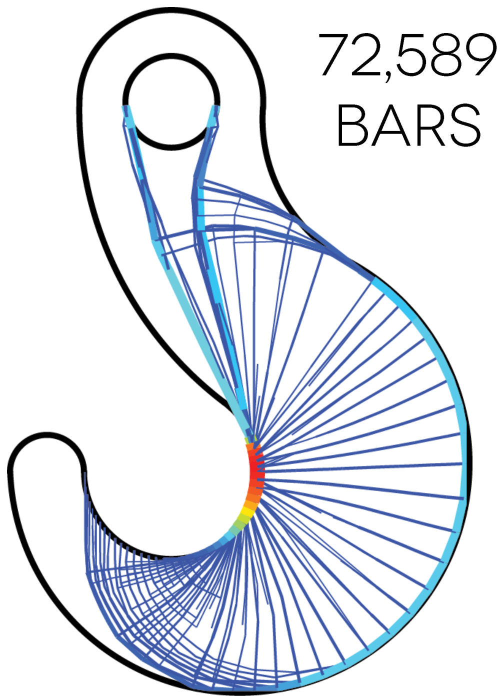

INTRO & MOTIVATION
2D GROUND STRUCTURES
3D GROUND STRUCTURES
DENSITY-BASED TOPOLOGY OPT.
MANUFACTURING
SUMMARY & CONCLUSIONS

6) SUMMARY AND CONCLUSIONS

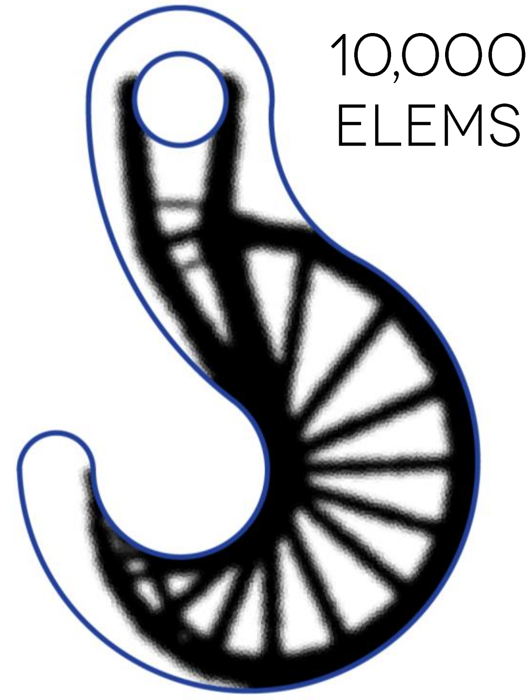
- HOOK PROBLEM



DOMAIN & BCs



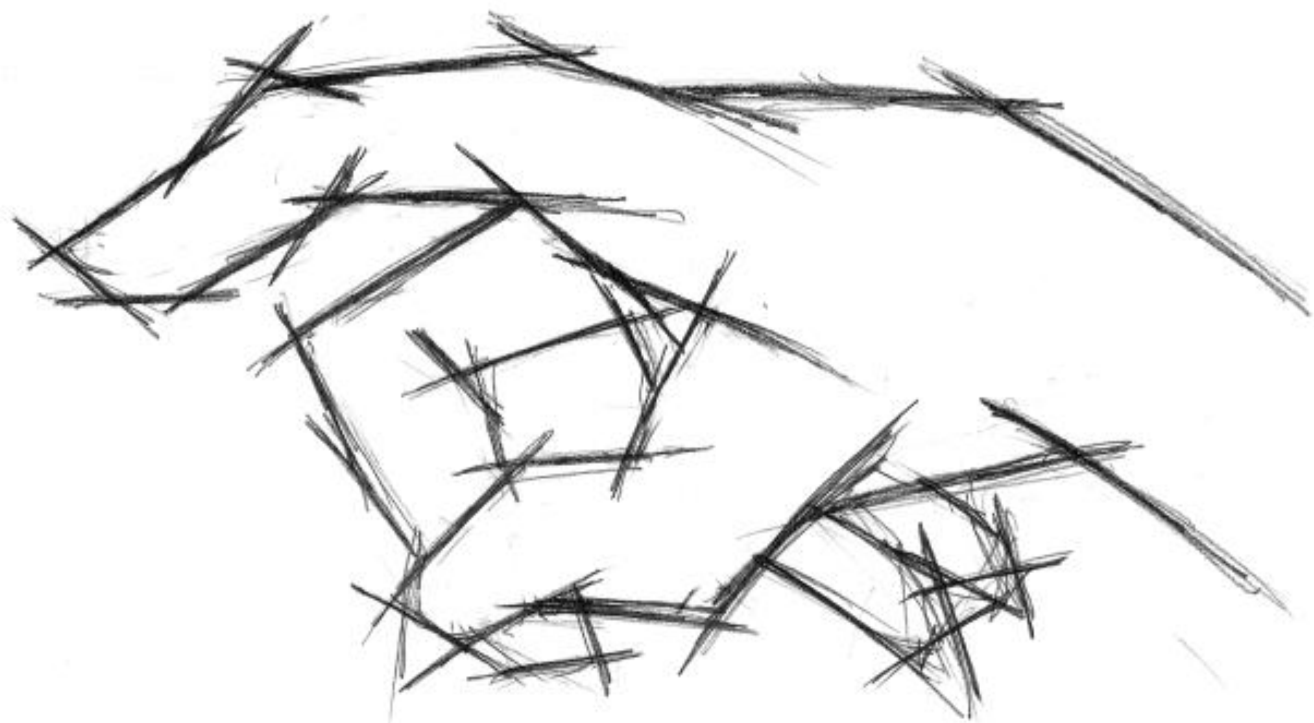
GROUND STRUCTURE



DENSITY-BASED METHOD

6) SUMMARY AND CONCLUSIONS

TOOLS FOR INSPIRATION



INFORMATION FOR DESIGNS

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