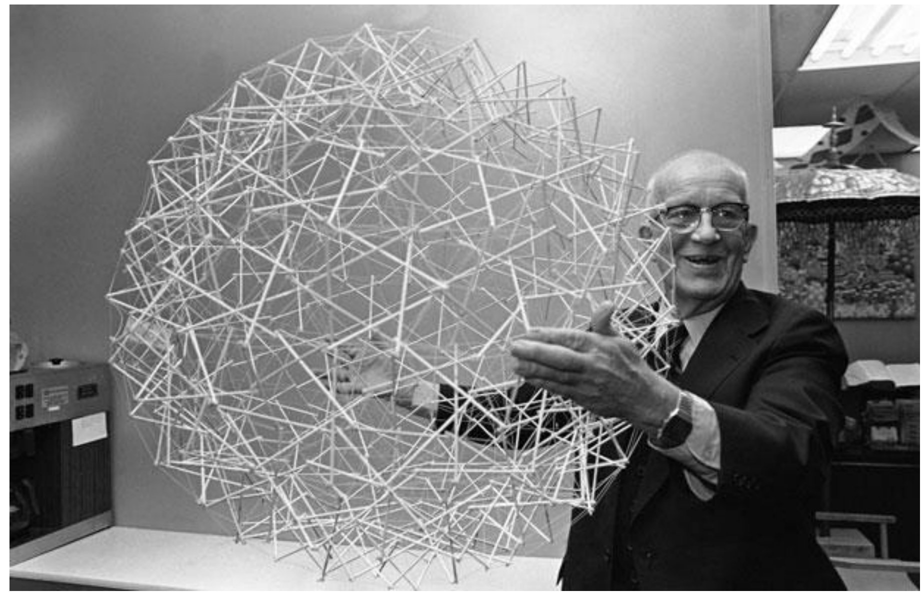
Tomás Zegard

- Structural Engineer
  - MSc from UIUC
  - PhD from UIUC
- Professional work @ SOM-Chicago
- Assistant Professor @ PUC-Chile

- Research Focus
  - Structural Optimization
  - Topology Optimization
  - Computational Mechanics

Tensegrity Tessellations

- Tensegrities

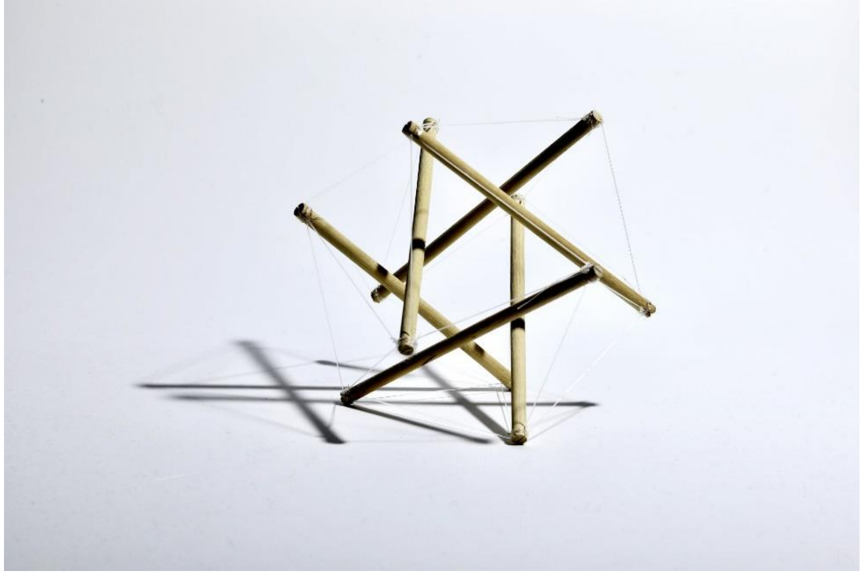


TOP: Buckminster Fuller (architect)

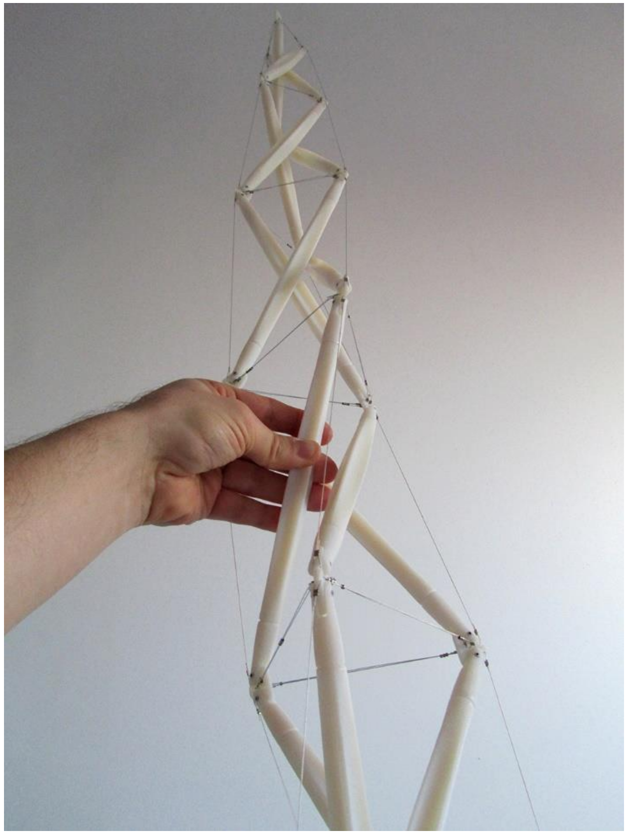
LEFT: Needle Tower by Kenneth Snelson

Tensegrity Tessellations

- Tensegrity Class
  - BOTTOM: Class-1
  - RIGHT: Class-2



Oliver Loesler



Tristan d'Estree Sterk

Automatic tensegrity generation

- MILP optimization (Liu & Paulino 2019)

$$\max_{\mathbf{t},\mathbf{c},\mathbf{s}} \mathbf{1}^{\mathrm{T}}(\mathbf{t}-\mathbf{c})$$

$$\text{s.t.} \quad \mathbf{B}(\mathbf{t}-\mathbf{c}) = \mathbf{0}$$

$$\mathbf{G}\mathbf{s} \leqslant n\mathbf{1}$$

$$\mathbf{G}_p\mathbf{s} \leqslant \mathbf{1}$$

$$\mathbf{1}^T\mathbf{s} \leqslant N_{S,max}$$

$$\mathbf{0} \leqslant \mathbf{t}$$

$$\mathbf{0} \leqslant \mathbf{c} \leqslant \mathbf{s} \leqslant \mathbf{1}$$

$$\mathbf{s} \in \mathbb{Z}^{N_{E_g}}$$

Automatic tensegrity generation

- MILP optimization (Liu & Paulino 2019)

True force in a member

$$\max_{\mathbf{t},\mathbf{c},\mathbf{s}} \mathbf{1}^{\mathrm{T}}(\mathbf{t}-\mathbf{c})$$

As little pre-stress as possible in all candidate members

$$\text{s.t.} \quad \mathbf{B}(\mathbf{t}-\mathbf{c}) = \mathbf{0}$$

$$\mathbf{G}\mathbf{s} \leqslant n\mathbf{1}$$

$$\mathbf{G}_p\mathbf{s} \leqslant \mathbf{1}$$

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$$\mathbf{0} \leqslant \mathbf{t}$$

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As little pre-stress as possible in all candidate members

$$\text{s.t.} \quad \mathbf{B}(\mathbf{t}-\mathbf{c}) = \mathbf{0}$$

Nodal equilibrium

$$\mathbf{G}\mathbf{s} \leqslant n\mathbf{1}$$

$$\mathbf{G}_p\mathbf{s} \leqslant \mathbf{1}$$

$$\mathbf{1}^T\mathbf{s} \leqslant N_{S,max}$$

$$\mathbf{0} \leqslant \mathbf{t}$$

$$\mathbf{0} \leqslant \mathbf{c} \leqslant \mathbf{s} \leqslant \mathbf{1}$$

$$\mathbf{s} \in \mathbb{Z}^{N_{Eg}}$$

Automatic tensegrity generation

- MILP optimization (Liu & Paulino 2019)

$$\max_{\mathbf{t},\mathbf{c},\mathbf{s}} \mathbf{1}^{\mathrm{T}}(\mathbf{t}-\mathbf{c})$$

As little pre-stress as possible in all candidate members

$$\text{s.t.} \quad \mathbf{B}(\mathbf{t}-\mathbf{c}) = \mathbf{0}$$

Nodal equilibrium

$$\mathbf{G}\mathbf{s} \leqslant n\mathbf{1}$$

$$\mathbf{G}_p\mathbf{s} \leqslant \mathbf{1}$$

G = element connectivity, s = “is strut” flag,
n = tensegrity class, G_p checks for strut collisions

$$\mathbf{1}^T\mathbf{s} \leqslant N_{S,max}$$

$$\mathbf{0} \leqslant \mathbf{t}$$

$$\mathbf{0} \leqslant \mathbf{c} \leqslant \mathbf{s} \leqslant \mathbf{1}$$

$$\mathbf{s} \in \mathbb{Z}^{N_{Eg}}$$

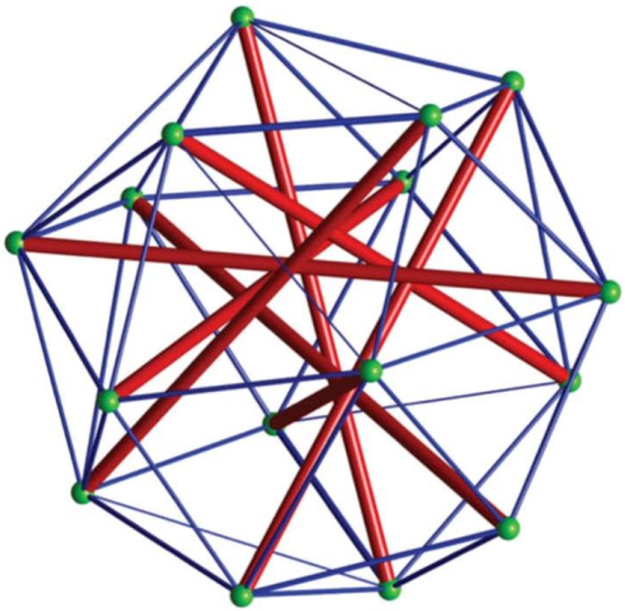
Automatic tensegrity generation

- MILP optimization (Liu & Paulino 2019)

$\max\limits_{\mathbf{t},\mathbf{c},\mathbf{s}} \mathbf{1}^{\mathrm{T}}(\mathbf{t}-\mathbf{c})$	As little pre-stress as possible in all candidate members
s.t. $\mathbf{B}(\mathbf{t}-\mathbf{c}) = \mathbf{0}$	Nodal equilibrium
$\mathbf{G}\mathbf{s} \leqslant n\mathbf{1}$ $\mathbf{G}_p\mathbf{s} \leqslant \mathbf{1}$	G = element connectivity, s = “is strut” flag, n = tensegrity class, G_p checks for strut collisions
$\mathbf{1}^T\mathbf{s} \leqslant N_{S,max}$	Limits the total number of struts
$\mathbf{0} \leqslant \mathbf{t}$	Tension slack variable must be positive
$\mathbf{0} \leqslant \mathbf{c} \leqslant \mathbf{s} \leqslant \mathbf{1}$	Compression slack variable requires s_k=1 to not be zero
$\mathbf{s} \in \mathbb{Z}^{N_{Eg}}$	S_k={0,1} is a integer variable

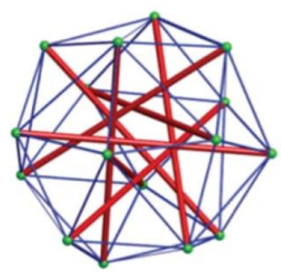
Automatic tensegrity generation

- Free-form tensegrity automatic generation

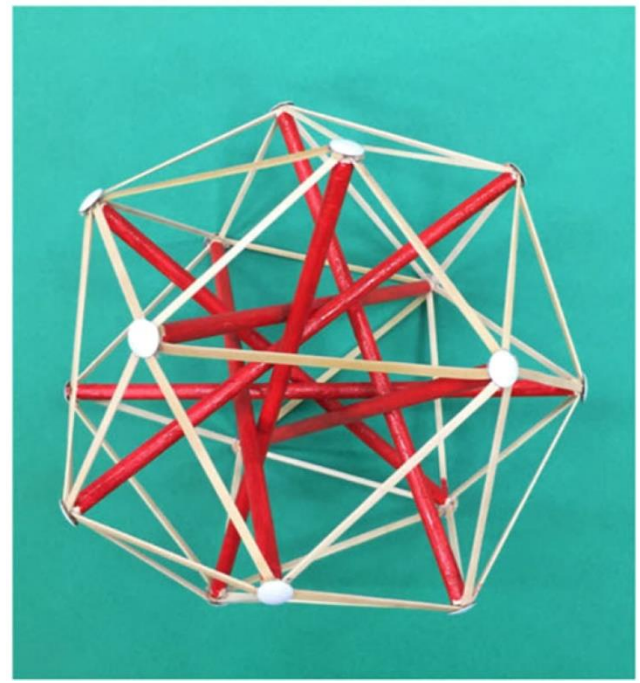


Iso view

Geometry

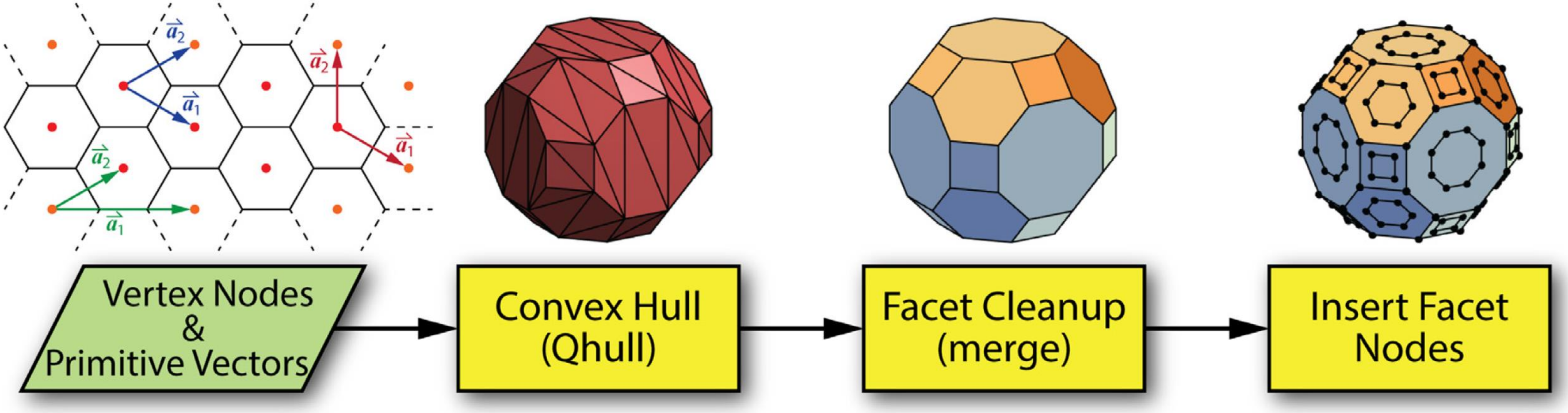


Top view



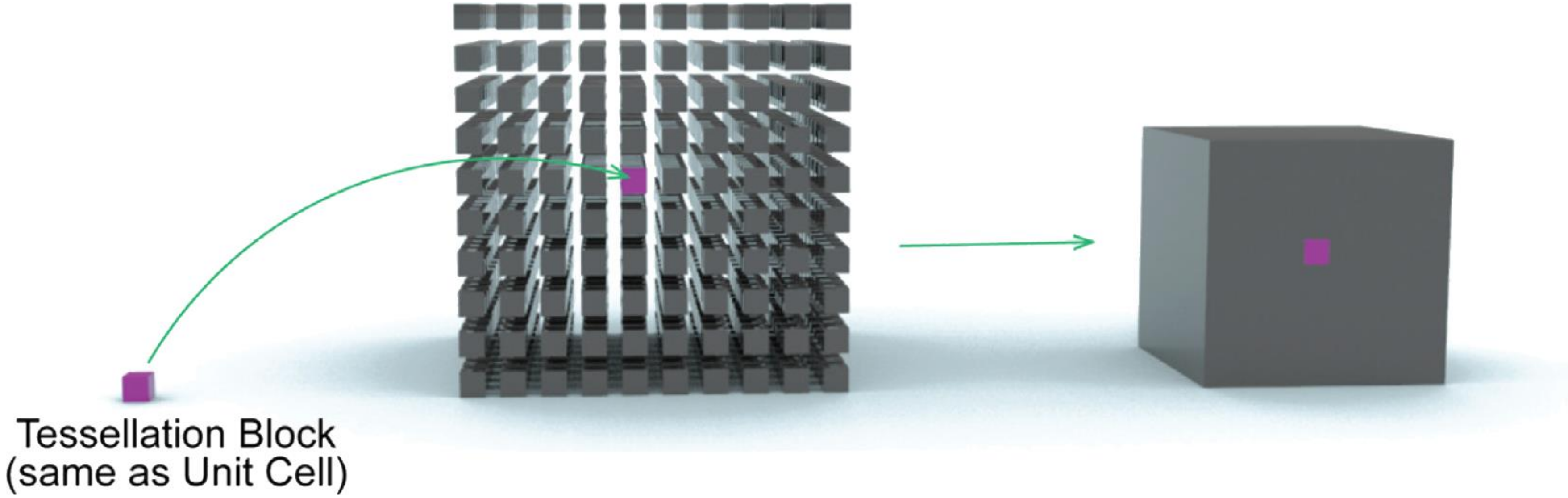
Tensegrity tessellations for metamaterials

- Periodic tensegrities



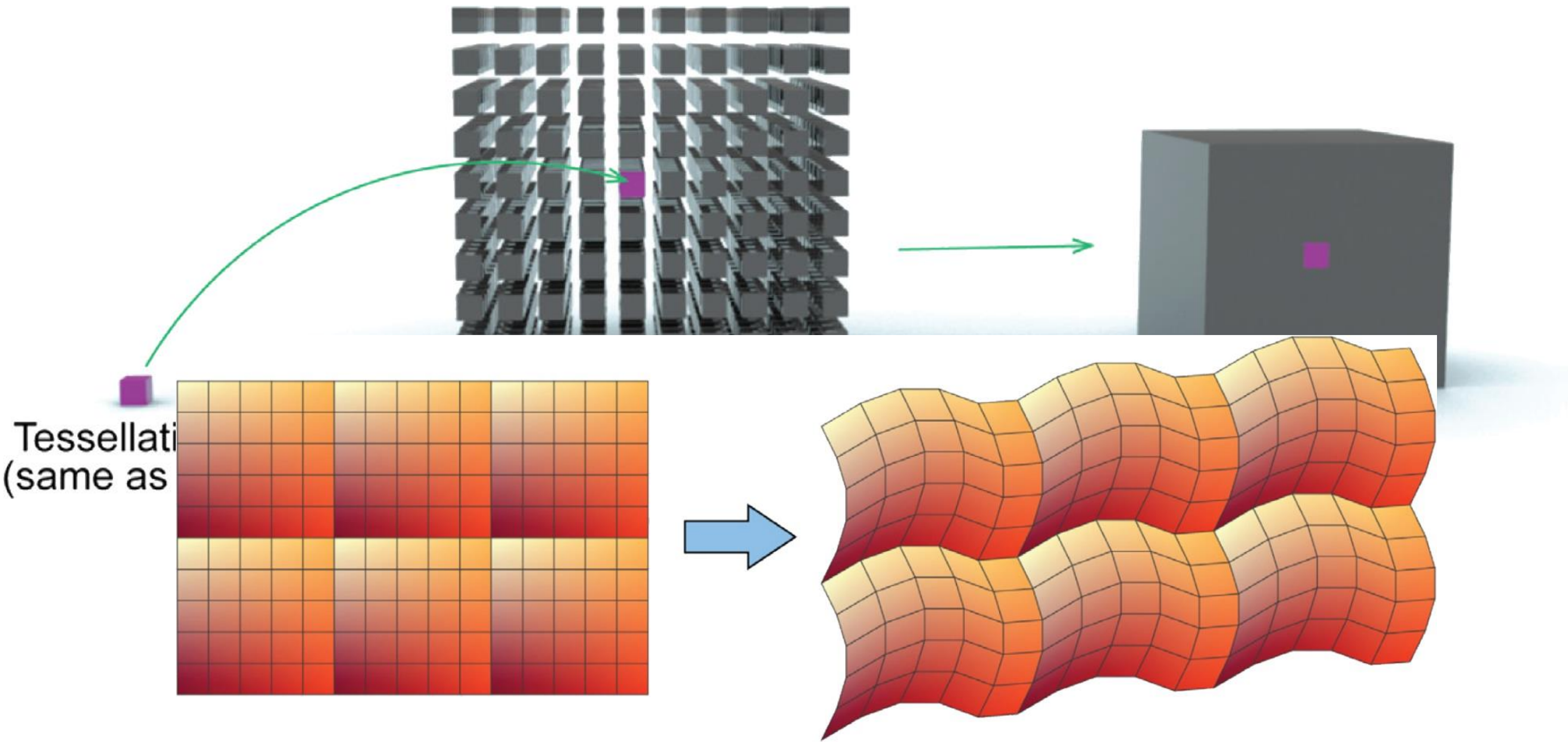
Tensegrity tessellations for metamaterials

- Periodic tensegrities



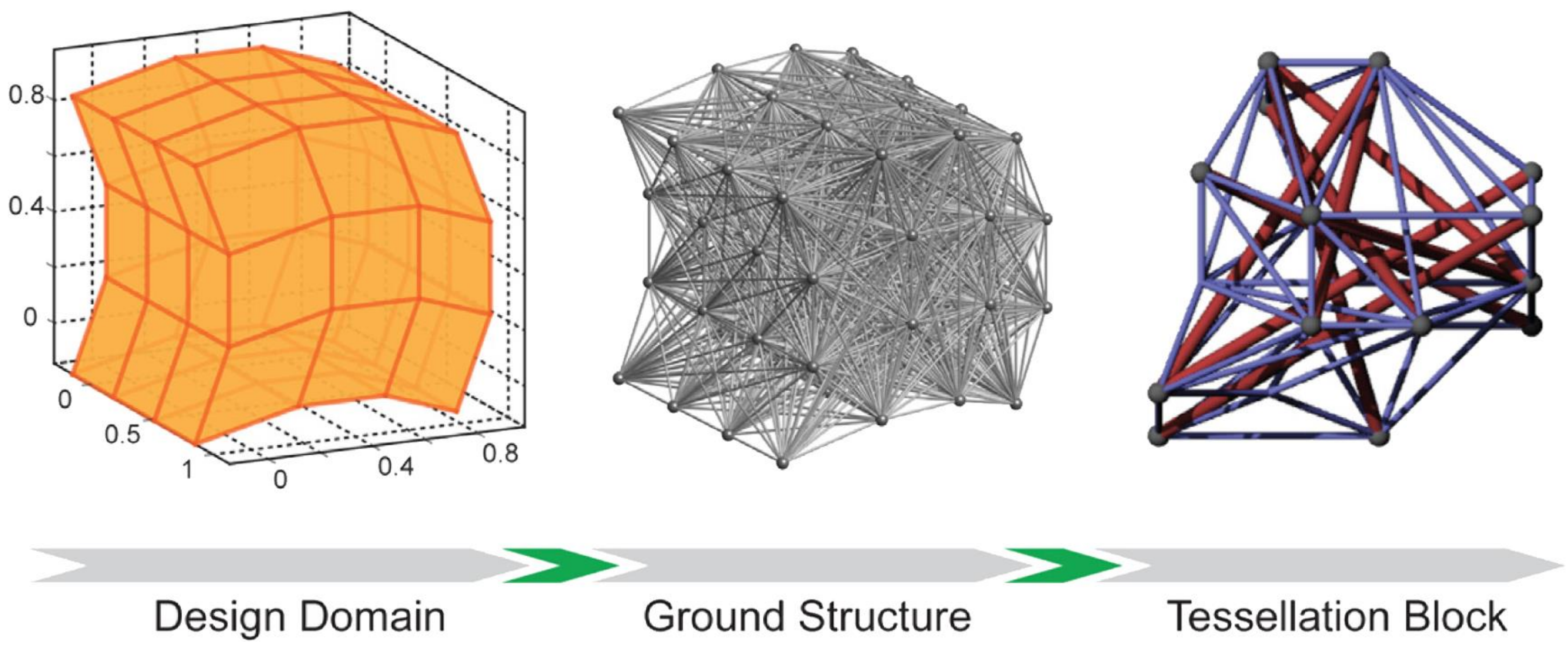
Tensegrity tessellations for metamaterials

- Periodic tensegrities



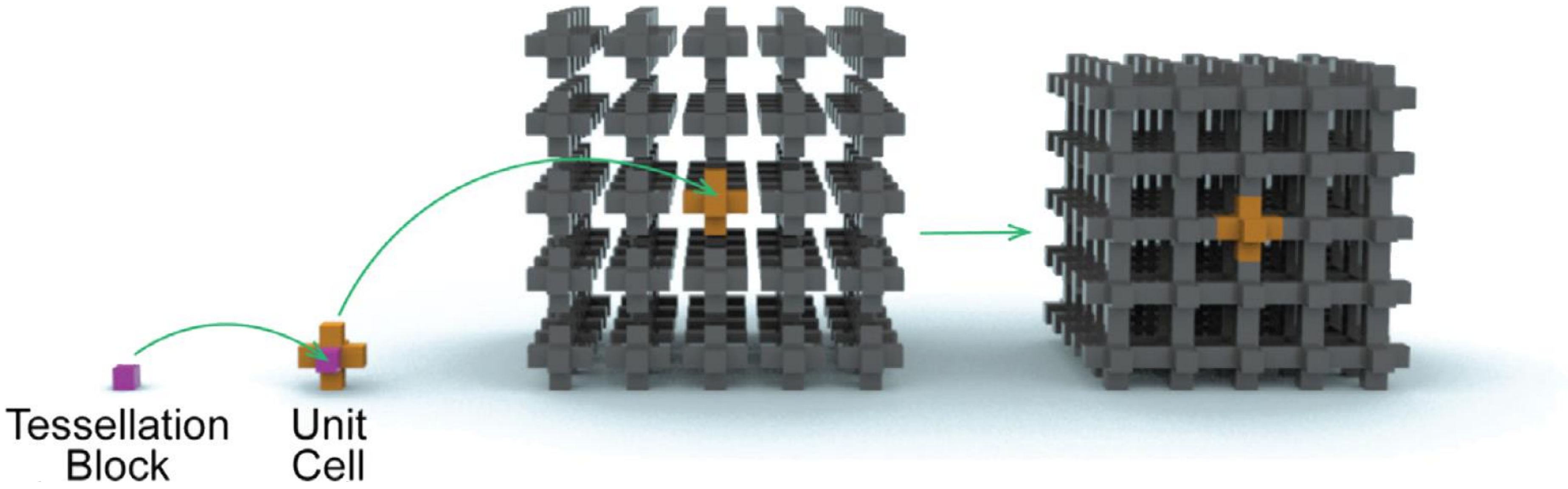
Tensegrity tessellations for metamaterials

- Periodic tensegrities



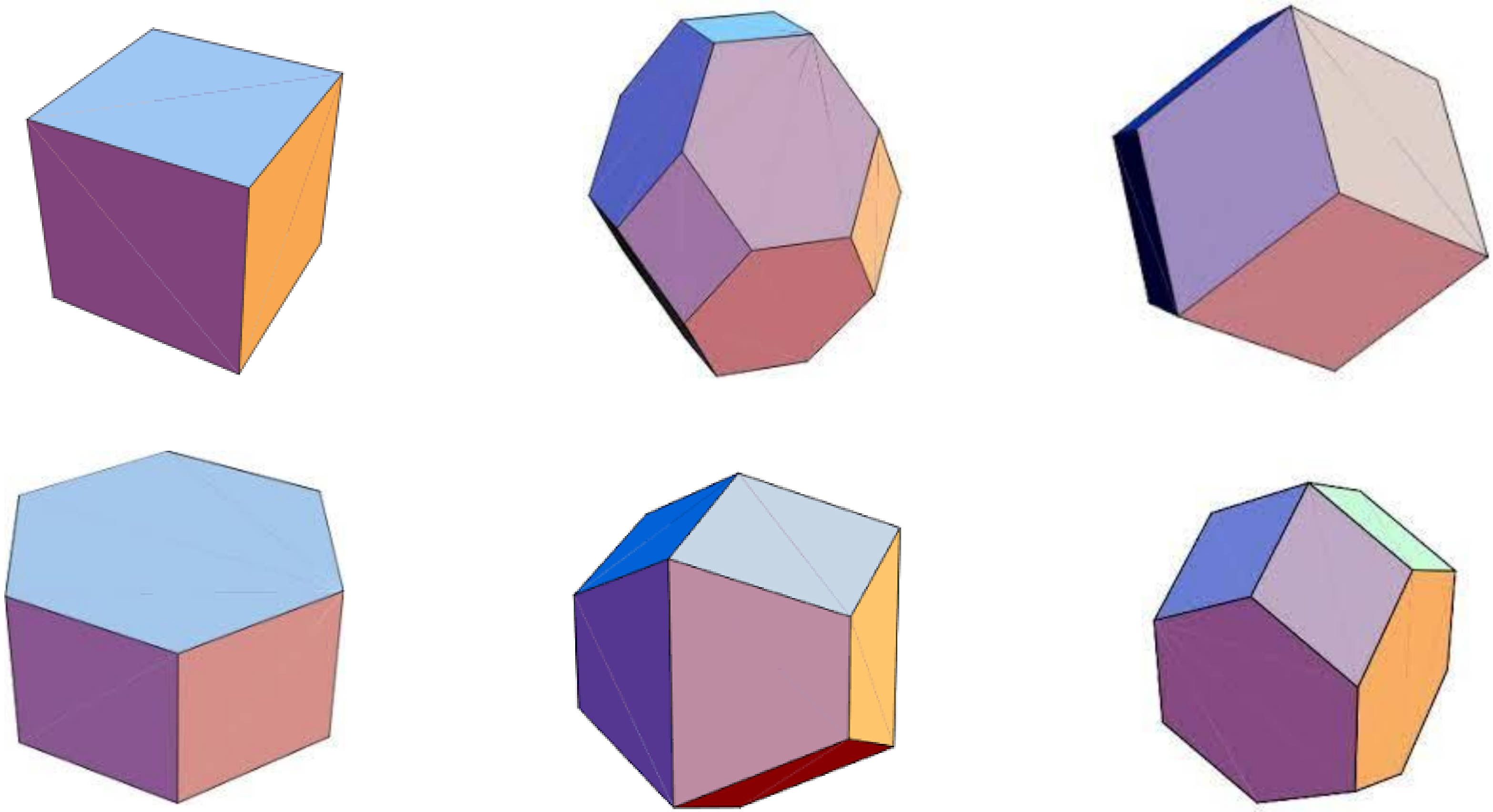
Tensegrity tessellations for metamaterials

- Periodic tensegrities

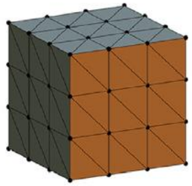
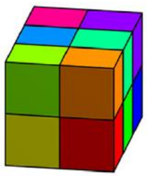
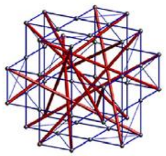
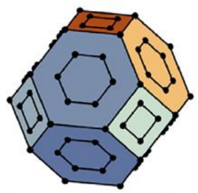
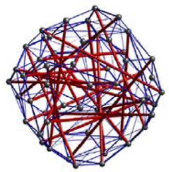
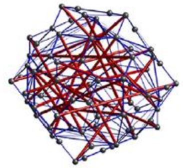
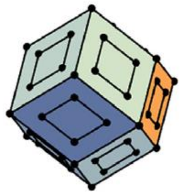
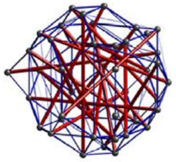
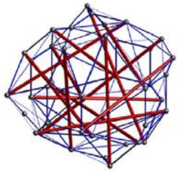


Space-filling polyhedra

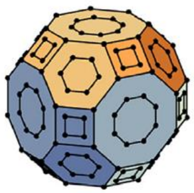
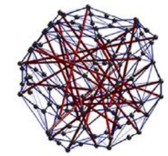
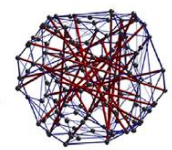
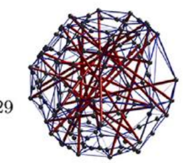
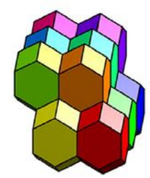
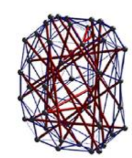
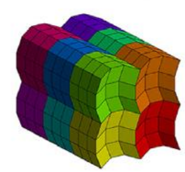
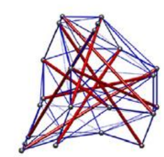
- Actually… we can have voids.



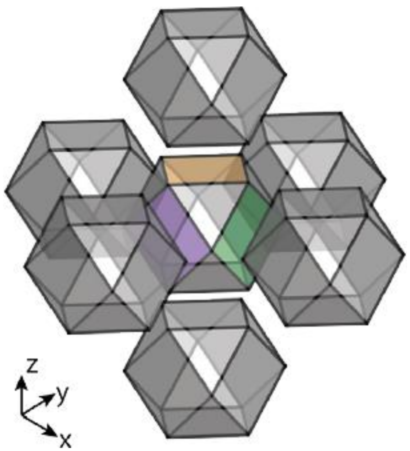
Library of examples

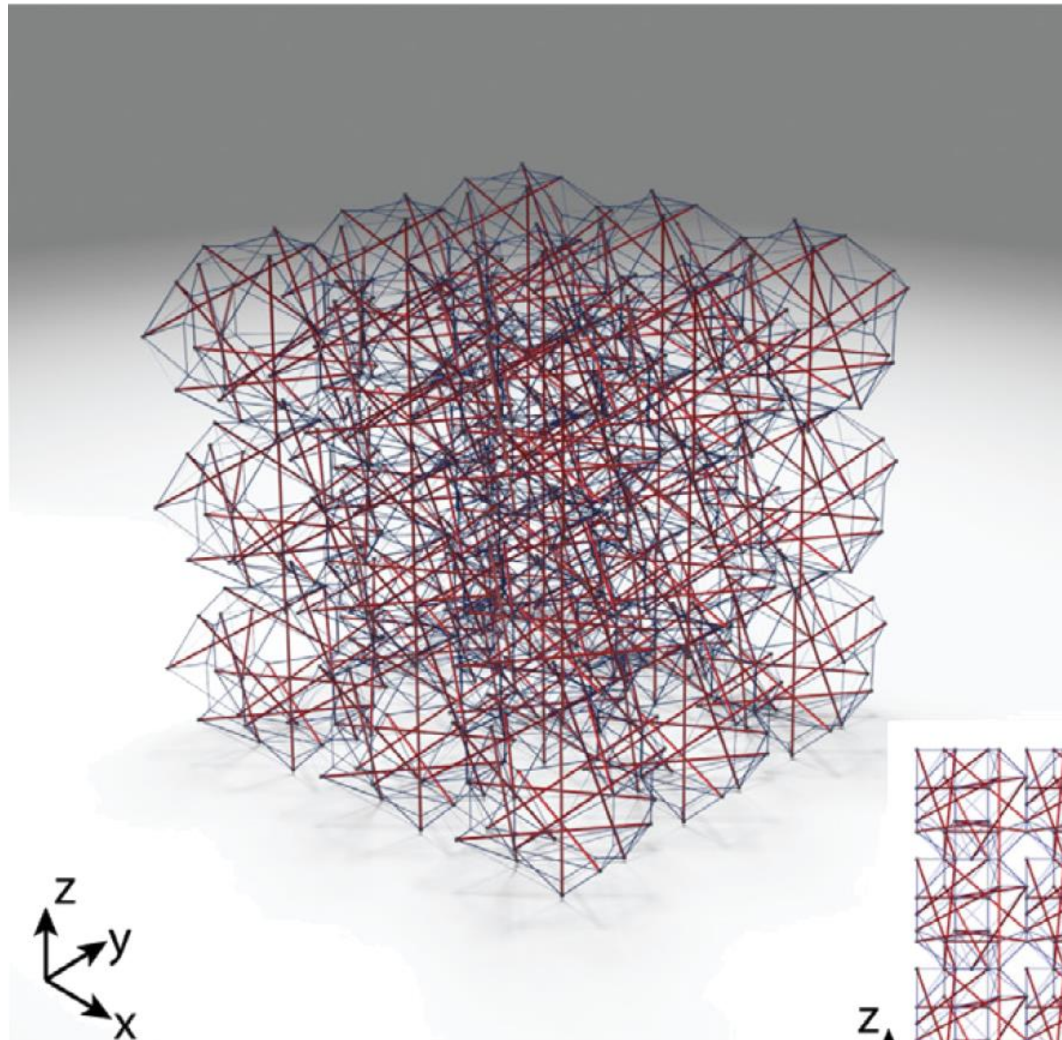
Design Domain	Vertices	Primitive Vectors (columns)	Assembly Sample	Specifications	Tessellation Block
 Cube	$\mathbf{v}_{1\to 8} = [\pm 1, \quad \pm 1, \quad \pm 1]$	$\mathbf{A}_1 = \begin{bmatrix} 2, & 0, & 0 \\ 0, & 2, & 0 \\ 0, & 0, & 2 \end{bmatrix}$		$R_{rz} = 0.1$ $N_S = 24$ Class-4 N_V=48,N_B=120 KI=25, PS=7	
 Truncated Octahedron	$\mathbf{v}_{1\to 24} = \text{perm}\,[0, \quad \pm 1, \quad \pm 2]$	$\mathbf{A}_1 = \begin{bmatrix} 2, & 2, & 2 \\ -2, & 2, & 2 \\ 2, & -2, & 2 \end{bmatrix}$		$R_{rz} = 0.75$ $N_S = 24$ Class-1 N_V=58,N_B=178 KI=14, PS=24	
		$\mathbf{A}_2 = 4 \cdot \begin{bmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & 1 \end{bmatrix}$		$R_{rz} = 0.75$ $N_S = 24$ Class-1 N_V=73,N_B=204 KI=32, PS=23	
 Rhombic Dodecahedron	$\mathbf{v}_{1\to 8} = [\pm 1, \quad \pm 1, \quad \pm 1]$ $\mathbf{v}_{9\to 10} = [\pm 1, \quad 0, \quad 0]$ $\mathbf{v}_{11\to 12} = [0, \quad \pm 1, \quad 0]$ $\mathbf{v}_{13\to 14} = [0, \quad 0, \quad \pm 1]$	$\mathbf{A}_1 = \begin{bmatrix} 2, & 2, & 0 \\ 2, & -2, & 0 \\ 2, & 0, & 2 \end{bmatrix}$		$R_{rz} = 0.5$ $N_S = 20$ Class-1 N_V=44,N_B=138 KI=8, PS=20	
 Cuboctahedron	$\mathbf{v}_{1\to 4} = [\pm 1 \quad , \quad \pm 1 \quad , \quad 0]$ $\mathbf{v}_{5\to 8} = [\pm 1 \quad , \quad 0 \quad , \quad \pm 1]$ $\mathbf{v}_{9\to 12} = [0 \quad , \quad \pm 1 \quad , \quad \pm 1]$	$\mathbf{A}_1 = \begin{bmatrix} 2 & , & 0 & , & 0 \\ 0 & , & 2 & , & 0 \\ 0 & , & 0 & , & 2 \end{bmatrix}$		$R_{rz} = 0.75$ $N_S = 13$ Class-1 N_V=40,N_B=109 KI=17, PS=12	

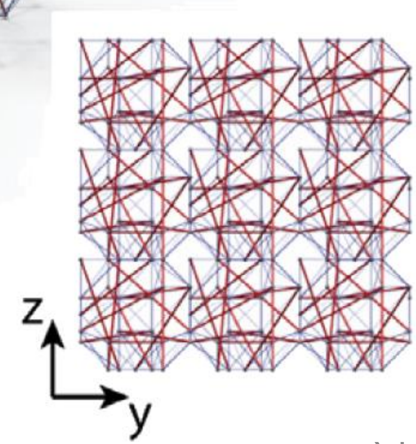
Library of examples

 Truncated Cuboctahedron	$\phi_1 = 1+\sqrt{2}$ $\phi_2 = 1+2\sqrt{2}$ $\phi_3 = 2+2\sqrt{2}$ $\phi_4 = 2+3\sqrt{2}$ $\mathbf{v}_{1\to 48} = \mathrm{perm}\,[\pm 1, \quad \pm\phi_1, \quad \pm\phi_2]$	$\mathbf{A}_1 = 2\phi_2 \cdot \begin{bmatrix} 1, & 0, & 0 \\ 0, & 1, & 0 \\ 0, & 0, & 1 \end{bmatrix}$		$R_{rz} = 1.5$ $N_S = 24$ Class-1 N_V=108,N_B=271 KI=70, PS=23	
		$\mathbf{A}_2 = \phi_3 \cdot \begin{bmatrix} 1, & 1, & 1 \\ -1, & 1, & 1 \\ 1, & -1, & 1 \end{bmatrix}$		$R_{rz} = 1.5$ $N_S = 24$ Class-1 N_V=128,N_B=310 KI=90, PS=22	
		$\mathbf{A}_3 = \phi_4 \cdot \begin{bmatrix} 1, & 1, & 0 \\ -1, & 1, & 0 \\ 1, & 0, & 1 \end{bmatrix}$		$R_{rz} = 1.5$ $N_S = 24$ Class-1 N_V=137,N_B=329 KI=100, PS=24	
 Elongated Dodecahedron	$\phi = 1/\sqrt{3}$ $\mathbf{v}_{1\to 8} = [\pm 1, \quad \pm 1, \quad \pm\phi]$ $\mathbf{v}_{9\to 12} = [\pm 1, \quad 0, \quad \pm 2\phi]$ $\mathbf{v}_{13\to 16} = [0, \quad \pm 1, \quad \pm 2\phi]$ $\mathbf{v}_{17\to 18} = [0, \quad 0, \quad \pm\sqrt{3}]$	$\mathbf{A}_1 = \begin{bmatrix} 2, & 0, & 0 \\ 0, & 2, & 0 \\ 1, & 1, & 4\phi \end{bmatrix}$		$R_{rz} = 0.5$ $N_S = 24$ Class-1 N_V=48,N_B=155 KI=5, PS=22	
 Morphed Cube	$\mathbf{v}_{1\to 8} = [\pm 1, \quad \pm 1, \quad \pm 1]$	$\mathbf{A}_1 = \begin{bmatrix} 2, & 0, & 0 \\ 0, & 2, & 0 \\ 0, & 0, & 2 \end{bmatrix}$		$N_S = 24$ Class-1 N_V=26,N_B=75 KI=5, PS=8	

Applied example







Why?

- Metamaterial with unique (and extreme) properties

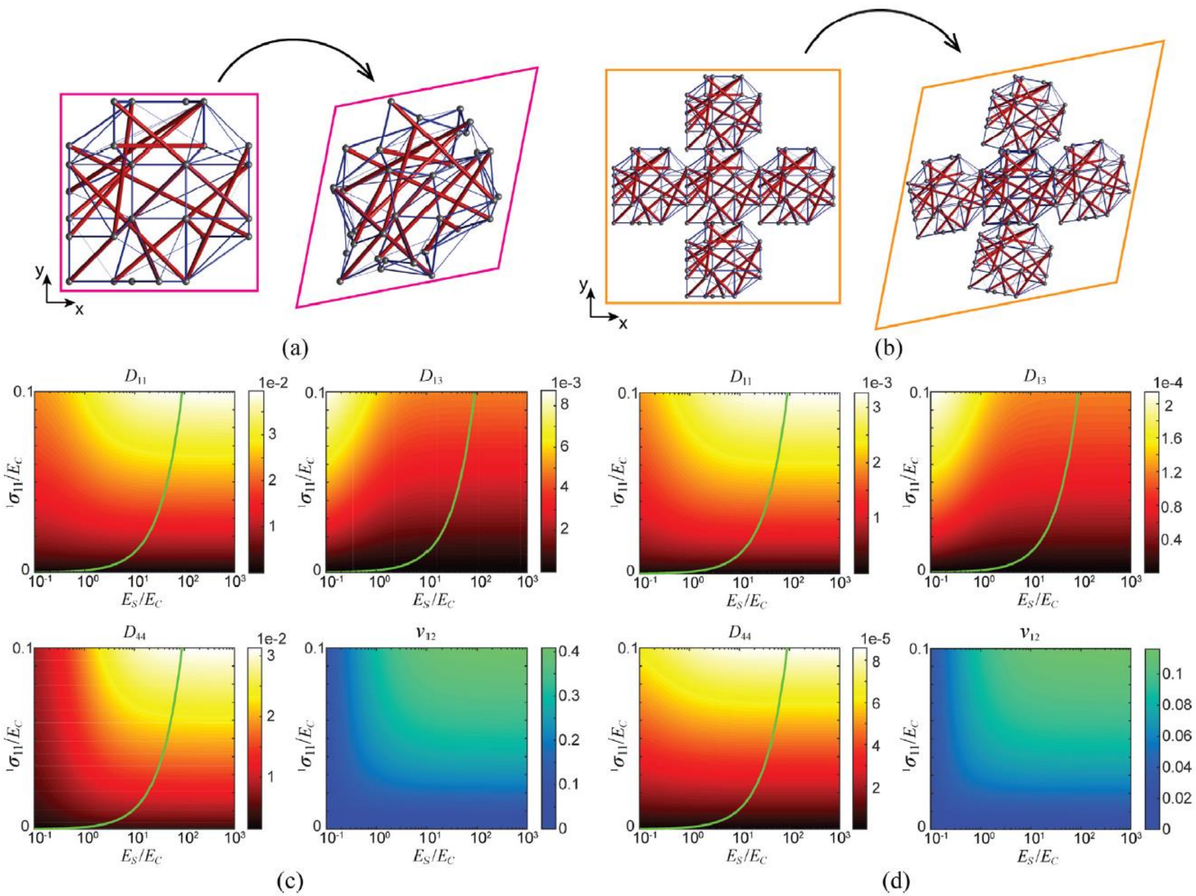


Fig. 6. Tensegrity metamaterial with tunable elastic properties. (**a**)-(**b**) Undeformed and simply sheared modes of the two metamaterials unit cells: (**a**) Densely tessellated metamaterial, (**b**) Porously tessellated metamaterial. (**c**)-(**d**) Tunable elastic properties (including elastic modulus D_{11}, D_{13}, D_{44}; and Poisson's ratio ν_{12}): (**c**) Densely tessellated metamaterial, (**d**) Porously tessellated metamaterial. The green lines indicate the limit of prestress that may cause buckling of struts. To the right of the green line, buckling is unlikely to happen. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)